Goal of empirical work:

- We observe bids $b_1, \ldots, b_n$, and we want to recover valuations $v_1, \ldots, v_n$.
- Why? Analogously to demand estimation, we can evaluate the "market power" of bidders, as measured by the margin $v - p$.

  Could be interesting to examine: how fast does margin decrease as n (number of bidders) increases?
- Useful for the optimal design of auctions:
  1. What is auction format which would maximize seller revenue?
  2. What value for reserve price would maximize seller revenue?
- Methodology: identification, nonparametric estimation

1 Laffont-Ossard-Vuong (1995): "Econometrics of First-Price Auctions"

- Structural estimation of 1PA model, in IPV context.
- Example of a parametric approach to estimation.
- Another exercise in simulation estimation

MODEL

- I bidders
- Information structure is IPV: valuations v^i, $i = 1, \ldots, I$ are *i.i.d.* from $F(\cdot|z_l, \theta)$ where l indexes auctions, and z_l are characteristics of l-th auctions
- θ is parameter vector of interest, and goal of estimation
- p^0 denotes "reserve price": bid is rejected if $< p^0$.
- Dutch auction: strategically identical to first-price sealed bid auction.

Equilibrium bidding strategy is:

$$b^i = e\left(v^i, I, p^0, F\right) = \begin{cases} v^i - \frac{\int_{p^0}^{v^i} F(x)^{I-1} dx}{F(v^i)^{I-1}} & \text{if } v^i > p^0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Note: (1) $b^i(v^i = p^0) = p^0$; (2) strictly increasing in v^i.

■■■

Dataset: only observe *winning bid* b_l^w for each auction l. Because bidders with lower bids never have a chance to bid in Dutch auction.

Given monotonicity, the winning bid $b^w = e\left(v_{(I)}, I, p^0, F\right)$, where $v_{(I)} \equiv \max_i v^i$ (the highest order statistic out of the I valuations).

Furthermore, the CDF of $v_{(I)}$ is $F(\cdot|z_l, \theta)^I$, with corresponding density $I \cdot F^{I-1} f$.

■■■

Goal is to estimate θ by (roughly speaking) matching the winning bid in each auction l to its expectation.

Expected winning bid is (for simplicity, drop z_l and θ now)

$$\begin{aligned} E_{v_{(I)} > p^0}(b^w) &= \int_{p^0}^{\infty} e\left(v_{(I)}, I, p^0, F\right) I \cdot F(v|\theta)^{I-1} f(v|\theta) dv \\ &= I \int_{p^0}^{\infty} \left(v - \frac{\int_{p^0}^{v} F(x)^{I-1} dx}{F(v)^{I-1}} \right) F(v|\theta)^{I-1} f(v|\theta) dv \\ &= I \int_{p^0}^{\infty} \left(v \cdot F(v)^{I-1} - \int_{p^0}^{\infty} F(x)^{I-1} dx \right) f(v) dv. \quad (*). \end{aligned}$$

■■■

If we were to estimate by simulated nonlinear least squares, we would proceed by finding θ to minimize the sum-of-squares between the observed winning bids and the predicted winning bid, given by expression (*) above. Since (*) involves complicated integrals, we would simulate (*), for each parameter vector θ.

How would this be done:

- Draw valuations v^s, $s = 1, \ldots, S$ i.i.d. according to $f(v|\theta)$. This can be done by drawint $u_1, \ldots, u_S$ i.i.d. from the $U[0,1]$ distribution, then transform each draw:

$$v_s = F^{-1}(u_s|\theta).$$

- For each simulated valuation v_s, compute integrand $\mathcal{V}_s = v_s F(v_s|\theta)^{I-1} - \int_{p^0}^{v_s} F(x|\theta)^{I-1} dx$. (Second term can also be simulated, but one-dimensional integral is that very hard to compute.)
- Approximate the expected winning bid as $\frac{1}{S}\sum_s \mathcal{V}_s$.

However, the authors do not do this— they propose a more elegant solution. In particular, they simplify the simulation procedure for the expected winning bid by appealing to the **Revenue-Equivalence Theorem**: an important result for auctions where bidders' signals are independent, and the model is symmetric. (See Myerson (1981); this statement is due to Klemperer (1999).)

