Attention for Sale: Endogenous Agenda Formation in Sequential vs. Simultaneous Debates

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Abstract: We model a competition between two debaters trying to capture the attention of a decision maker. In the model, the decision maker can only choose alternatives that are on the agenda and the agenda is endogenous. The debaters compete to put their alternatives on the agenda by participating in all-pay-auctions for a finite number of attention slots. We compare simultaneous debates, in which all investments are taken before the debate, to sequential debates, in which investments can be taken along with the evolution of the agenda. In a simple model with complete information we highlight why sequential debates are preferable for the decision maker. We show the robustness of this result to several generalizations of the model including asymmetric information between the debaters.

1 Introduction

In many decision making processes, a decision maker does not have the ability to consider or to observe all feasible alternatives, due to both time and attention constraints. This *scarcity* implies that interested parties, who wish to influence the decision maker to choose or at least consider their preferred alternative, may compete to capture the decision maker's limited attention. Thus, the final set of options available to a decision maker - the agenda - may be a result of a series of behind-the-scenes investments and counter-investments.

In this paper we are interested in modelling the competitive process of endogenous agenda formation. While debates may include many phases such as agenda formation, argumentation and information exchange, we focus on the first phase, which is largely ignored in the literature.[1]

We model a competition between two debaters who are facing a time and attention constrained decision maker. The decision maker can only choose among alternatives that are on the agenda and the agenda is endogenously determined by the efforts of the two debaters. Specifically, agenda formation is modelled as all-pay-auctions over a finite number of time/attention slots of the decision maker. Some motivating examples are:

(i) The deliberation of a hiring committee in an academic department: the committee (or more generally the department) can only consider candidates which have been placed on the agenda. Some members of the committee may be more successful than others in "making a case" for some candidate to be on the agenda. The department then chooses one candidate from the set proposed by the committee.

(ii) A media campaign trying to influence public opinion: typically the public forms an opinion or demands that the government will take some action only on issues that

[1] Most of the literature (see for example Austen-Smith and Feddersen (2006) and Gerardi and Yariv (2007)) consider debates as information exchange. Spiegler (2006) and Glazer and Rubinstein (2001) focus on the role of argumentation in debates.

have been brought to its attention, usually via advertisements or other costly lobbying efforts of interested parties.

(iii) Presidential TV debates: these involve two or more candidates vying to make an impression on swing voters. While the identity of the candidates is known, the nature of the policies they will pursue is often revealed in these contentious debates, where the candidates' war-chest includes well-prepared and rehearsed arguments, anecdotes, and sometimes, (costly) provocative revelations.

In these examples, the competing parties spend time, money and effort to try and capture the attention of a third party. In Presidential debates it seems that a substantial part of this effort is invested before the debate takes place and without knowing exactly what the other party will pursue. In contrast, deliberations that take place over time, such as recruitment or some public opinion campaigns, allow for the different sides to react to the evolving agenda.

In this paper, we compare between two procedures of debate. In a *simultaneous debate* all bids are placed before the debate starts. In a *sequential debate* bids can be placed along with the evolution of the agenda during the debate. In both procedures the decision maker chooses his preferred alternative among all the ones that have been placed on the agenda.

In a simple model with complete information we highlight why sequential debates are preferable for the decision maker. In particular, we focus on a situation in which one alternative, represented by one of the debaters, is favoured by the decision maker. That is, if both alternatives are on the agenda, the decision maker always chooses this alternative. This implies that to win, the less advantaged debater must "buy" all time/attention slots. We further assume that this advantage is common knowledge among the debaters. Finally we assume that debaters have the same willingness to pay. We show that while simultaneous debates lead to strictly positive probabilities of both alternatives being chosen by the decision maker, in a sequential debate the preferred alternative is chosen with probability one.

To see the intuition consider the sequential debate and in particular the subgame after the disadvantaged debater has already managed to place her alternative on the agenda. The continuation of this subgame is identical to a symmetric all-pay-auction for one good; whoever wins this round, wins the debate. In the unique equilibrium of this game, both players are "squeezed" out of their rents. As a result, the continuation value of the disadvantaged debater from putting her alternative on the agenda is equal to that of losing the debate outright. Thus, for the less advantaged, attention is much harder to sustain than to achieve in the first place and she will not be willing to buy all time/attention slots to win the debate.

In contrast, in the subgame in which the advantaged debater's alternative is already on the agenda, she wins the debate and no bidding wars will arise. Therefore, the advantaged debater has a strictly positive willingness to pay to place her alternative on the agenda early on. As a result, in the unique equilibrium, the advantaged debater wins the debate with probability one.

In a simultaneous debate players cannot place an advantaged alternative on the agenda early on in order to commit to avoid wasteful bidding wars. In this case, both debaters have a strictly positive willingness to pay and each can win the debate with a strictly positive probability.

We show the robustness of this result for a wide range of generalizations of the model. In Section 4.1 we analyse the case of asymmetric willingness to pay. When the willingness to pay of the disadvantaged candidate is larger than that of the advantaged player, a trade-off arises. In particular we show that in this case, both candidates have a strictly positive probability of wining the debate in any procedure. However, for any level of asymmetry in the willingness to pay, it is still the case that in a sequential procedure the advantaged debater wins with a higher probability than in a simultaneous procedure. In section 4.2 we show that when debaters care only about winning, our results hold for any finite number of debaters.

