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# Uncertainty about Relative Power and War*

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**Danger! Preliminary!**

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**Abstract**

The traditional and formal literatures in international relations have placed uncertainty at the center of their explanations for war. The formal literature has largely focused on uncertainty regarding the costs of war while the traditional literature focuses on uncertainty about relative power. In this paper we analyze a class of formal models of conflict where players choose between fighting and a negotiated settlement, players are uncertain about their likelihood of winning a war, and where each side has private information about the probability of winning. Unlike the few existing formal papers modeling uncertainty about relative power, we find that there are different and important strategic incentives that arise in this interdependent values setting. First, we find that in these formal models of conflict, mutual optimism regarding the players' prospects in war and the occurrence of war are simply coincidental. That is, mutual optimism is neither necessary nor minimally sufficient for war in equilibrium. Second, we find that strategic incentives in the presence of uncertainty about relative power leads decision-makers to choose war even when both sides' private information tells them that the expected outcome of war is worse than opting for the efficient settlement. That is, there is rational war with mutual pessimism.

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# 1 Introduction

The question of why and under what conditions countries choose to fight costly wars when less costly negotiated settlements are available is one central to the study of international relations. Traditional and formal theories of war have long put uncertainty at the center of their stories. But while the formal game theoretic literature on war has tended to focus on private information regarding costs or resolve, the traditional literature has also emphasized uncertainty about countries' prospects in war. As is well known, when leaders face uncertainty and have special or private knowledge that informs their assessment regarding the likely outcome of war, inconsistent expectations might be a cause of conflict. If leaders expectations are inconsistent in that both antagonists think they are likely to win a costly war, then there may not exist any peaceful settlement that both prefer to war. In these circumstances, the cause of war might then be inconsistent expectations that arise from private information (Fearon 1995, Blainey 1988, Wagner 1994, Kim and Bueno de Mesquita 1995).

In this paper we analyze a conflict game between two countries where both sides have private information regarding the likely outcome of war. In particular, we analyze the strategic incentives decision-makers face when choosing between fighting a war or opting for a peaceful settlement. The strategic problem for decision-makers in these circumstances is complicated by the interdependence of the players' payoffs in war. That is, unlike conflict games in which uncertainty is about "private valued" elements of a decision-maker's uncertainty, when the uncertainty concerns the probability of winning a war a player must account for the fact that their opponent's private information (or type) is relevant for their own payoff to war directly.

In its most common form, uncertainty and private information about the probability of war is associated with discussion of optimism and mutual optimism. In Fey and Ramsay (2007), this mutual optimism argument is formalized and shown to be an inconsistent rationalist explanation for war. That analysis, by far, did not exhaust what may be learned about the strategic incentives in conflict games with uncertainty about the probability of winning a war. For example, a critical assumption in Fey and Ramsay's (2007) argument was that wars only occurred with mutual agreement. That is, war was a public event and could only occur when both sides agreed to fight. It was argued that for the concept of "war by mutual optimism" to have any substantive content one needed to assume that both sides to the conflict choose war, that everyone know that everyone knows war is going to occur, and that decision-makers are satisfied with their decision–i.e., making a expected utility maximizing choice. Otherwise, it is not clear what is "mutual" in mutual optimism story.

Clearly, the set of assumptions in Fey and Ramsay (2007), linking mutual optimism to war, are not the only assumptions one could make regarding the strategic interaction of countries in the shadow of war. In fact, given the anarchic nature of the international system, it seems natural to assume that countries can always choose to break off negotiations and use force instead. This paper pushes further to understand how uncertainty about the probability of success in war influence strategic incentives in conflict. Specifically, we relax the important assumption that each country has some action that they can take to get the negotiated settlement instead of war, and characterize its consequences for the set of strategies and outcomes that are consistent equilibrium behavior.

In particular, we are interested in how the mutual optimism argument fairs as a causal story under the assumption that any single country can start a war. Following Fearon (1991), we say that mutual optimism causes war if there are cases in which mutual optimism is present and war occurs and [the counterfactual] in which mutual optimism is not present, war does not occur. When war is a mutual act, the argument is simple. War does not occur, so mutual optimism cannot be a cause for war. When war is a unilateral act, the argument is more subtle. Our first set of results show that if war occurs in equilibrium, there must be states of the world in which war occurs and only one side thinks it is going to win, i.e., is optimistic. Thus, the counterfactual claim that mutual optimism is a necessary condition for war is false even when war is a unilateral act. As might seem obvious, mutual optimism is more or less irrelevant where war is concerned when it only takes one side fighting to get into a war. As a result, we go further and also show that mutual optimism is not even a minimally sufficient cause of war. We also analyze the prospect of war as an equilibrium phenomenon in this class of games by looking at the setting where players are boundedly rational in their information processing. Here we find, like the results in Fey and Ramsay 2007, the weak link between mutual optimism and war is not fragile. It extends to situations where players are boundedly rational information processors.

Following the discussion of mutual optimism and war, we explore the nature of equilibrium strategies in conflict games with uncertainty about the probability of war and the unilateral war assumption. We show an interesting consequence of the unilateral war assumption with common values and independent types; it can be impossible to maintain peace in equilibria even with weakly undominated strategies and where players have expected payoffs from war, given their private information, that are worse for them than agreeing to a peaceful settlement. That is, the ability to impose unilateral war can lead to war with what might be called *mutual pessimism*. Finally, we generalize this result to the situation where there are a continuum of independent types and find that 1) there is no peace in equilibirium when costs are sufficiently small and 2) if the probability of winning function is sufficiently steep, war can occur with probability close to 1 for reasonably large costs of war.

# 2 Model and Assumption

In this section, we describe our model and introduce the appropriate notation. We will follow the notation in Fey and Ramsay (2007) closely.[1]

Two countries face a potential conflict that can be settled either by force or by a negotiated settlement. For our initial results we suppose that any negotiated settlement is efficient, but that war is inefficient. We also suppose that a war can be started by either side, unilaterally. To explore the role that private information plays in this choice, we assume that there is a set $\Omega$ of possible states of the world. Each possible state of the world, denoted $\omega$, is a complete description of both countries' capabilities and prospects for war. As is standard, we suppose both countries share a *common prior* $\pi$ on $\Omega$ and focus on how differences in information might lead to the choice of war.