Theorem 1 *(Revenue Equivalence) Assume each of N risk-neutral bidders has a privately-known signal X independently drawn from a common distribution F that is strictly increasing and atomless on its support $[\underline{X}, \bar{X}]$. Any auction mechanism which is (i)* efficient *in awarding the object to the bidder with the highest signal with probability one; and (ii) leaves any bidder with the lowest signal $\underline{X}$ with zero surplus yields the same expected revenue for the seller, and results in a bidder with signal x making the same expected payment.*

From a mechanism design point of view, auctions are complicated because they are multiple-agent problems, in which a given agent's payoff can depend on the reports of all the agents. However, in the independent signal case, there is no gain (in terms of stronger incentives) in making any given agent's payoff depend on her rivals' reports, so that a symmetric auction with independent signal essentially boils down to independent contracts offered to each of the agents individually.

Furthermore, in any efficient auction, the probability that a given agent with a signal x wins is the same (and, in fact, equals $F(x)^{N-1}$). This implies that each bidder's expected surplus function (as a function of his signal) is the same, and therefore that the expected payment schedule is the same.

■■■

By RET:

- expected revenue in 1PA same as expected revenue in 2PA
- expected revenue in 2PA is $Ev^{(I-1)}$

- with reserve price, expected revenue in 2PA is $E\max(v^{(I-1)}, p^0)$. (Note: with IPV structure, reserve price r screens out same subset of valuations $v \leq r$ in both 1PA and 2PA.)

■■■

Hence, we have that

$$Eb^*(v_{(I)}) = E\left[\max\left(v_{(I-1)}, p^0\right)\right]$$

which is insanely easy to simulate:

For each parameter vector θ, and each auction l

- For each simulation draw $s = 1, \ldots, S$:
  - Draw $v_1^s, \ldots, v_{I_l}^s$: vector of simulated valuations for auction l (which had I_l participants)
  - Sort the draws in ascending order: $v_{1:I_l}^s < \cdots < v_{I_l:I_l}^s$
  - Set $b_l^{w,s} = v_{I-1_l:I_l}$ (ie. the second-highest valuation)
  - If $b_l^{w,s} < p_l^0$, set $b_l^{w,s} = p_l^0$. (ie. $b_l^{w,s} = \max\left(v_{I-1_l:I_l}^s, p_l^0\right)$)
- Approximate $E\left(b_l^w; \theta\right) = \frac{1}{S}\sum_s b_l^{w,s}$.

Estimate θ by simulated nonlinear least squares:

$$\min_\theta \frac{1}{L}\sum_{l=1}^{L}\left(b_l^w - E\left(b_l^w; \theta\right)\right)^2.$$

Results.

■■■

Remarks:

- Problem: bias when number of simulation draws S is fixed (as number of auctions $L \to \infty$). Propose bias correction estimator, which is consistent and asymptotic normal under these conditions.
- This clever methodology is useful for independent value models: works for all cases where revenue equivalence theorem holds.

- Does not work for affiliated value models (including common value models)

■■■

2 Application: internet used car auctions

- Consider Lewis (2011) paper on used cars sold on eBay (simplified exposition)
- Question: does information revealed by sellers lead to high prices? (Question about the credibiltiy of information revealed by sellers.)
- Observe transsactions price in ascending auction. Assume that transaction price is equal to

$$v(X_{n-1:n}, X_{n-1:n})$$

  (as in second-price auction).

- Consider pure common value setup with conditionally independent signals. Log-normality is assumed:

$$\begin{aligned} \tilde{v} \equiv \log v = \mu + \sigma\epsilon_v \;\; &\sim \;\; N(\mu, \sigma^2) \\ x_i|\tilde{v} = \tilde{v} + r\epsilon_i \;\; &\sim \;\; N(\tilde{v}, r^2) \end{aligned}$$

  These are, respectively, the prior distribution of valuations, and the conditional distribution of signals.

- Allow seller information variables z to affect the mean and variance of the prior distribution:

$$\begin{aligned} \mu &= \alpha' z \\ \sigma &= \kappa(\beta' z). \end{aligned}$$

  $\kappa(\cdot)$ is just a transformation of the index $\beta' z$ to ensure that the estimate of $\sigma > 0$.

- z includes variables such as: number of photos, how much text is on the website. (Larger z denotes better information.)
- Question: is $\alpha > 0$?
- Results.

3 Guerre-Perrigne-Vuong (2000): Nonparametric Identification and Estimation in IPV First-price Auction Model

The recent emphasis in the empirical literature is on *nonparametric* identification and estimation of auction models. Motivation is to estimate bidders' unobserved valuations, while avoiding parametric assumption (as in the LOV paper).