In Section 4.3 we consider the case of private information. In particular we focus on a political application in which alternatives are the ideal policies of debaters with single peaked preferences. We assume that a debater's ideal policy is her private information. This model differs from our basic framework in several respects. First, as alternatives are private information, being on the agenda first (in a sequential debate) implies a loss of information rent vis-a-vis the opponent. Second, the advantage of a debater is uncertain, as the debater whose ideal policy is closer to that of the decision maker is the favourable one. Finally, the willingness to pay is endogenous as debaters care about policy; how much they are willing to pay to avoid losing depends on their ideal policies. In this framework as well we show that in sequential debates, with probability one, the debater who is closest to the decision maker is chosen. In contrast, in a simultaneous debate both debaters have a strictly positive probability of winning the debate.

Our paper is related to the literature on strategic information transmission in groups. Austen-Smith and Feddersen (2006) and Gerardi and Yariv (2007) model deliberation and voting with a fixed agenda. Gerardi and Yariv (2007) show that any equilibrium outcome that arises in sequential debates can arise in a simultaneous debate as well.[2] In contrast, our paper highlights a difference between sequential and simultaneous debates albeit in a different context.

More closely related is Glazer and Rubinstein (2001). They analyse a model in which, due to time constraints, debaters cannot present all of their information to a decision maker. Using a mechanism design approach, they show that sequential debates are better for the decision maker than simultaneous ones. Sequential protocols work better as they allow the debater with "truth" on her side to better reveal her opponent's lack of evidence. In our model sequential protocols are beneficial as they provide higher incentives for the preferred debater to outbid her opponent.

A number of papers have analysed the role of auctions in political competition. Notable examples are Becker (1983) who analyses influence functions to study political pressure, and Grossman and Helpman (1994) who introduced menu auctions to model lobbying activity.[3] One related strand of this literature concerns impressionable voters. Specifically, Grossman and Helpman (2001) assume that money buys votes, that is, that some voters are affected by advertising. Thus, the more a candidate spends, the more she is likely to gain these votes. Another strand of this literature assumes that voters are rational and that campaign spending provides information to voters, either directly (as

[2] A related paper is Dekel and Piccione (2001) who have shown an equivalence in the set of equilibria arising in simultaneous and sequential voting mechanisms.

[3] In a recent paper, Jackson *et al* (2007) evaluate all-pay-auctions as a mechanism to choose candidates in party primaries.

in Ashworth (2003), Coate (2004), or Schultz (2003)), or indirectly, as in Prat (2002). In contrast, the attention-constrained decision maker in our model is not fully rational and money/effort buys his attention but do not secure a particular decision.

Our focus on an attention-constrained decision maker is also related to how consumers are often modelled in the marketing literature and in the behavioural IO literature (see for example Eliaz and Spiegler (2007)). The term "consideration set" is used in this literature to refer to the set of alternatives that the consumer takes into account when making a decision. Moreover, it is often assumed that this consideration set is a subset of what actually is available to the consumer. In our model the "consideration set" of the consumer is the endogenous agenda which is determined by the costly efforts of the two debaters.

The remainder of the paper is organized as follows. We present the simple complete information model in the next Section. In Section 3 we derive our main results. We next consider the extensions of the model in Section 4, and conclude in Section 5. All proofs which are not in the text are provided in the appendix.

2 The Model

A decision maker (DM) is facing a decision problem in which he has to take an action. The DM does not know what are the feasible alternatives he can choose from. Assume that there are only two such alternatives. We assume that two players (debaters) $i \in \{1, 2\}$, are representing these alternatives, with player 1 (2) representing x_1 (x_2). We assume that the debaters know the feasible alternatives. In Section 4.3 we relax this assumption and analyse the case in which the alternatives are the private information of debaters.

Alternatives may be revealed to the DM via an endogenous agenda formation process, i.e., the debate, which we describe below. Once the agenda forms, the DM picks his preferred alternative from the agenda. We assume that in the case in which the DM does not choose any of these alternatives, his utility would be worse than the utility from any of the feasible alternatives x_1 or x_2. We consider an asymmetric situation, in which player 1 is an advantaged player: If x_1 is on the agenda, then the DM chooses it, whereas he chooses x_2 only if it is the only alternative on the agenda.

We assume that the DM faces time and attention constraints. In particular, the DM has two (subsequent) time/attention slots: In each such slot he can register only one alternative on the agenda.[4] In the debate, debaters will compete to capture the attention of the DM by "winning" these slots. We model this competition by an all-pay-auction in which the winner captures all the attention of the decision maker for that time/attention slot. The winner-takes-all assumption is not important; our main result holds if we assume instead that the probability that a player captures the attention is a function of the relative size of the bids. In addition, we allow each debater to yield the slot to her opponent if she doesn't want to participate in the bidding. The agenda at the end of the debate is the set of alternatives that are registered on the two time/attention slots.

Although the time/attention slots are (naturally) sequential, we distinguish between

[4]The number two is of no consequence; as long as we assume that there are a finite number of these slots, our results will follow through.

two kinds of debate procedures depending on the timing of the submission of the bids. In the first procedure, debaters decide on their bids for both slots in advance and so cannot react to whatever occurred in the preceding time slot. We call this procedure *simultaneous debate*. This procedure captures debates such as presidential TV debates in which the important investment is the preparation done prior to the debate.

The second debate we examine is one in which debaters can react after the outcome of the first time/attention slot was observed. We call this a *sequential debate*. This procedure is relevant for debates that take place over time. For example, this procedure could be relevant to model the debate in a hiring committee during the course of the job market in an academic department.