In order to incorporate these states of the world into a conflict game, Fey and Ramsay (2007) present a general model of knowledge, which we summarize briefly here. In this model of knowledge, we associate information or knowledge with the ability to distinguish between various states $\omega$ in $\Omega$. More specifically, we are interested in knowledge of *events*, which are naturally defined as subsets of $\Omega$. We represent a player's knowledge by a *possibility correspondence* $P_i(\omega)$, that maps every state $\omega$ to a non-empty set of states $P_i(\omega) \subseteq \Omega$. For each $\omega \in \Omega$, $P_i(\omega)$ is interpreted as the collection of states that individual $i$ thinks are possible when the true state is $\omega$. A possibility correspondence $P_i(\omega)$ for $\Omega$ is *partitional* if there is a partition of $\Omega$ such that for any $\omega \in \Omega$ the set $P_i(\omega)$ is the element of the partition that contains $\omega$. As discussed in Fey and Ramsay (2007) (and proved by others), a fully rational player must have a partitional possibility correspondence. For later use, we define the *join* of two partitions as the coarsest common refinement of the two partitions. In terms of knowledge, the join of the possibility correspondences of two players represents what players would know if their information were public instead of private.

Certain kinds of events have special importance. First, we say that an event $E$ is *self-evident* event for a possibility correspondence $P_i$ if and only if for all $\omega \in E$, $P_i(\omega) \subseteq E$. In

[1] Interested readers can find the explanation of some well known results in the our 2007 article.

other words, an event $E$ is self-evident if, for any state in $E$, a player knows $E$ has occurred. Second, a *public* event, unlike a private signal, is known to all players when it happens. Specifically, if $E$ is a public event, then $E$ is self-evident to all players.

We know turn to incorporating this model of knowledge into a general model of war. We define two functions, $p_1(\omega)$ and $p_2(\omega)$, that specify the probability that country 1 and 2 will win a war, given the true state of the world $\omega$. Of course, $p_1(\omega) + p_2(\omega) = 1$ and $0 \leq p_i(\omega) \leq 1$ for all values $\omega \in \Omega$. Consider an arbitrary event $E$. If a country knows an event $E \subseteq \Omega$ has occurred, it can combine this information with the prior $\pi$ via Bayes's Rule to form a posterior belief about the value of $p_i$ as follows:

$$\mathrm{E}[p_i|E] = \frac{\sum_{\omega \in E} p_i(\omega)\pi(\omega)}{\sum_{\omega \in E} \pi(\omega)} \tag{1}$$

From this expression, it is easy to verify that if $\mathrm{E}[p_i|E'] \geq x$ and $\mathrm{E}[p_i|E''] \geq x$ for disjoint sets of states $E'$ and $E''$, then $\mathrm{E}[p_i|E' \cup E''] \geq x$.

We represent the private information of country $i$ by a possibility correspondence $P_i : \Omega \to 2^{\Omega}$, which we assume is partitional. Recall that $P_i(\omega)$ is the set of states that country $i$ views as possible, given the true state $\omega$. Given a true state $\omega$, a country can combine its knowledge of $P_i(\omega)$ with the prior $\pi$ and equation 1 to construct its posterior belief about the probability it will win, $\hat{p}_i(\omega) = \mathrm{E}[p_i|P_i(\omega)]$. In this setting, we say that *mutual optimism* occurs when $\hat{p}_1(\omega) + \hat{p}_2(\omega) > r + 1 - r = 1$, while *unilateral optimism* occurs when for any player $\hat{p}_i(\omega) > r_i$, where $r_i$ is player $i$'s payoff to a settlement.

We end this section by describing the class of games that we analyze. In order to be as general as possible and to cover as many different varieties of strategic interaction, we describe an abstract class of games. Let the set of actions for player $i$ in some two-player strategic form game be given by the set $A_i$. The result of the choice of actions for the two sides will be either war or a peaceful settlement. We normalize the utility of countries to be 1 for victory in war and 0 for defeat, and we suppose there is a cost $c_i > 0$ of fighting a war for country $i$. We assume that war is a unilateral act, so that either side can start a war. Formally, war is a unilateral act if, for each $i$, there is an action $\bar{a}_i \in A_i$ such that whatever action is chosen by the opponent, the outcome is war.

Finally, we define a (pure strategy) strategy $s_i \in S_i$ as a function $s_i : \Omega \to A_i$ with the restriction that

$$P_i(\omega) = P_i(\omega') \quad \Rightarrow \quad s_i(\omega) = s_i(\omega').$$

This condition states that if a country cannot distinguish state $\omega$ from state $\omega'$, then its action must be the same in both states.

# 3 Mutual Optimism and Unilateral War

Let $G$ denote any strategic form game of incomplete information that satisfies our assumptions on the information structure, payoffs, and strategies. We then get our main result, which states that there is no game in which mutual optimism is a necessary condition for war.

**Theorem 1** *Suppose countries have a common prior, war is a unilateral act, and $P_i$ is partitional for $i = 1, 2$. In any Bayesian-Nash equilibrium of $G$ in which war occurs, there is a state $\omega$ at which war occurs but mutual optimism does not hold.*

*Proof*: The proof of this theorem is similar to the proof of Theorem 1 in Fey and Ramsay (2007). Suppose that the strategy profile $(s_1^*, s_2^*)$ is a Bayesian-Nash equilibrium in which war occurs. Let the set of states for which the outcome of the game is war be $W$. Define the following two events

$$O_1 = \{\omega \in \Omega \mid \hat{p}_1(\omega) > r + c_1\}$$
$$O_2 = \{\omega \in \Omega \mid \hat{p}_2(\omega) > 1 - r + c_2\}.$$

As war is a unilateral act, at any state in either $O_1$ and $O_2$, war will be the outcome. To see this, note that if we are at one of the states in $O_1$ or $O_2$ and war were not the equilibrium outcome, then at least one player could start a war and have a higher expected payoff. Thus war is the outcome for all states in $O_1 \cup O_2$, and $W \supseteq O_1 \cup O_2$.

To prove the theorem, suppose that the conclusion is false. That is, suppose that in every state that war occurs, mutual optimism also occurs. Formally, this requirement is that $W \subseteq O_1 \cap O_2$. Combining this with the above yields $O_1 \cup O_2 \subseteq W \subseteq O_1 \cap O_2$. This can only be true if $O_1 \cup O_2 = W = O_1 \cap O_2$, implying $O_1 = O_2 = W$.

