Problem Set: Single-agent Dynamic Discrete-Choice Models

In this problem set, we will explore computation and estimation of single-agent dynamic discrete-choice models, with an emphasis on the Harold Zurcher model.

1. Compute the Harold Zurcher model.

   - Use the parameter estimates θ from the top of Table X in Rust's (1987) paper.
   - Compute $EV(x, i; \theta)$ using the value iteration procedure, described in Rust paper (and lecture notes).
   - Graph $EV(x, i; \theta)$, separately for $i = 0, 1$.

2. Simulate the Harold Zurcher model.

   - Assume there are $N = 100$ homogeneous buses, and you observe each for $T = 100$ weeks. HZ makes replacement decision every week.
   - Initial values: take $x_{n0} = 0$, $i_{n0} = 0$ for all buses n.
   - For each week t, simulate the utility shocks $\epsilon_{0nt}, \epsilon_{1nt}$, the mileage x_{nt}, and replacement decision i_{nt}:
     - Draw $\epsilon_{0nt}, \epsilon_{1nt}$, independently from Type I extreme value distribution, with CDF $F(\epsilon) = \exp[-\exp(-(\epsilon - 0.577))]$.*
     - Draw mileage x_{nt} from transition $G(x|x_{n,t-1}, i_{n,t-1})$, which is multinomial as given in Rust paper.
     - Compute replacement decision

       $$i_{nt} \equiv \text{argmax}_{i=0,1} \left(u(x_{nt}, i; \theta) + \epsilon_{int} + \beta \cdot EV(x_{nt}, i; \theta) \right)$$

       where you use $EV(x_{nt}, i; \theta)$ as computed in problem #1.
   - After sequences of x, i are simulated for all buses, provide summary statistics of your simulated data.

3. Estimate the model using Rust's MLE/nested-fixed-point algorithm.
4. Estimate the model using the indirect Hotz-Miller method.

*To simulate from a desired CDF $F(x)$, draw uniform random variables $u \sim U[0, 1]$, and transform $x = F^{-1}(u)$.