Formally, in the simultaneous debate, each debater i, simultaneously, submits two bids (b_i^1, b_i^2) where $b_i^k \in [0, \infty) \cup \{yield\}$ is the bid for time/attention slot $k \in \{1, 2\}$. For each time/attention slot k the DM registers the alternative of the highest bidder when both have submitted bids in $[0, \infty)$. If both debaters submitted the same bid in $[0, \infty)$ for some slot k, the decision maker registers one of the alternatives (each with a strictly positive probability which is not necessary to specify). If only one debater yields slot k, the DM registers the alternative of the other debater. If both debaters yield slot k, the DM doesn't register any alternative. If neither alternative was registered for both slots, the DM receives his default utility.

In the sequential debate, debaters compete over two time/attention slots, one at a time, in a sequence of two all-pay auctions. At each period $t \in \{1, 2\}$, each debater simultaneously bids $b_i^t \in [0, \infty) \cup \{yield\}$. At each period t, the DM registers the alternative of the highest bidder for slot t when both have submitted bids in $[0, \infty)$. The rest of the rules determining the agenda formation for each slot are as above. In contrast to the simultaneous debate, players submit bids in the second period after one alternative is already known to be on the agenda. Note that we allow players to keep on bidding even though their alternative has been registered; by bidding the highest amount at each round a player insures that only her alternative is on the agenda. Thus, in both types of debates, the agenda could comprise of the two alternatives, only one of the alternatives or no alternative at all.

Finally, debater $i's$ utility from winning is $v_i - \sum_{h=1,2} b_i^h$ and is $-\sum_{h=1,2} b_i^h$ otherwise, where h is the time/attention slot. To illustrate our results, we consider first the simplest case where $v_i = v_j = v$. This utility value can be interpreted as either a pure winning motivation or as a policy motivation. For example if alternatives are policies on a unidimensional policy space, $v = |x_1 - x_2|$ could be the gain from one's alternative winning over the second alternative.

3 Analysis

3.1 A simultaneous debate

As an illustration of the mechanism of the all-pay-auction, let us consider first a benchmark case in which the DM has only one time/attention slot, and each player is allowed to place one bid.[5] Note that this renders the debaters to be fully symmetric: whoever wins the time/attention slot also wins the debate, and both value winning the debate in the same way.

[5] This is obviously analogous to a sequential debate with only one round.

Note first that there cannot be a pure strategy equilibrium in this game. In equilibrium, no player can lose with probability one with a costly bid. Hence, players either tie or one player loses, either with a zero bid or by yielding. But if a player loses, then if she places a zero bid the winning player cannot be best responding with a positive bid, and if she yields, then the winning player must place a zero bid which implies that the yielding player is not best responding. Thus players must tie. But then, as long as $b < v$, each player can deviate to $b+\varepsilon$ for a small enough ε and win with probability one rather than with a lower probability. On the other hand players cannot tie with $b \geq v$ as in this case they would rather lose.

Players must play a mixed strategy equilibrium. Moreover, these strategies must include zero in their support as neither player wishes to lose with probability one with a positive bid. In the appendix we establish that these strategies are continuous and that there are no gaps in their support. It is also easy to see that they must have the same support.

Let $f^i(b)$ be the density function used by player i. Solving the maximization problem of player j, we have

$$\max_{b_j} F^i(b)v - b_j \rightarrow f^i(b) = \frac{1}{v}$$

for any b in the support of $f^i(.)$, which implies that player i uses a uniform distribution. The maximization problem of player i is analogous which implies that the players use the same strategies. Finally, it is easy to see that the maximum bid is v, which yields the same utility as a bid of zero. We then have:

LEMMA 1 *In a game with one time/attention slot and one bid per player, in the unique* *equilibrium both players use mixed strategies with bids drawn from a uniform distribution over* $[0, v]$.

The main features of the all-pay-auction equilibrium described above will also arise when we consider more general environments later on; players must use mixed strategies and in particular they uniformly mix among the bids in the support. The support includes zero and the maximum bid that the players are willing to place in order to win.[6]

Note that the equilibrium of such a game is highly inefficient: the DM is as likely to "choose" the advantaged and the disadvantaged position. Thus, in terms of information aggregation, he cannot be guaranteed to choose the right option. Moreover, the players' expected utility is zero (as they are mixing, this utility is easily computed for a bid of zero which is in the support); both exert resources up to their willingness to pay for winning the debate. This is a feature of the inefficient bidding war.

Consider now the debate with two time/attention slots and two bids per debater, one for each slot. Player 1 needs to win just one slot, whereas player 2 needs to win both of them in order to win the debate. It is easy to see that players have to mix, as above. Note that as player 1 needs to win just one slot, she cannot place more than one positive bid. Suppose she draws two positive bids, one per each slot. She can clearly increase her utility by placing the sum of the bids in one of the slots. Next note that player 1 cannot place her positive bid at one of the slots with probability one. In this case, player 2 wins

[6] For more general environments in which the players are not symmetric, the maximum bid is determined by what the weakest player is willing to pay, and this player will also place an atom on the bid of zero.

the second slot at almost no cost, and hence player 1 can deviate and place a slightly higher bid in this second slot. Therefore player 1 mixes, to "hide" the slot she competes on. In the appendix we establish:

LEMMA 2 *In the simultaneous debate, in the unique equilibrium, player 1 draws one costly bid from a uniform distribution over [0,v/2], and her second bid is zero. Player 1 places the strictly positive bid in time/attention slot 1 (2) with probability $\frac{1}{2}$ ($\frac{1}{2}$). Player 2 draws two bids from a joint distribution over [0,v/2], and places each at one slot. The probability that each player wins is strictly positive.*

The probability that player 2 wins the debate is lower in this game then in the benchmark with one time/attention slot.[7] Note however that although player 2 wins with a lower probability, she still wins the simultaneous debate with a substantial probability and is able sometimes to "capture" the debate.