Now, because $O_i$ is defined in terms of $\hat{p}_i$, $r_i$ and $c_i$, it clear that for all $\omega' \in P_i(\omega)$, $\omega' \in O_i$ if and only if $\omega \in O_i$. Therefore, $O_1 = O_2 = W$ implies that

$$W = \bigcup_{\omega \in W} P_i(\omega) = \bigcup_{\omega \in W} P_2(\omega).$$

That is, as $W$ is self-evident for nondeluded $P_i$, $W$ is the union of $P_i(w)$ for all $\omega$ in $W$, and this is true for each player $i$. As the correspondence $P_i$ is partitional, we can further write $W$ as the union of disjoint sets $P_1(\omega)$, defined by some collection of states $D^*$ with $D^* \subseteq W$. Since $D^* \subseteq W = O_1$, we have $E[p_1 \mid P_1(\omega)] \geq r + c_1$ for every $\omega \in D^*$. Because each disjoint set $P_1(\omega)$ has conditional expectation $E[p_1 \mid P_1(\omega)]$ of at least $r + c_1$, then the conditional expectation over the union of these disjoint sets (i.e., $E[p_1 \mid W]$) is also at least $r + c_1$. That is, $E[p_1 \mid W] \geq r + c_1$. By a symmetric argument for player 2, $E[p_2 \mid W] \geq 1 - r + c_2$. Therefore

$$E[p_1 \mid W] + E[p_2 \mid W] \geq 1 + c_1 + c_2. \tag{2}$$

But as $p_1(\omega) + p_2(\omega) = 1$, $r + 1 - r = 1$, and $c_i > 0$ for all $\omega \in \Omega$, it follows from Bayes's Rule that $E[p_1 \mid W] + E[p_2 \mid W] = 1$. This yields a contradiction, which proves the result. ∎

The theorem shows that there cannot be an equilibrium to any game, in a broad class where countries with the ability to unilateral start a war, where mutual optimism is a necessary condition for costly conflict. At first glance, this result seems obvious. Why would mutual optimism have to be present for war to occur when any single state can start a war? But the result actually shows more than this. To be clear, this does not mean that mutual optimism and war cannot occur together in equilibrium, but rather that it is never necessary. Any game with such an equilibrium must have other realizations of the state of the world–in the particular equilibrium–where war occurs and there is no mutual optimism.

Next, to show that this coincidence of mutual optimism and war is not important in any such equilibrium, we present the following result.

**Theorem 2** *Suppose countries have a common prior, war is a unilateral act, and $P_i$ is partitional for $i = 1, 2$. Let $P^*$ be the join of the partitions $P_1$ and $P_2$.*

*In any Bayesian-Nash equilibrium of $G$ in which war occurs, if $\omega$ is a state in which war occurs and mutual optimism holds, then there exists a player $i^* \in \{1, 2\}$ such that letting $P_i = P^*$, in every equilibrium of this new game, war occurs at $\omega$ but mutual optimism does not hold at $\omega$.*

*Proof*: Let $\omega$ be a state in which war occurs and mutual optimism holds. Then $\omega \in O_1 \cap O_2$, where $O_1$ and $O_2$ are the events described in Theorem 1 and thus $E[p_i \mid P_i(\omega)] > r_i + c_i$ for $i = 1, 2$. Clearly, war occurs at $\omega$ in every equilibrium to $G$ because otherwise any single player could deviate and get a higher expected payoff.

To prove the theorem, let $P^*$ be the join of the partitions $P_1$ and $P_2$. That is, $P^*$ is the coarsest common refinement of $P_1$ and $P_2$. We first show that there exists $i^* \in \{1, 2\}$ such that if $P_i = P^*$, then mutual optimism does not hold. If $E[p_1 \mid P^*(\omega)] \leq r + c_1$, then choosing $i^* = 1$ makes this statement true. So suppose this inequality does not hold. Then

$$
\begin{aligned}
E[p_1 \mid P^*(\omega)] &> r + c_1 \\
E[1 - p_2 \mid P^*(\hat{\omega})] &> 1 - r_2 + c_1 \\
E[p_2 \mid P^*(\hat{\omega})] &< r_2 - c_1,
\end{aligned}
$$

from which it follows that $E[p_2 \mid P^*(\hat{\omega})] < r_2 + c_2$. Therefore, in this case, mutual optimism does not hold if we choose $i^* = 2$.

Note, however, that whichever player we assign to $i^*$, the other player has unilateral optimism at $\omega$ because nothing has changed for this player. Therefore, war must occur at $\omega$ in every equilibrium of this new game because this player prefers war to peace. ∎

Notice what is happening here. In every state where there was war and mutual optimism, the unique equilibrium outcome in the new game without mutual optimism is also war. As one might expect, mutual optimism is more than sufficient in a conflict game with unilateral war, but coupled with the result of Fey and Ramsay 2007, we have the rather general result that mutual optimism is not a strong theoretical explantion for war. Next we show this result is robust to some bounded rationality in the learning process of decision-makers.

## 3.1 Mutual Optimism, Bounded Rationality, and Unilateral War

While Theorem 1 is true for any of the class of games in which the decision-makers rationally process information, one may wonder if the results depend on strictly rational learning. In this section, we consider a class of games where, again, two countries are choosing whether to fight a war or resolve the dispute by some other means. Here, we show that even if players' information processing suffers from cognitive biases the link between mutual optimism and war is still quite weak. In particular, even if both players ignore "bad news" or are inattentive, then war is only coincidently related to mutual optimism.

When it comes to information processing, a rational Bayesian may be able to deduce much more information from a "signal" than the signal carries at face value. That is, the rational Bayesian, like Sherlock Homes, learns from the dog that does not bark. There are, however, many cases in which we think that decision-makers, particularly the leaders of countries, may not be processing information rationally. Consider the information processing errors found in the psychological international relations literature (Jervis, Lebow and Stein 1985, Jervis 1976). For example, a decision-maker who has many responsibilities may face a volume of information that induces flaws in their learning. In particular, such a decision-maker may not update their beliefs when the state of the world is not explicitly brought to their attention. This error may occur because of a flaw in human psychology or it could be an information shortcut that allows decision-makers to deal with a world far more complex than the two state example above.