- Start with first-order condition:

$$\begin{aligned} b'(x) &= (x - b(x)) \cdot (n-1) \frac{F(x)^{n-2} f(x)}{F(x)^{n-1}} \\ &= (x - b(x)) \cdot (n-1) \frac{f(x)}{F(x)}. \end{aligned} \tag{2}$$

- Now, note that because equilibrium bidding function $b(x)$ is just a monotone increasing function of the valuation x, the change of variables formulas yield that (take $b_i \equiv b(x_i)$)

  -
$$G(b_i) = F(x_i)$$
  -
$$g(b_i) = f(x_i) \cdot 1/b'(x_i)$$

  .

  Hence, substituting the above into Eq. (2):

$$\begin{aligned} \frac{1}{g(b_i)} &= (n-1) \frac{x_i - b_i}{G(b_i)} \\ \Leftrightarrow x_i &= b_i + \frac{G(b_i)}{(n-1) g(b_i)}. \end{aligned} \tag{3}$$

  Everything on the RHS of the preceding equation is observed: the equilibrium bid CDF G and density g can be estimated directly from the data *nonparametrically.* Assuming a dataset consisting of T n-bidder auctions:

$$\begin{aligned} \hat{g}(b) &\approx \frac{1}{T \cdot n} \sum_{t=1}^{T} \sum_{i=1}^{n} \frac{1}{h} \mathcal{K}\left(\frac{b - b_{it}}{h} \right) \\ \hat{G}(b) &\approx \frac{1}{T \cdot n} \sum_{t=1}^{T} \sum_{i=1}^{n} \mathbf{1}(b_{it} \leq b). \end{aligned} \tag{4}$$

The first is a *kernel density estimate* of bid density. The second is the *empirical distribution function (EDF).*

- In the above, $\mathcal{K}$ is a "kernel function". A kernel function is a function satisfying the following conditions:
  1. It is a probability density function, ie: $\int_{-\infty}^{+\infty} \mathcal{K}(d)du = 1$, and $\mathcal{K}(u) \geq 0$ for all u.
  2. It is symmetric around zero: $\mathcal{K}(u) = \mathcal{K}(-u)$.
  3. h is bandwidth: describe below
  4. Examples:
     (a) $\mathcal{K}(u) = \phi(u)$ (standard normal density function);
     (b) $\mathcal{K}(u) = \frac{1}{2}\mathbf{1}(|u| \leq 1)$ (uniform kernel);
     (c) $\mathcal{K}(u) = \frac{3}{4}(1-u^2)\mathbf{1}(|u| \leq 1)$ (Epanechnikov kernel)
- To get some intuition for the kernel estimate of $\hat{g}(b)$, consider the histogram

$$h(b) = \frac{1}{Tn} \sum_t \sum_i \mathbf{1}(b_{it} \in [b-\epsilon, b+\epsilon])$$

  for some small $\epsilon > 0$. The histogram at b, $h(b)$ is the frequency with which the observed bids land within an ϵ-neighborhood of b.

- In comparison, the kernel estimate of $\hat{g}(b)$ replaces $\mathbf{1}(b_{it} \in [b-\epsilon, b+\epsilon])$ with $\frac{1}{h}\mathcal{K}\left(\frac{b-b_{it}}{h}\right)$. This is:
  - always ≥ 0
  - takes large values for b_{it} close to b; small values (or zero) for b_{it} far from b
  - takes values in $\mathbb{R}+$ (can be much larger than 1)
  - h is bandwidth, which blows up $\frac{1}{h}\mathcal{K}\left(\frac{b-b_{it}}{h}\right)$: when it is smaller, then this quantity becomes larger.

    Think of h as measuring the "neighborhood size" (like ϵ in the histogram). When $T \to \infty$, then we can make h smaller and smaller.

    Bias/variance tradeoff.
  - Roughly speaking, then, $\hat{g}(b)$ is a "smoothed" histogram,
- For $\hat{G}(b)$, recall definition of the CDF:

$$G(\tilde{b}) = Pr(b \leq \tilde{b}).$$

  The EDF measures these probabilities by the (within-sample) frequency of the events.