3.2 A sequential debate

We now analyse the sequential debate. In this debate, the alternative that was registered on the agenda in the first time/attention slot, may end up as the only one on the agenda. This is particularly beneficial for the less advantaged debater and may possibly induce her to bid more aggressively in the first period. We now explore whether the less advantaged debater is able to buy attention both in the first and the second round.

[7] Indeed, one can easily show that as the number of time/attention slots increases, the probability that player 2 wins the simultaneous debate decreases. As this number goes to infinity, her chances of winning converge to zero.

We start by analysing the second round game. Note first that if player 1 is on the agenda at this time, the game is over; debater 2 has no incentive to bid as if she were to be on the agenda as well, she would lose. On the other hand, if player 2 is on the agenda, then both players engage in a bidding war. Whoever wins the slot, wins the debate; if player 2 wins, her alternative will be the only one on the agenda, whereas if player 1 wins, her alternative will be chosen as it is preferred by the decision maker. This symmetry implies that the equilibrium that is played is exactly as described in Lemma 1, in the auction with one time/attention slot. We can therefore conclude that when x_2 is on the agenda, the expected utility of each player is zero - the utility from losing.

Given the equilibrium of the second round, it is easy to derive the following observations. First, player 2 gains the same utility from being on the agenda in the first period as she does when player 1 is on the agenda. As a result, she will not be willing to place any positive bid in the first period. Second, player 1 strictly benefits from placing herself on the agenda in the first stage in comparison with letting player 2 do so. She will therefore be willing to place a positive bid in the first round in order to insure she wins straight away instead of engaging in a bidding war in the second stage. Given that player 2 does not place any positive bid, player 1 does the same and the equilibrium is sustained by player 2 yielding in the first period:

LEMMA 3 *In equilibrium, in the first period, the advantaged debater places a zero bid and the disadvantaged debater yields. There is no bidding in the second round and the advantaged player's alternative is the unique one on the agenda.*[8]

[8] The same equilibrium arises for a sequential debate with any number of slots, with the advantaged player revealing her alternative in the first period at no cost.

Proof of Lemma 3: Player 2 has no incentive to place a zero bid and a strong incentive not to place a positive bid as her continuation utility is zero. Player 1 wins by placing a zero bid. If she yields or increases her bid her utility decreases. Uniqueness follows from the discussion above.■

The advantaged debater has a higher utility from being on the agenda early on, as she can avoid bidding wars in the second period. The disadvantaged debater, on the other hand, cannot improve her utility by putting her alternative on the agenda early on, even if it is costless. Once on the agenda, she will face a relatively fierce competition - which does not make it worthwhile for her to be on the agenda first. Attention, for the less advantaged, is much harder to sustain than to achieve in the first place. Such debaters are therefore not able to buy both time/attention slots to win the debate.

3.3 Sequential vs simultaneous debates

We now formally compare between the sequential and the simultaneous debates.

Proposition 1 *(i) A sequential debate allows the advantaged player to win with a higher probability than a simultaneous debate; (ii) The debaters exert less resources in the sequential debate than in the simultaneous one.*

The proof of Proposition 1 follows from Lemma 2 and Lemma 3. From the point of view of the DM, a sequential debate is preferable, as it allows him to pick the right option with a higher probability.

The intuition for this result is straightforward. Bidding wars are inefficient, and players would rather avoid them. A sequential debate allows for one alternative to be on the agenda before the other and thus allows players to commit to avoid wasteful bidding wars. In a simultaneous debate this commitment is not possible. This means that both players engage in a rather competitive bidding war in which each stands a chance to win. Accordingly, they also exert more resources in this form of debate.

These conclusions will turn out to be general for many other specifications. We proceed to consider several extensions of the model as well as to apply it to more specific situations.

4 Extensions

4.1 Asymmetric willingness to win

In our analysis so far we have examined the case in which both debaters have the same willingness to win the debate. This assumption allowed us to focus on how an advantage in popularity of a debater translates into the her success in debates. In this section we examine the case in which debaters have asymmetric willingness to pay. In particular assume that the value of winning the debate is v_i, for player $i = 1, 2$.

The interesting case to consider is when $v_1 < v_2$. In this case there is a trade-off between the popularity advantage of player 1 and the higher willingness to pay of player 2. In the next Lemma we show that this trade-off implies that player 2 can win the sequential debate with a strictly positive probability.

LEMMA 4 *If $v_2 > v_1$ then in the sequential debate: (i) there is strictly positive spending in the first round, (ii) player 2 wins with a strictly positive probability.*

The main difference here is that in the second round of the sequential debate, if x_2 is already on the agenda, the players are not symmetric any more. Although each wins the debate if her bid is the highest, player 2 values winning more. This implies that she fights more fiercely than player 1, and is more likely to win. Her expected utility is therefore higher than her utility from losing outright to player 1, which implies that both players exert costly resources in the first round.

Our general intuition as well as the results about the difference between simultaneous and sequential debates still hold. Mainly, a sequential debate allows player 1 to materialize her advantage more effectively; she will not be challenged once exposed. The next Proposition shows that sequential debates are more favourable for the advantaged for any $v_1 \neq v_2$:

PROPOSITION 2 *For any v_1 and v_2, (i) the probability with which the advantaged player wins the debate is strictly higher in the sequential debate than it is in the simultaneous debate, (ii) as player 2 becomes infinitely more willing to win the debate (as $\frac{v_2}{v_1} \rightarrow \infty$), the probability that player 2 wins any debate goes to one.*

4.2 Many debaters

In our basic model we have considered the case of two debaters representing two alternatives. It is easy to see that our results hold for any $K > 1$ debaters representing K alternatives that are allowed to bid over K (or m such that $2 \leq m \leq K$) time/attention slots.