Alternatively, due to what Jervis, Lebow and Stein (1985, p.4) call *motivated bias*, a player's knowledge may be partly a matter of choice. So given that some people have strong predispositions to believe certain things to be true, this may prevent them from recognizing new information inconsistent with their world view. That is, sometimes decision-makers may consciously, or subconsciously, choose to ignore unpleasant information.

Next we consider a game with players whose information processing is flawed in ways consistent with the learning processes described above. A common component of these cognitive biases is that the player's information processing allows them to learn from new information in some states of the world, but not in others. To capture this idea formally, we define a new restriction on the players' possibility correspondences, $P_i$. In particular, while we still assume $P_i$ is nondeluded, we now allow players to "ignore" or "throw out" information at a given state of the world that would be known to a fully rational Bayesian. To allow such pathologies, we must allow for the possibility that some information sets are nested within, or subsets of, other information sets. To do this, we define a new weaker possibility correspondence to capture these important departures from rationality.[2]

**Definition 1** *A player's possibility correspondence is* nested *if for all* $\omega, \omega' \in \Omega$*, either (1)* $P_i(\omega) \cap P_i(\omega') = \emptyset$ *or (2)* $P_i(\omega) \subseteq P_i(\omega')$ *or (3)* $P_i(\omega') \subseteq P_i(\omega)$*.*

Possibility correspondences that satisfy nondeluded and nestedness represent a generalization of rational learning. That is, a decision-maker with a nested possibility correspondence may processes information in a rational way or she may ignore new information at a number of different states. Such a formalization is consistent with many forms of bias, because it is agnostic to the reason information is ignored. Players could fail to learn in

[2]For more on decision-theoretic approaches to bounded rationality in models of knowledge see Geanakoplos (1989) or Rubinstein (1998) for game theoretic approaches. The concept of nestedness is taken from Geanakoplos's (1989).

some states because acquiring information is costly, because they are inattentive, or because they would rather not think about the implications of the information in front of them. We now verify the robustness of Theorem **??**

**Theorem 3** *Suppose countries have a common prior and $P_i$ is nondeluded and nested for $i = 1, 2$, then there is no game $G$ where mutual optimism is a necessary condition for the existence of a Bayesian Nash in which war occurs.*

*Proof*: The core of the proof of this theorem is a combination of the proof of Theorem 1 above and Theorem 2 in Fey and Ramsay (2007). Again let the set of states for which the outcome of the game is war be $W$. Define the following two events

$$O_1 = \{\omega \in \Omega \mid \hat{p}_1(\omega) > r + c_1\}$$
$$O_2 = \{\omega \in \Omega \mid \hat{p}_2(\omega) > 1 - r + c_2\}.$$

As war is a unilateral act, at any state in either $O_1$ and $O_2$, war will be the outcome. AS before war is the outcome for all states in $O_1 \cup O_2$, and $W \supseteq O_1 \cup O_2$.

To prove the theorem, suppose that the conclusion is false. That is, suppose that in every state that war occurs, mutual optimism also occurs. By the argument in Theorem 1, this can only be true if $O_1 \cup O_2 = W = O_1 \cap O_2$, implying $O_1 = O_2 = W$. Clearly, $P_i(\hat{\omega}) \subseteq W$ for all $\hat{\omega} \in W$ and we can then write

$$W = \bigcup_{\omega \in W} P_i(\omega) = \bigcup_{\omega \in W} P_2(\omega).$$

So the assumption of unilateral war, the necessity of mutual optimism, and the measurability of the players strategies imply the event $W$ is a self-evident (public) event. As a self-evident event is well defined for any nondeluded possibility correspondence, $W$ is self-evident to 1 and 2, and therefore

$$W = \bigcup_{\omega \in W} P_1(\omega) = \bigcup_{\omega \in W} P_2(\omega). \tag{3}$$

Let $\bar{P}_i(\omega)$ be the set with the largest cardinality such that $\omega \in P_i(\omega')$ for some $\omega' \in W$. By nestedness $\bar{P}_i(\omega)$ is unique and $\bar{P}_i(\omega) \subseteq W$ for all $\omega \in W$. Because $W$ is self-evident,

$$W = \bigcup_{\forall \omega \in W} \bar{P}_i(\omega)$$

and for all $\omega, \omega' \in W$, either $\bar{P}_i(\omega) = \bar{P}_i(\omega')$ or $\bar{P}_i(\omega) \cap \bar{P}_i(\omega') = \emptyset$. Therefore, country $i$'s nondeluded, and nested $P_i(\omega)$ induces a $\bar{P}_i(\omega)$ partition on $W$. We can then write $W$ as the union of disjoint sets $\bar{P}_i(\omega)$, defined by some collection of states $\hat{D}^*$ all contained in $W$, i.e., $\hat{D}^* \subseteq W$. The result then follows as in Theorem 1. ∎

Theorem 3 show that for some plausible types of "boundedly rational" actors, mutual optimism cannot be necessary for war. Or, in other words, if war and mutual optimism occur simultaneously at some state in an equilibrium in $G$, then there must be some other state where there is war and no mutual optimism.

As mentioned above, considering information structures that relax the requirements of strict Bayesian rationality can help us understand just how general our coincidence result is. On the one hand, the analysis in this section shows that the mutual optimism result in Theorem 1 is not fragile. Clearly, some departure from rational Bayesian learning is acceptable and consistent with our results. In particular, if decision-makers sometimes ignore unpleasant information or behave as if they have imperfect memory, then our result survives.

# 4 Unilateral War and the Unraveling of Peace

Moving beyond mutual optimism arguments, we now turn to characterizing other aspects of Bayesian Nash equilibria to conflict games. In this section we show the unilateral war assumption has some interesting implications in the setting where players have interdependent values for war. To get an intuition for the incentives decision-makers face we consider two simple examples. The first is a version of the highest die game found in Fey and Ramsay (2007) that changes the underlying assumption regarding how players participate in the game. In the original version, both players had to agree to play, whereas in this incarnation any single player can force the play of the game. Even though war is costly, the example shows that strategic incentives interact with the unilateral war assumption to produce a surprising implication: rational war with mutual pessimism.