- Hence, the IPV first-price auction model is *nonparametrically identified.* For each observed bid b_i, the corresponding valuation $x_i = b^{-1}(b_i)$ can be recovered as:

$$\hat{x}_i = b_i + \frac{\hat{G}(b_i)}{(n-1)\hat{g}(b_i)}. \tag{5}$$

Hence, GPV recommend a two-step approach to estimating the valuation distribution $f(x)$:

1. In first step, estimate $G(b)$ and $g(b)$ nonparametrically, using Eqs. (4).
2. In second step, estimate $f(x)$ by using kernel density estimator of recovered valuations:

$$\hat{f}(x) \approx \frac{1}{T \cdot n} \sum_{t=1}^{T} \sum_{i=1}^{n} \frac{1}{h} \mathcal{K}\left(\frac{x - \hat{x}_{it}}{h}\right). \tag{6}$$

3.1 Athey-Haile (2002): identification with incomplete observations of bids

Athey and Haile (2002) shows many nonparametric identification results for a variety of auction models (first-price, second-price) under a variety of assumption on the information structure (symmetry, asymmetry). They focus on situations when only a subset of the bids submitted in an auction are available to a researcher.

As an example of such a result, we see that identification continues to hold, even when only the highest-bid in each auction is observed. Specifically, if only $b_{n:n}$ is observed, we can estimate $G_{n:n}$, the CDF of the maximum bid, from the data. Note that the relationship between the CDF of the maximum bid and the marginal CDF of an equilibrium bid is

$$G_{n:n}(b) = G(b)^n$$

implying that $G(b)$ can be recovered from knowledge of $G_{n:n}(b)$. Once $G(b)$ is recovered, the corresponding density $g(b)$ can also be recovered, and we could solve Eq. (5) for every b to obtain the inverse bid function.

4 Affiliated values models

Can this methodology be extended to affiliated values models (including common value models)?

However, Laffont and Vuong (1996) nonidentification result: from observation of bids in n-bidder auctions, the affiliated private value model (ie. a PV model where valuations are dependent across bidders) is indistinguishable from a CV model.

- Intuitively, all you identify from observed bid data is joint density of $b_1, \ldots, b_n$. In particular, can recover the correlation structure amongst the bids. But correlation of bids in an auction could be due to both affiliated PV, or to CV.

4.1 Affiliated private value models

Li, Perrigne, and Vuong (2002) proceed to consider nonparametric identification and estimation of the affiliated private values model. In this model, valuations $x_i, \ldots, x_n$ are drawn from some joint distribution (and there can be arbitrary correlation amongst them).

First-order condition for equilibrium bid in general affiliated values case:

$$b'(x) = (v(x,x) - b(x)) \cdot \frac{f_{y_i|x_i}(x|x)}{F_{y_i|x_i}(x|x)}; \quad y_i \equiv \max_{j \neq i} x_i. \tag{7}$$

where $y_i \equiv \max_{j \neq i} x_i$ (highest among rivals' signals) and $b(\cdot)$ denotes the equilibrium bidding strategy. Also, $v(x,x) = E[V_i | X_i = x, Y_i = x]$, the "value conditional on winning" (see theory notes, part 1).

Under (affiliated) private values, still have $v(x,x) = x$.

Procedure similar to GPV can be used here to recover, for each bid b_i, the corresponding valuation $x_i = b^{-1}(b_i)$. Let b_i^* denote the maximum among bidder i's rivals bids: $b_i^* = \max_{j \neq i} b_j$. Then there is a monotonic transformation $b_i^* = b(y_i)$ so that, as before, we exploit the following change of variable formulas:

- $$G_{b^*|b}(b|b) = F_{y|x}(x|x)$$
- $$g_{b^*|b}(b|b) = f_{y|x}(x|x) \cdot 1/b'(x)$$

Note that the conditioning event $\{X = x\}$ (on right-hand side) is equivalent to $\{B = b\}$ (on left-hand side). To prepare what follows, we introduce n subscript (so we index distributions according to the number of bidders in the auction).

Li, Perrigne, and Vuong (2000) suggest nonparametric estimates of the form

$$\begin{aligned} \hat{G}_n(b;b) &= \frac{1}{T_n \times h \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} K\left(\frac{b - b_{it}}{h}\right) \mathbf{1}\left(b_{it}^* < b, n_t = n\right) \\ \hat{g}_n(b;b) &= \frac{1}{T_n \times h^2 \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} \mathbf{1}(n_t = n) K\left(\frac{b - b_{it}}{h}\right) K\left(\frac{b - b_{it}^*}{h}\right). \end{aligned} \tag{8}$$