Consider therefore K debaters, where each represents one alternative in $\{x_1, x_2, .., x_i, ..., x_k\}$, with the feature that from any agenda, the DM chooses the alternative x_i with the lowest index i. Debaters all value winning in the same way, i.e., $v_i = v$ for all i, and have complete information about all other alternatives.

In the last round of the sequential debate, player x_i for $i > 1$, whose index is lowest among those on the agenda, will face competition from $\{x_1, ..., x_{i-1}\}$, which is a symmetric all pay auction with i players. This implies, as in the two players case, that $x_i's$ utility from being exposed in the last round equals her utility from losing outright. She will therefore yield in the first round. On the other hand, when x_1 is exposed no player will challenge her in any round of the game. As a result she has a strict incentive to expose her position in the first round. We therefore have:

Proposition 3 *Debater 1 wins the sequential debate with probability one and loses the simultaneous debate with a strictly positive probability.*

Note that the analysis carried above was under the assumption that each debater cares only about winning. While in the two debaters case this was analogous to assuming that players care about the winning policy, this is not the case for $K > 2$ players. With many players, negative externalities might exist when players care also about the identity of the alternative chosen and not only about winning. In such cases the analysis becomes more complicated. The main element that is added to the analysis is that once x_i is on the agenda, players who are less favourable than x_i may still engage in costly bidding wars. A player representing x_{i+1} may keep on bidding if she prefers x_i to x_{i-1}, for example, and thus would rather be on the agenda with no chance of winning than have x_{i-1} on the agenda. This implies that the utility of moderately advantaged players is not necessarily their utility from losing outright, as other, less advantaged players, may "fight" for them once they are on the agenda. Such a model would be rather complicated as free riding issues contribute to the possibility of multiple equilibria. In the next section we analyse the case of negative externalities when we consider a model with two debaters and private information.

4.3 Private information

So far we have maintained the assumption that both debaters know each other's type. This assumption might be too strong in many situations in which the relative popularity of the different debaters is not known or when debaters do not know exactly the willingness of the other debater to fight it out. In this section we analyse the debate between two players with single peaked preferences and privately known ideal policies.[9]

Suppose that the debaters' positions are their ideal policies. These two policies are drawn from some continuous and symmetric distribution $g(x)$ over $[-1, 1]$. Each debater learns her position x_i but not the other debater's. We assume that the DM's ideal policy is zero, and that he chooses the position on the agenda that is closest to his ideal policy. Denote his policy choice by y. Note that neither player can know for sure whether she is the more moderate player (i.e., the advantaged one) or whether she is the extreme one

[9] Our analysis in this section is related to the citizen candidate literature (Osborne and Slivinski (1996), Besley and Coate (1997)). In that literature, politicians are ideological, their positions are known, and they endogenously choose whether to offer them to voters or not. In our model, politicians also choose whether to offer their position to voters, but to do so they must compete and win an all-pay-auction first.

(unless her ideal policy is 0, -1, or 1). Debaters do not observe each other's bids, but only the revealed positions.

We assume that the utility of player i from the game is

$$-|x_i - y| - \sum_{h=1,2} b_i^h$$

(where the results can be extended to other single peaked utilities). We will focus on continuous bidding functions, and on "double" symmetric equilibria, i.e., equilibria that satisfy (i) $b_i(x) = b_j(x)$; (ii) $b_i(x) = b_i(-x)$. Finally, we make a slight modification to the model which allows us to insure equilibrium existence in the sequential game for continuous types. Instead of assuming that a player can "yield", we allow each player to place a costless bid β_i^t. This bid will determine who wins slot $h \in \{t, k\}$ when the costly bids are equal. The rest of the model remains the same.

There are two key changes from the model with complete information. First, when a player is exposed, she loses information rents as she reveals all information about herself. Second, the ideal policy of a player is also linked with her desire to win. Specifically, a player with a more extreme ideal policy is more keen to win relative to a moderate player, as the extremist's utility from losing to another (unknown) player is lower than the moderate's. As an illustration of this point, note that in a one period debate, in which only one time/attention slot is available and only one costly bid is allowed, then:

Proposition 4 *In the game with private information, there is a unique equilibrium in a one-round debate, in which b(0)=0 and the costly bidding function b(x) strictly increases in* $|x|$. *Thus, the player with the more extreme ideal policy wins.*

The intuition for this result is the following. The closer is a player to the mean of the distribution of the alternatives, which is at 0, the less she is afraid of losing to the other, unknown, debater. Thus, moderate players are less willing to bid highly. Conversely, the intensity of preferences of extremists is higher, and they are willing to fiercely compete in the bidding wars. Extremists are therefore more likely to win.

This model embeds the trade-off we have discussed above in Section 4.1. Moderates are more popular, but extremists have a higher willingness to win the debate. We find that the results obtained in the complete information model continue to hold in this model as well.

PROPOSITION 5 *In the game with private information, (i) in the unique equilibrium of the sequential debate, first period costly bids are zero for all players, costless bids are decreasing in the distance of players from the decision maker, and there are no bids in the second round. (ii) in the sequential debate the more moderate debater always wins; (iii) in the simultaneous debate, there is a strictly positive probability that an extreme debater wins against a moderate opponent.*

Consider the sequential debate and suppose we are in the second stage when a debater, say debater 2 with some position x_2, was exposed. In this round, debater 1 submits a positive bid if her type is in $[-x_2, x_2]$. In particular, there exists a type of debater 1 (namely $-x_2$) whose willingness to pay to win the debate equals the maximum willingness of debater 2 to win over this set of types. This game is therefore analogous to the game in which both players have the *same willingness to win.* As we show in the appendix, the expected utility of the exposed player is her utility from losing to all players who are moderate than herself. She is therefore indifferent between exposing her position and losing outright to more moderate players. On the other hand, as in the complete information model, a player strictly prefers to win outright against more extreme players (or the less advantageous). If she wins outright she will face no additional bids from these players whereas otherwise she will have to engage in a bidding war in the second round.