**Example:** [1] **The highest die game recast**. Consider a game between two players, Alice and Bob, who have a choice between engaging and not engaging in a contest whose outcome is determined by chance. To start, each player is given a (fair) die to roll and the die are compared. If Alice's die generates a higher number than Bob's, Alice wins. If the number on Bob's die is the larger, then he wins, and Bob and Alice tie otherwise. Suppose that each player maximizes expected utility and assigns a utility value of $+1$ to winning, $-1$ to losing, and 0 to a tie. Before deciding whether to engage in the contest or not, each player receives some information about the roll of the dice. In particular, Alice observes the result of her die and Bob observes the result of his die. After receiving this information, the two players simultaneously announce whether or not they want to play the game of chance. If *either* player decides to engage in the contest, then the contest occurs and payoffs are awarded as above and, in addition, each player pays a small cost $c$. If *both* players decide not to engage in the contest, then they both receive a payoff of 0.

To begin, lets consider the naive strategy. As any single player can initiate the contest, a player will clearly choose to do so if their expected payoff from the contest is higher than their payoff of 0 from not having the contest. A simple expected utility calculation shows that both Alice and Bob prefer to initiate the contest, rather than sit out, when they see a 4, 5, or 6 on their die. But that is not the end of the story. Think about Alice's play when she sees a 3 on her die. If Bob gets a 4, 5, or 6 he will unilaterally initiate the contest and Alice's payoff does not depend on her choice. If, however, Alice's choice is "pivotal", that is, Bob has not chosen to start the contest, then it means Bob sees a 1,2, or 3 on his die. But in such a case, Alice's die value of 3 is likely to prevail in the contest. In particular, for $c < 1/4$, Alice strictly prefers to initiate the contest when she sees a 3. The same is true for Bob. Further, if either sees a 2, for $c < 1/4$ they prefer to initiate the contest even though their unconditional expectation of their chances of prevailing in the contest are pessimistic. In fact, for both players, their unconditional expected utility for the contest is strictly less than what they would get if the contest did not occur.

In this example, the unraveling stops when both players see a 1. In this case, there is no chance of winning and the best they can do is pay a cost to get a lottery with the same expected value as not having the contest. What is striking here is that the unilateral war assumption leads to war when both players believe they will be worse off, given their private information. This effect can be quite prevalent. In the highest die example given here, we see that the *ex ante* probability of war is 35/36. In other words, the only state of the world without war in this game is the one in which both players receive their worst possible signal regarding their prospects in war. This is true even if we restrict ourselves to equilibria in weakly undominated strategies.[3] □

But unlike the case where both players must agree to play the game, which results in no play in every Bayesian Nash Equilibrium, with unilateral play there exist counter examples

[3]Games with unilateral war always have an equilibrium where both players choose to fight at every state.

to the unraveling process. That is, it is not always the case that war occurs with mutual pessimism. Our next example illustrates the point.

**Example:** [2] **An example without unraveling of peace**. Consider a slightly more stylized model of the strategic problem of war than the highest die game. In this game each player can choose to fight for a prize of value 1 or agree to a settlement. If either player chooses to fight then the winner of the prize depends on the state of the world. Let the states of the world be $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$. If $\omega_k$ is odd then player 1 wins the prize. If $\omega_k$ is even then player 2 wins the prize. In this game there is a positive cost $c > 0$ to contest a prize. If neither player chooses to fight, then a settlement of 1/2 is awarded to each player.

Suppose that the players' information partitions have the following structure:

$$P_1 = \{\omega_1\}\{\omega_2, \omega_3, \omega_4, \omega_5\}$$
$$P_2 = \{\omega_1, \omega_2, \omega_3\}\{\omega_4, \omega_5\}$$

and that the players have a common prior belief that is uniform on $\Omega$.

Clearly, this information structure leads player 1 to choose to fight at $\omega_1$ if $c < 1/2$. But given that choice, what does player 2 want to do at the $P_2(\omega_2)$ information set? The expected payoff to fighting here, even when conditioning on being pivotal, is $1/2 - c$, which is smaller than what she expects to get from a settlement. In this example the unraveling stops and peace occurs with probability 4/5. In fact, this is true for any cost and the choice not to fight almost everywhere remains an equilibrium even if the cost of war is zero. □

These examples are suggestive in two ways. First, we see that there may be a general tendency for the unilateral war condition to lead to wide spread conflict in equilibrium. The second example suggests, however, that the prevalence of conflict is not completely general. So what kinds of conditions result in war with mutual pessimism? A closer examination of the second example suggests an answer. Specifically, in this example, if the settlement value is $(x, 1 - x)$ with $x \neq 1/2$, then for sufficiently small $c$, all equilibria involve war at every state of the world. Thus, the assumption that the settlement is *exactly* 1/2 is critical for this example.

Our next result generalizes this observation and presents a sufficient condition on the expected probability of victory and the value of the settlement such that in every equilibrium war occurs at every state of the world.

Let $G$ denote any strategic form game of incomplete information that satisfies our assumptions on information structure, payoffs, and strategies given in Section 2. Here, a peaceful settlement gives payoff $v_i$ to country $i$, with $v_1 + v_2 = 1$.

**Theorem 4** *Suppose countries have a common prior, war is a unilateral act, and $P_i$ is partitional for $i = 1, 2$. Suppose $E[p_i(w)|Y] \neq v_i$ for all $Y \subseteq \Omega$. Then, for sufficiently small $c_1, c_2$, in every pure strategy Bayesian Nash equilibrium of $G$ war occurs with probability one.*

*Proof*: Suppose not. Let the strategy profile $(s_1^*, s_2^*)$ be such a Bayesian Nash equilibrium, where peace occurs with positive probability. Let $T$ be the non-empty set of states where the outcome of $g(s_1^*(\omega), s_2^*(\omega))$ is not war. For every $\hat{\omega} \in T$, it must be that the $s_i^*(\hat{\omega}) \neq \bar{a}_i$.

To simplify notation, let $I^*(\omega) = 1$ if the outcome of the equilibrium $(s_1^*, s_2^*)$ at $\omega$ is peaceful settlement and $I^*(\omega) = 0$ if the outcome at $\omega$ is war. Then the expected utility to player $i$ of the strategy profile $(s_1^*, s_2^*)$ for a given $\omega$ is

$$u_i(s_1^*(\omega), s_2^*(\omega)) = I^*(\omega)v_i + (1 - I^*(\omega))[p_i(\omega) - c_i]$$

Noting that each player can impose war by playing $\bar{a}_i$, and this deviation changes the payoff to player $i$ only if war would not have occurred anyway, equilibrium requires

$$E[u_i(s_1^*(\omega), s_2^*(\omega)) \mid P_i(\omega') \cap T] \geq E[p_i(\omega) - c_i \mid P_i(\omega') \cap T] \tag{4}$$

for every $\omega' \in T$.