Here h and h are bandwidths and $K(\cdot)$ is a kernel. $\hat{G}_n(b;b)$ and $\hat{g}_n(b;b)$ are nonparametric estimates of

$$G_n(b;b) \equiv G_n(b|b) g_n(b) = \frac{\partial}{\partial b} \Pr(B_{it}^* \leq m, B_{it} \leq b)|_{m=b}$$

and

$$g_n(b;b) \equiv g_n(b|b) g_n(b) = \frac{\partial^2}{\partial m \partial b} \Pr(B_{it}^* \leq m, B_{it} \leq b)|_{m=b}$$

respectively, where $g_n(\cdot)$ is the marginal density of bids in equilibrium. Because

$$\frac{G_n(b;b)}{g_n(b;b)} = \frac{G_n(b|b)}{g_n(b|b)} \tag{9}$$

$\frac{\hat{G}_n(b;b)}{\hat{g}_n(b;b)}$ is a consistent estimator of $\frac{G_n(b|b)}{g_n(b|b)}$. Hence, by evaluating $\hat{G}_n(\cdot,\cdot)$ and $\hat{g}_n(\cdot,\cdot)$ at each observed bid, we can construct a pseudo-sample of consistent estimates of the realizations of each $x_{it} = b^{-1}(b_{it})$ using Eq. (7):

$$\hat{x}_{it} = \frac{\hat{G}_n(b_{it}; b_{it})}{\hat{g}_n(b_{it}; b_{it})} + b_{it}. \tag{10}$$

Subsequently, joint distribution of $x_1, \ldots, x_n$ can be recovered as sample joint distribution of $\hat{x}_1, \ldots, \hat{x}_n$.

4.2 Common value models: testing between CV and PV

Laffont-Vuong did not consider variation in n, the number of bidders.

In Haile, Hong, and Shum (2003), we explore how variation in n allows us to test for existence of CV.

Introduce notation:

$$v(x_i, x_i, n) = E[V_i | X_i = x_i, \max_{j \neq i} X_j = x_i, n].$$

Recall the winner's curse: it implies that $v(x, x, n)$ is invariant to n for all x in a PV model but strictly decreasing in n for all x in a CV model (see theory notes, part 1).

Consider the first-order condition in the common value case:

$$b'(x,n) = (v(x,x,n) - b(x,n)) \cdot \frac{f_{y_i|x_i,n}(x|x)}{F_{y_i|x_i,n}(x|x)}; \quad y_i \equiv \max_{j\neq i} x_i.$$

Hence, the Li, Perrigne, and Vuong (2002) procedure from the previous section can be used to recover the "pseudovalue" $v(x_i, x_i, n)$ corresponding to each observed bid b_i. Note that we cannot recover $x_i = b^{-1}(b_i)$ itself from the first-order condition, but can recover $v(x_i, x_i, n)$. (This insight was also articulated in Hendricks, Pinkse, and Porter (2003).)

In Haile, Hong, and Shum (2003), we use this intuition to develop a test for CV:

$$\begin{aligned} H_0 \text{ (PV)}: \; & E[v(X,X;\underline{n})] = E[v(X,X;\underline{n}+1)] = \cdots = E[v(X,X;\bar{n})] \\ H_1 \text{ (CV)}: \; & E[v(X,X;\underline{n})] > E[v(X,X;\underline{n}+1)] > \cdots > E[v(X,X;\bar{n})] \end{aligned}$$

Problem: bias at boundaries in kernel estimation of pseudo-values. The bid density $g(b,b)$ is estimated inaccurately for bids close to the boundary of the empirical support of bids.

Solution: use *quantile-trimmed means*: $\mu_{n,\tau} = E[v(X,X;n)\mathbf{1}\{x_\tau < X < x_{1-\tau}\}]$

above $\Rightarrow$

$$\begin{aligned} H_0 \text{ (PV)} &: \quad \mu_{\underline{n},\tau} = \mu_{\underline{n}+1,\tau} = \cdots = \mu_{\overline{n},\tau} \\ H_1 \text{ (CV)} &: \quad \mu_{\underline{n},\tau} > \mu_{\underline{n}+1,\tau} > \cdots > \mu_{\overline{n},\tau}. \end{aligned}$$

Theorem 3 Let $\hat{\mu}_{n,\tau} = \frac{1}{n\times T_n}\sum_{t=1}^{T_n}\sum_{i=1}^{n}\hat{v}_{it}\,\mathbf{1}\{b_{\tau,n} \le b_{it} \le b_{1-\tau,n}\}$ and assume [...conditions for kernel estimation...]. Then
(i) $\hat{\mu}_{n,\tau} \xrightarrow{p} E[v(X,X,n)\;\mathbf{1}\{x_\tau < X < x_{1-\tau}\}]$;
(ii)$\sqrt{T_n h}\,(\hat{\mu}_{n,\tau} - \mu_{n,\tau}) \xrightarrow{d} N(0,\omega_n)$, where
$\omega_n = \left[\int\left(\int K(v)K(u+v)dv\right)^2 du\right]\left[\frac{1}{n}\int_{F_b^{-1}(\tau)}^{F_b^{-1}(1-\tau)} \frac{G_n(b;b)^2}{g_n(b;b)^3} g_n(b)^2 db\right]$.