These features imply that results in the sequential debate are analogous to the ones in the complete information (and symmetric valuations) case, as we specify in (i) and (ii). Thus, even though a player does not know in advance whether she is the more moderate one, information is fully aggregated and the more moderate player wins the debate. The simultaneous model, as before, does not allow the more extreme players to commit to participate less fiercely in the debate, and hence extreme players can win with some positive probability.

5 Conclusion

In our paper we have suggested a model of debates that is based on the scarcity of time and attention of decision makers. This scarcity creates a competition between debaters to outbid each other in the race to win slots on the agenda. We have highlighted how sequential debates outperform simultaneous debates. In particular, we have shown that under sequential debates, alternatives that are favoured by the decision maker have a higher probability of securing a slot on the agenda.

Our framework could be useful for the analysis of other applications beyond the study of debates. One such example is the timing of the introduction of new products to the market by firms producing goods with different qualities. One firm's marketing decisions are affected by the uncertainty about other firms' strategies, and are often adjusted when rival firms have already been successful in introducing their products. Our model can shed light on how these considerations affect the quality of the goods that are introduced to consumers.

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Appendix

(i) A useful Lemma

Lemma A1 *Consider two bidders competing in a one-round all-pay-auction with bids in* $[0,\infty)\cup\{yield\}$. *Assume that bidder 1's valuation from winning is* V_1 *and that bidder's 2 valuation is* V_2, *that valuations are common knowledge, and that without loss of generality* $V_1 \geq V_2$. *The unique equilibrium has bidder 1 mixing uniformly on* $[0,V_2]$, *and bidder 2 mixing according to* $f(b)=\frac{1}{V_1}$ *on* $[0,V_2]$ *with an atom of on* 1-$\frac{V_2}{V_1}$ *on the zero bid. Bidder 2 wins with probability* $\frac{V_2}{2V_1}$.

Proof of Lemma A1 We first establish that players cannot play pure strategies. Note that no player can lose with probability 1 with a costly bid. Hence, players either tie or one player loses with a zero bid or yields. But if a player loses with a zero bid then the winning player cannot win with any positive bid. If the losing player yields, the winning player must place a zero bid which implies that yielding is not a best reply. Thus players must tie. But then, as long as $b<V_2$, each player can deviate to $b+\varepsilon$ for a small enough ε and win with probability 1 rather than with a lower probability. On the other hand players cannot tie with $b \geq V_2$ as in this case bidder 2 would rather lose.

Consider now mixed strategies. We now establish that: (i) there are no gaps in the support, the support includes zero; (ii) both players cannot place atoms at zero; (iii) both players must have the same support, $[0,V_2]$; (iv) there are no atoms but possibly at zero.

(i) Suppose there is a gap in the density, i.e., player i does not place bids in (b',b''). In this case, player j should not place bids on (b',b'') as well. In particular, player j cannot place an atom on b''; if i does not place an atom on b'' then $j's$ utility converges to be higher on $b' + \varepsilon$ than on $b'' + \varepsilon$, and if i places an atom on b'' then $j's$ utility is higher from a bid $b'' + \varepsilon$ than b''. As j does not place bids in (b', b'') and has no atom on b'' it must be that the utility of i from $b'' + \varepsilon$ must converge to be lower than her utility from $b' + \varepsilon$, a contradiction. One can repeat the same argument with $b' = 0$ to conclude the infimum bid in the support must be zero.

(ii) By arguments as in (i), both players cannot have atoms at zero, as then for at least one player a bid of ε provides a higher utility.

(iii) From (i), both players must have zero in their support. As at least one player loses for sure with a bid of zero (as by (ii), only one can place an atom), then the maximum bid in the support must give this player the utility from losing. Note that by arguments similar to (i), both players have the same maximum bid. As the highest bid must be weakly lower than V_2, we therefore establish that the maximum bid is equal to V_2. As there are no gaps, both players have the same support.

(iv) We now show that the densities do not include any atoms for positive bids. Suppose player i places an atom on some b. Then, as above, it cannot be that player j places an atom on b. As there are no gaps and the support is the same, player j must place some bids below b and some above b. Thus player j's utility converges to be higher at $b + \varepsilon$ than at b, a contradiction. If an atom is placed on the maximum bid, then if player 2 places the atom then player 1 can have the same deviation as above, whereas if player 1 places the atom, then the utility of player 2 from $V_2 - \varepsilon$ converges to be lower than the utility from a bid zero, a contradiction.

Finally, we find the equilibrium strategies. Suppose that bidder i uses some $f_i(b)$.

Maximization implies:

$$\max_b F_2(b)V_1 - b \quad \rightarrow \quad f_2(b) = \frac{1}{V_1}, \ \forall b \in (0, V_2]$$
$$\max_b F_1(b)V_2 - b \quad \rightarrow \quad f_1(b) = \frac{1}{V_2}, \ \forall b \in (0, V_2]$$

Bidder 1 is therefore using a uniform strategy, whereas bidder 2 mixes uniformly over $[0,V_2]$ and places an atom on 0 of $1 - \frac{V_2}{V_1}$. Bidder's 2 expected utility is 0 and bidder's 1 expected utility is $V_1 - V_2$. The probability of winning for player 1 is

$$1 - \frac{V_2}{V_1} + \int_0^{V_2} \frac{b}{V_2 V_1} db = 1 - \frac{V_2}{2V_1}$$

and the probability of player 2 winning is therefore $\frac{V_2}{2V_1}$.■

(ii) Proofs for Section 3

Proof of Lemma 1 follows from Lemma A1.