As the correspondence $P_i$ is partitional, we can define a set of states $D_i^*$ with $D_i^* \subseteq T$ such that the sets $\{P_i(\omega)\}_{\omega \in D_i^*}$ are disjoint and their union is equal to $\bigcup_{\hat{\omega} \in T} P_i(\hat{\omega})$. Since $D_i^* \subseteq T$, we have from inequality (4) that

$$E[u_i(s_1^*(\omega), s_2^*(\omega)) \mid P_i(\hat{\omega}) \cap T] \geq E[p_i(\omega) - c_i \mid P_i(\hat{\omega}) \cap T]$$

for every $\hat{\omega} \in D_i^*$. As this holds for each disjoint set $P_1(\hat{\omega})$, then the same conditional expectation inequality holds over the union of these disjoint sets. Therefore,

$$E[u_i(s_1^*(\omega), s_2^*(\omega)) \mid \bigcup_{\hat{\omega} \in D_i^*} P_i(\hat{\omega}) \cap T] \geq E[p_i(\omega) - c_i \mid \bigcup_{\hat{\omega} \in D_i^*} P_i(\hat{\omega}) \cap T].$$

Because $\bigcup_{\hat{\omega}\in D_i^*} P_i(\hat{\omega}) = \bigcup_{\hat{\omega}\in T} P_i(\hat{\omega})$, it follows that $\bigcup_{\hat{\omega}\in D_i^*} P_i(\hat{\omega}) \cap T = T$. Thus, we have

$$E[u_i(s_1^*(\omega), s_2^*(\omega)) \mid T] \geq E[p_i(\omega) - c_i \mid T].$$

By definition, at every state $\omega \in T$, the strategy profile $s_1^*(\omega), s_2^*(\omega)$ results in a peaceful settlement. Thus,

$$\begin{aligned} E[v_i \mid T] &\geq E[p_i(\omega) - c_i \mid T] \\ v_i &\geq E[p_i(\omega) \mid T] - c_i \\ v_i - E[p_i(\omega) \mid T] + c_i &\geq 0. \end{aligned} \tag{5}$$

for $i = 1, 2$. This last expression must be true for every equilibrium and every cost $c_i$. Now because $v_1 + v_2 = 1 = p_1(\omega) + p_2(\omega)$ for all $\omega \in \Omega$ and $E[p_i(w)|Y] \neq v_i$ for all $Y \subset \Omega$, it follows that $v_i - E[p_i(\omega) \mid T] < 0$ for some $i$. For sufficiently small $c_i$, this contradicts inequality (5), which proves the result. ∎

The sufficient condition given in this result clarify the situations in which the strategic incentives of the conflict game lead to equilibria with only war.

Our result describes a world in which a decision-maker's choice only matters when the other has not chosen to fight. But an opponent only prefers not to fight when war is not good for him. But that means war is good for you. So conditional on there not being a war started by your opponent, as the cost of war get small, the expected utility of war is good for the decision-maker. That is, decision-makers expect to win the wars they start. A consequence of this incentive, however, is war under mutual pessimism. As in the dice game example when both players see a 2, they can know with probability one that they expect to do worse by playing than from not playing, but conditional on their choice being relevant they prefer to play. In conflict games, such an incentive could lead to apparently irrational wars between countries that are pessimistic about their prospects.

In the next section, we show for the case of a continuum of independent types, but interdependent values, we can prove a general result about the probability of war as the costs of war go to zero. We are also able to give examples of how different contest functions influence the equilibrium cut-points of players and how the *ex ante* probability of war changes with costs. Most interestingly, we can show that there exist situations where the *ex ante* probability of war is close to one for positive costs. That is, even though our proofs are based on a limiting case—as is true in Theorem 4—the limiting case can be a good approximation for situations with small positive cost of war.[4]

### 4.1 Unilateral War and Peaceful Equilibria for a Continuum of Types

Suppose types are drawn independently. For player 1, type $t_1$ is drawn from a distribution $G_1$ on $[0,1]$ and likewise $t_2$ is drawn from $G_2$, also a distribution on $[0,1]$, with each $G_i$ continuous with full support. Each player simultaneously chooses an action $a_i \in \{F, N\}$. If both players choose $a_i = N$, then the conflict is settled without war and the players receive payoffs $(v_1, v_2)$, where $v_1 + v_2 = 1$. If either player chooses $a_i = F$, then war ensues. In this case, the payoff of a player is $p_i(t_1, t_2) - c_i$, where $p_1(t_1, t_2) + p_2(t_1, t_2) = 1$.

We assume that the types of players can be ordered such that the probability of victory is (strictly) monotone in the countries' types. That is, higher types have a greater chance of winning, all other things being equal. Formally this assumption is that, for $i = 1, 2$, $t_i > t'_i$ implies $p_i(t_i, t_j) \geq p_i(t'_i, t_j)$, for all $t_j \in [0,1]$. Also, to ensure there is uncertainty, we assume that $p_i$ is not everywhere constant. In this way, the type $t_i$ reflects the "strength" of country $i$ and thus the probability of victory depends on the relative strength of the two combatants. We also require that $p_i$ be is continuous.

A (pure) strategy for player $i$ is a function $s_i : [0,1] \to \{F, N\}$. We begin by showing that in any Bayesian-Nash equilibrium to this game, both players must use cutpoint strategies. To see this, fix a strategy profile $(s_1, s_2)$ and define $F_j = \{t_j : s_j(t_j) = F\}$ and $N_j = \{t_j : s_j(t_j) = N\}$. These are types of player $j$ that choose Fight and Not Fight, respectively. Given this, the expected utility to type $t_i$ of player $i$ for choosing $F$ is

$$Eu_i(F \mid t_i) = \int_{[0,1]} (p_i(t_i, y) - c_i)\, dG_j(y)$$

and the expected utility to type $t_i$ of player $i$ for choosing $F$ is

$$Eu_i(N \mid t_i) = \int_{F_j} (p_i(t_i, y) - c_i)\, dG_j(y) + \int_{N_j} v_i\, dG_j(y).$$

We then have the following proposition.