Test statistic Now use standard multivariate one-sided LR test (Bartholomew, 1959) for normally distributed parameters $\hat{\mu}_{n,\tau}$

- $a_n = \frac{T_n h}{\omega_n}$ (inverse variance weights)

- $\overline{\mu} = \frac{\sum_{n=\underline{n}}^{\bar{n}} a_n \, \hat{\mu}_{n,\tau}}{\sum_{n=\underline{n}}^{\bar{n}} a_n}$ (MLE under null)
- $\mu^*_{\underline{n}}, \ldots, \mu^*_{\bar{n}}$ solves

$$\min_{\mu_{\underline{n}},\ldots,\mu_{\bar{n}}} \sum_{n=\underline{n}}^{\bar{n}} a_n \left(\hat{\mu}_{n,\tau} - \mu_n\right)^2 \quad s.t. \quad \mu_{\underline{n}} \geq \mu_{\underline{n}+1} \geq \cdots \geq \mu_{\bar{n}}. \tag{13}$$

- $\bar{\chi}^2 = \sum_{n=\underline{n}}^{\bar{n}} a_n \left(\mu^*_{n,\tau} - \bar{\mu}\right)^2$
  - distributed as mixture of χ^2_k rv's, $k = 0, 1, \ldots, \overline{n} - \underline{n}$
  - mixing weights: $\Pr_{H_0}\{$soln to (13) has exactly k slack constraints$\}$ (obtain by simulation)
- estimate ω_n using asymptotic formula or with bootstrap

4.3 Endogenous participation

The validity of this test relies crucially on the assumption that variation in n, the number of bidders, across auction is exogenous. Next, we consider how this can be relaxed.

Idea: bidder participation determined by unobservable (to us) factors, denoted W, which are also correlated with bidder valuations.

Problems:

1. valuations varying with N ($\Longrightarrow$ *second-stage test may be invalid*). Extreme case: if N is decreasing in W, then $\mu_{N=2} > \mu_{N=3}$, even under PV. "Usual" problem that endogeneity can confound results.
2. to estimate pseudo-values using the FOC, we must condition on all info (both N and W, e.g.) bidders do ($\Longrightarrow$ *first stage estimation invalid too*!). We must estimate equilibrium bid distributions g and G conditional on both N and W.

IV approach: assume there is an instrument Z which satisfies

Assumption 1 $N = \phi(Z, W)$, with ϕ nonconstant in Z and strictly increasing in W. (Implies W uniquely determined given N and Z, and discrete.)

This assumption is strong, but we will see why we need it.

Assumption 2 Z is independent of $(U_1, \ldots, U_n, X_1, \ldots, X_n, W)$.

Assumption 3 The support of $N|Z$ consists of a set of contiguous integers.

With these assumptions, it turns out there is no loss in generality from taking $\phi(\cdots)$ to be additive, and equal to:

$$\phi(Z_t, W_t) = \texttt{int}[E(N|Z_t)] + W_t.$$

Hence, the unobserved factor in auction t, is essentially "observed" after we run a first-stage nonparametric regression of N_t on Z_t:

$$\hat{W}_t = N_i - \texttt{int}[\widehat{E(N|}Z_t)].$$

This suggests that we can adapt the test in the following way:

1. Estimate bid distributions $G(b, b|n, w)$ and $g(b, b|n, w)$ conditional on both n and w.

2. For bid b_{it} in auction t, we can recover the corresponding pseudovalue as:

$$\hat{v}(x_i, x_i|n_t, w_t) = b_{it} + \frac{\hat{G}(b_{it}, b_{it}|n_t, w_t)}{\hat{g}(b_{it}, b_{it}|n_t, w_t)}.$$

3. Now the winner's curse implies that under PV, the conditional expectation $E_x v(x, x|n, w)$ conditional on (n, w) is invariant in n, for all w. However, under CV, it is decreasing in n, for all w.

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