Proof of Lemma 2 As explained in the text, and by arguments similar to the ones in Lemma A1, the equilibrium must involve player 1 submitting one positive bid and one zero bid. The positive bid is placed either on slot 1 or on slot 2, on each with a strictly positive probability. For each slot k, the positive bid, if placed, is drawn from a continuous and atomless distribution $f_1^k(.)$ on $[0, \frac{v}{2}]$. Player 2 on the other hand places two bids, each at one slot. The bids for both slots are drawn from some continuous and atomless distributions $f_2^k(.)$ on $[0, \frac{v}{2}]$ that might be correlated.

To characterize the equilibrium behaviour, consider first the utility of player 1. Let $\tilde{F}_2(b^1, b^2)$ be the probability that either $b_2^1 < b^1$ or $b_2^2 < b^2$ (where we use the convention that $yield < 0$). Player 1's utility from submitting bids b_1^1 and b_1^2 is then

$$\tilde{F}_2(b^1, b^2)v - (b_1^1 + b_1^2)$$

Remember that player 1 submits one strictly positive bid, b, and must be indifferent between slots 1 and 2. To be willing to mix between the two slots, player 1's utility from placing the positive bid at each slot must be equal for any b in the support:

$$\tilde{F}_2(b,0) = \tilde{F}_2(0,b).$$

For player 1 the first order conditions are:

$$\begin{aligned} \frac{\partial \tilde{F}_2(b,0)}{\partial b^1} v &= 1, \\ \frac{\partial \tilde{F}_2(0,b)}{\partial b^2} v &= 1. \end{aligned}$$

And this holds for all $b \in [0, \frac{v}{2}]$, for player 1 to be indifferent.

Therefore we have that

$$\tilde{F}_2(b,0) = \tilde{F}_2(0,b) = \frac{b}{v} + \frac{1}{2}$$

Player 2's utility from (positive) b_2^1 and b_2^2 is:

$$\alpha F_1^1(b_2^1)v + (1-\alpha)F_1^2(b_2^2)v - (b_2^1 + b_2^2)$$

where α is the probability that player 1 places her positive bid in slot 1. The first order conditions are

$$\alpha f_1^1(b_2^1))v = 1; \ (1-\alpha)f_1^2(b_2^2))v = 1$$

This must hold for any b_2^1 and b_2^2 in the support, and as player 1 cannot have any atoms in her distribution, this implies that $\alpha = \frac{1}{2}$ and $f_1^1(b) = f_1^2(b) = \frac{2}{v}$.

Finally the probability of player 2 winning is

$$\int_0^{\frac{v}{2}} [\frac{1}{2}(1 - \tilde{F}_2(b,0)) + \frac{1}{2}(1 - \tilde{F}_2(0,b))]\frac{2}{v} db = \frac{1}{4}. \blacksquare$$

(iii) Proofs for Section 4.1

PROOF OF LEMMA 4: If player 1 is exposed, in the second stage no one bids any positive bids (as before). When x_2 was exposed, whoever wins the stage wins the debate. By Lemma A1, the expected utility of player 1 from this stage is 0 and that of player 2 is $(1 - \frac{v_1}{v_2})v_2 = v_2 - v_1$. This implies that the expected utility of player 1 from winning the first round (i.e., exposing her position rather than that of player 2) is v_1 whereas the expected utility of player 2 from winning the first round is $v_2 - v_1$. By Lemma A1, the equilibrium in the first round is a mixed strategy equilibrium on $[0, \min\{v_1, v_2 - v_1\}]$. As $\min\{v_1, v_2 - v_1, v_2\} > 0$, and there is mixing in each stage with positive probability, the probability of player 2 winning the debate is strictly positive. Also, with strictly positive probability players place strictly positive bids in the first round.■

PROOF OF PROPOSITION 2:

Case 1: $v_1 < v_2$.

Consider the sequential debate. By Lemma A1, the probability that player 2 wins the debate in the second stage is $1 - \frac{v_1}{2v_2}$. By Lemma A1 and Lemma 4, the probability that player 2 wins the first round is $\frac{v_2 - v_1}{2v_1}$ if $v_1 > v_2 - v_1$ and $1 - \frac{v_1}{2(v_2 - v_1)}$ otherwise. To win the debate, player 2 needs to win both rounds. The probability of this is

$$\Pr(2\text{wins}|\text{seq}) = \begin{cases} \frac{(2v_2 - v_1)(2v_2 - 3v_1)}{4v_2(v_2 - v_1)} & \text{if } \min\{v_1, v_2 - v_1\} = v_1 \\ \frac{(v_2 - v_1)(2v_2 - v_1)}{4v_2 v_1} & \text{if } \min\{v_1, v_2 - v_1\} = v_2 - v_1 \end{cases} \tag{1}$$

We now consider the simultaneous debate. The equilibrium is analogous to the one described in Lemma 2, and the solutions can be derived as in the proof of Lemma 2.