**Proposition 1** *If $s^* = (s_1^*, s_2^*)$ is a Bayesian-Nash equilibrium, then there exists a pair of*

[4] We will also see that what constitutes a "small" enough cost depends on the technology of war $p$.

*cutpoints* $(t_1^*, t_2^*) \in [0,1]^2$ *such that, for* $i = 1, 2$,

$$s_i^*(t_i) = \begin{cases} F & \text{if } t > t_i^* \\ N & \text{if } t < t_i^* \end{cases}$$

*Proof*: For type $t_i$ of player $i$, the action $F$ is optimal if and only if

$$Eu_i(F \mid t_i) \geq Eu_i(N \mid t_i)$$
$$\int_{N_j} (p_i(t_i, y) - c_i) - v_i \, dG_j(y) \geq 0. \tag{6}$$

As the function $p_i$ is strictly monotonic in $t_i$, it follows that the left hand side of this inequality is strictly monotonic in $t_i$. Therefore, there must exist a value $t_i^* \in [0, 1]$ such that $F$ is optimal if and only if $t \geq t_i^*$. We conclude that a best response for player $i$ must be a cutpoint strategy and therefore, in equilibrium, both players must be playing a cutpoint strategy. This establishes the proposition. ∎

Notice that the condition given by inequality (6) implies that for any pair of cost values $(c_1, c_2)$, the strategy defined by cutpoints $t_1^* = t_2^* = 0$ is a Bayesian-Nash equilibrium. Therefore, an equilibrium exists for every pair of cost values.

We next state our main result for a continuum of types. The result characterizes the set of equilibria to this game as costs become small. Formally, let $s^*$ be an equilibrium of this game, with associated cutpoints $t_1^*, t_2^*$.

**Theorem 5** *For any sequence* $c^k = (c_1^k, c_2^k) \to (0, 0)$, *if, for all* $k$, $t^{*k} = (t_1^{*k}, t_2^{*k})$ *is a convergent sequence such that* $t^{k*}$ *is a pair of cutpoints of a Bayesian-Nash equilibrium to the game with costs* $c^k$, *then* $t_i^{*k} \to 0$ *for some* $i \in \{1, 2\}$.

*Proof*: Fix a sequence $c^k = (c_1^k, c_2^k) \to (0, 0)$ and let $t^{*k} = (t_1^{*k}, t_2^{*k})$ be a convergent sequence such that $t^{k*}$ is a pair of cutpoints of a Bayesian-Nash equilibrium to the game with costs $c^k$ for all $k$. Let $l_i = \lim_k t_i^{*k}$. For a proof by contradiction, suppose that $l_i > 0$ for $i = 1, 2$.

We begin with the case that $l_i \in (0, 1)$ for $i = 1, 2$. This implies that for sufficiently large $k$, $t_i^{*k} < 1$ for $i = 1, 2$. As each threshold is interior, inequality (6) holds with equality.

Therefore, for $i = 1, 2$ and sufficiently large $k$,

$$\int_0^{t_j^{*k}} (p_i(t_i^{*k}, y) - c_i^k) - v_i \, dG_j(y) = 0$$

$$\int_0^{t_j^{*k}} p_i(t_i^{*k}, y) \, dG_j(y) = G_j(t_j^{*k})(c_i^k + v_i).$$

Letting $I(x, z)$ be the indicator function defined by

$$I(x,z) = \begin{cases} 1 & \text{if } x \in [0, z] \\ 0 & \text{otherwise,} \end{cases}$$

we have

$$\int_0^1 I(y, t_j^{*k}) p_i(t_i^{*k}, y) \, dG_j(y) = G_j(t_j^{*k})(c_i^k + v_i).$$

As the integrand on the left hand side is bounded, we can take limits as $c^k \to 0$ to arrive at

$$\int_0^1 I(y, l_j) p_i(l_i, y) \, dG_j(y) = G_j(l_j) v_i$$

$$\int_0^{l_j} p_i(l_i, y) \, dG_j(y) = G_j(l_j) v_i \tag{7}$$

for $i = 1, 2$.

Now, as $p_1(l_1, l_2) + p_2(l_1, l_2) = 1 = v_1 + v_2$, it must be that $p_i(l_1, l_2) \geq v_i$ for some $i$. Without loss of generality suppose that $p_1(l_1, l_2) \geq v_1$. But as $p_1(t_1, t_2)$ is strictly decreasing in $t_2$, it follows that

$$\int_0^{l_2} p_1(l_1, y) \, dG_2(y) > G_2(l_2) v_1.$$

But this contradicts equation (7), which establishes the result for the case that $l_i \in (0, 1)$ for $i = 1, 2$.

The remaining possibility is that $l_i = 1$ for some $i$. In this case, it possible that $t_i^{*k} = 1$ for arbitrarily large $k$. Thus, we have the following inequality for $i = 1, 2$:

$$\int_0^{t_j^{*k}} p_i(t_i^{*k}, y) \, dG_j(y) \leq G_j(t_j^{*k})(c_i^k + v_i).$$

By the same argument as above, for $i = 1, 2$,

$$\int_0^{l_j} p_i(l_i, y)\, dG_j(y) \leq G_j(l_j) v_i.$$

But then this expression gives rise to the same contradiction as before, by way of the same argument. This proves the theorem. ∎

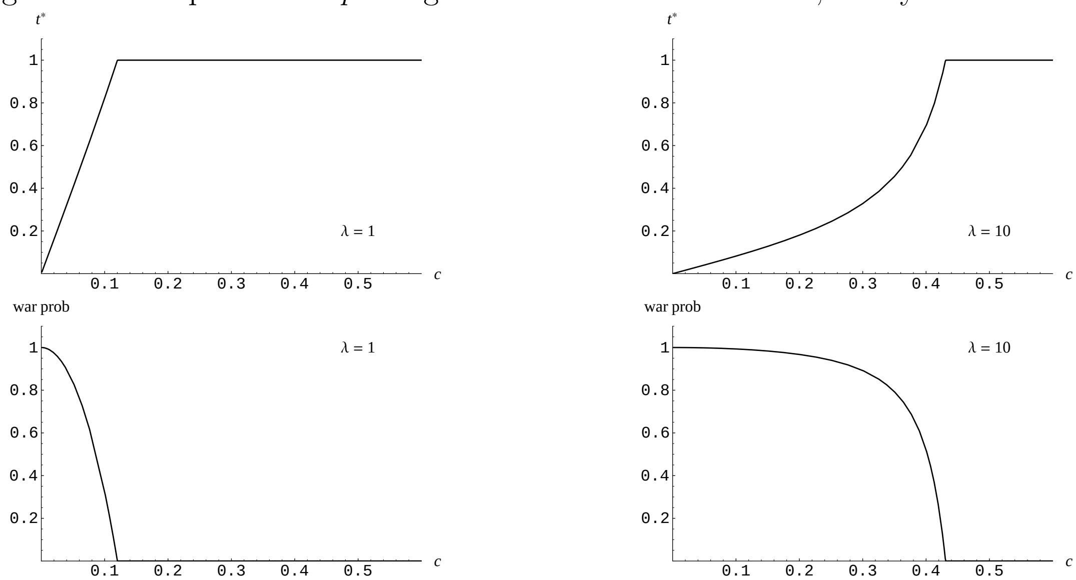
This result states that if we take a sequence of costs going to zero and a sequence of equilibrium cutpoints going to $t^*$, then at least one player's limiting cutpoints is also zero. Thus the probability of war must be going to 1 in the limit. Why is this the case? Consider the case of the strongest type of player $i$ that chooses to negotiate, $t_i^*$. We know that at $p_i(t_i^*, t_j^*) + p_j(t_i^*, t_j^*) = v_1 + v_2 = 1$. So for at least one player $p_i(t_i^*, t_j^*) \geq v_i$. But this player, when deciding whether to fight or not knows he is going to do well in the any war he starts. As he is the strongest type not starting a war, it must be his average payoff from wars where his decision is pivotal is strictly larger than his payoff at $p_i(t_i^*, t_j^*)$, which equals $G_j(t_j^*)v_i$. So for some positive cost, this player strictly prefers to deviate to fighting, even though he was chosen to be the type that is indifferent between fighting and not fighting.

Given this result, one might wonder, how small do the small costs have to be for such an unraveling to occur? What will the *ex ante* probability of war be for some given cost and technology of war? We explore this question as follows. First, note that if $G_1 = G_2$, $v_1 = v_2$, $c_1 = c_2$ and $p_1(t_1, t_2) = p_2(t_2, t_1)$, then the game is symmetric and there will exist a symmetric equilibrium given by cutpoints $t_1^* = t_2^*$. To illustrate our point, we assume that $G_i$ is uniform and $v_i = 1/2$ for $i = 1, 2$. We also assume that the probability of winning is logistic with parameter $\lambda$. That is,

$$p_i(t_i, t_j) = \frac{e^{\lambda t_i}}{e^{\lambda t_i} + e^{\lambda t_j}}.$$

Figure 1 gives the symmetric equilibrium cutpoint and the ex ante probability of war for two particular values of $\lambda$. The top left image gives the equilibrium cutpoint for this game with $\lambda = 1$. This results in a relatively flat $p$ function and requires costs to be less than approximately .1 before war occurs with significant *ex ante* probability. If, however, we consider the same game with $\lambda = 10$, $p$ becomes steeper and there is a positive probability of war for cost less than .4, and once cost reaches .2, the *ex ante* probability of war is very close to one. This is similar to the results in Theorem 4 and the die example. The examples also suggests that even though the results presented are limiting cases, proven as costs go to

Figure 1: Examples where $p$ is logistic with $\lambda = 1$ and $\lambda = 10$, the symmetric case.




Note: The top two pictures map the symmetric cutpoint $t^*$ as a function of $c$. The bottom two figures give the ex ante probability of war as a function of $c$.

zero, the substantive implication of the results hold for positive costs, at least with relatively well behaved war winning functions.

# 5 Conclusion

While much of the traditional literature on uncertainty and war in international relations focuses on uncertainty regarding the distribution of power and success in war, with few exceptions (Fey and Ramsay 2007, Powell 2004, Slantchev 2003, Wagner 1994) the formal literature on the causes of war has focused on uncertainty about the costs of war. In this paper we have focused on the special incentives that arise when uncertainty regarding the likelihood of winning a war is modeled directly. A few conclusions can be drawn. First, whether war is a unilateral act or some contest both players must agree to join, mutual optimism is not a good rationalist explanation for war. In particular, we show that there are no Bayesian Nash equilibria to a conflict game where players must choose between war and a negotiated settlement and where war occurs only when there is mutual optimism. Moreover, Theorem 2 says the distribution of equilibrium outcomes from an equilibrium where war and mutual optimism are coincidental can be replicated exactly in another equilibrium where there is no mutual optimism at any state of the world. These results hold even if we relax the assumption that decision-makers are perfect Bayesian learners.

Our second set of results show that there is an interesting strategic consequence of the interaction between uncertainty over the probability of winning in a war and the unilateral war assumption. It seems almost obvious that in international politics any single country can start a war rather than accept a peaceful settlement, but unlike the results in Powell (2004), there does seem to be a real difference between situations where uncertainty is about a common valued element of the strategic environment ($p$) and not a private valued element ($c_i$). When there is uncertainty and private information about the probability of winning a war, strategic incentives lead to a surprisingly large amount of war in equilibrium when the cost of war are not too high. While at first the statement that a low cost of war should lead to high *ex ante* probability of war seems obvious, the strategic incentives generating the results are not. We find that the unilateral war assumption in the interdependent value setting produces war with mutual pessimism. That is, players initiate wars when their private information tells them they would be better off with a settlement. This happens for two reasons. First, when one's opponent chooses to start a war, a decision-maker's choice between war and peace is irrelevant. That said, the wars a player starts are the ones she is more likely to win. Because the other player's type affects a player's payoff directly, if player $j$ does not want to fight–and chooses peace–it must mean $j$'s private information says war is good for $i$, and peace unravels.

This result is quite general, but that does not mean that the assumptions that have been made cannot be questioned. There are a few assumptions that qualify our results. First, the settlement in this model is efficient, but fixed. While this does not seem essential, if $v_i$ is able to vary with the state of the world like $p(\omega)$, our examples for the continuous types case suggest that there will be conditions on the relationship between changes in $p$ and changes in $v$ that will be necessary for unraveling to occur.

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