Specifically, player 1 submits one positive bid, drawn from the distribution $f_1(.) = \frac{2}{v_2}$ on $[0, \min\{\frac{v_2}{2}, v_1\}]$ with an atom $F_1(0) = 1 - \frac{2v_1}{v_2}$ when $\min\{v_2 - v_1, v_1\} = v_1$, and one zero bid. Her strictly positive bid is placed on each slot, 1 or 2, with equal probability. Player 2 places two bids. The level of each bid is drawn from a joint distribution which is represented by $\tilde{F}_2(b^1, b^2)$ as in Lemma 2, and which satisfies $\tilde{F}_2(0, b) = \tilde{F}_2(b, 0) = \frac{b}{v_1} + C$ on $[0, \min\{\frac{v_2}{2}, v_1\}]$ with an atom $C = 1 - \frac{v_2}{2v_1}$ when $\min\{v_2 - v_1, v_1\} = v_2 - v_1$ and $C = 0$ otherwise.

The probability of player 2 winning is:

$$\Pr(\text{2wins}|\text{sim}) = \begin{cases} 1 - \frac{v_1}{v_2} & \text{if } \min\{v_1, v_2 - v_1\} = v_1 \\ \frac{v_2 s}{4v_1} & \text{if } \min\{v_1, v_2 - v_1\} = v_2 - v_1 \end{cases} \tag{2}$$

Comparing (1) to (2) for each of the two possible cases, we have that the probability that player 2 wins is lower in the sequential case: (i) if $\min\{v_1, v_2 - v_1\} = v_1$, then $\Pr(\text{2wins}|\text{sim}) - \Pr(\text{2wins}|\text{seq}) = \frac{1}{4v_2}\frac{v_1^2}{v_2 - v_1} > 0$; (ii) if $\min\{v_1, v_2 - v_1\} = v_2 - v_1$, then $\Pr(\text{2wins}|\text{sim}) - \Pr(\text{2wins}|\text{seq}) > 0 \Leftrightarrow (v_2 - v_1)^2 - v_2 v_1 < 0$ which holds as $(v_2 - v_1)^2 - v_2 v_1 < v_1^2 - v_2 v_1 = v_1(v_1 - v_2) < 0$. Moreover, note that all these probabilities converge to 1 when $\frac{v_2}{v_1} \to \infty$.

Case 2: $v_1 \geq v_2$.

It is easy to see that by Lemma A1, when x_2 is exposed in the second round, her utility is zero. Thus, the equilibrium analysis of the sequential debate remains as in the case of $v_1 = v_2 = v$, which implies that player 1 wins with probability one. It remains to show that player 2 has a positive probability of winning in the simultaneous debate. The equilibrium there, similar to the case above, involves the players submitting bids in

$[0, \frac{v_2}{2}]$. Player 1 draws this bid from $f_1() = \frac{2}{v_2}$, submits only one bid and mixes between placing it in slot 1 and slot 2, whereas player 2 draws her bids from a joint distribution represented by $\tilde{F}_2(b^1, b^2)$ as in Lemma 2, and which satisfies $\tilde{F}_2(0, b) = \tilde{F}_2(b, 0) = \frac{b}{v_1} + C$ with $C = 1 - \frac{v_2}{2v_1}$. The probability of player 2 winning is therefore $\int_0^{\frac{v_2}{2}} (\frac{v_2}{2v_1} - \frac{b}{v_1}) \frac{2}{v_2} db = \frac{v_2}{4v_1} > 0.$■

(iv) Proofs for Section 4.2

Proof of Proposition 3 The sequential case was established in the text. Consider the simultaneous game. It is easy to see that there is no equilibrium in which player 1 wins with probability one. In that case, all other players would have to yield and player 1 must place a bid of zero. But then any of the other players can place a bid of ε in all K slots and win the debate, a contradiction.■

(v) Proofs for Section 4.3

Proof of Proposition 4:[10] (i) Uniqueness: it is easy to see that there is no interval with positive measure in which the costly bidding function is neither increasing not decreasing. If this is the case, then we can find an infinitesimally small ε, so that any x in this interval can increase her bid by ε and her probability of winning by a positive probability and hence her utility.

We now show that there cannot be $x'' > x' > 0$ with $b(x)$ decreasing over $[x', x'']$. In such a case, let $X = \{x \notin [x', x''] \cup [-x'', -x'] | b(x) \in [b(x''), b(x')]\}$. Consider the

[10]With some slight modifications, the proof below holds for any concave loss function $V(|x - y|)$.

incentive compatibility constraint for both x' and x''. These satisfy:

$$\begin{aligned}\int_X |v-x'|g(v)dv + \int_{x'}^{x''} |v-x'|g(v)dv) + \int_{-x''}^{-x'} |v-x'|g(v)dv &\geq b(x') - b(x'')\\ \int_X |v-x''|g(v)dv + \int_{x'}^{x''} |v-x''|g(v)dv) + \int_{-x''}^{-x'} |v-x''|g(v)dv &\leq b(x') - b(x'')\end{aligned}$$

Note that $\int_{x'}^{x''} |v-x'|g(v)dv = \int_{x'}^{x''} |v-x''|g(v)dv$ and that $\int_{-x''}^{-x'} |v-x'|g(v)dv \leq \int_{-x''}^{-x'} |v-x''|g(v)dv$. Finally, note also that by symmetry of the equilibrium bidding function,

$$\int_X |v-x''|g(v)dv = \int_{X\cap[0,1]} 2vg(v)dv = \int_X |v-x'|g(v)dv,$$

a contradiction to the IC constraints.

(ii) Existence: The following increasing bidding function

$$b(x) = \int_0^x 2vg(v)dv.$$

satisfies the incentive compatibility constraints:

$$\int_{x'}^{x''} -(|v-x''| + |x''+v|)g(v)dv \leq b(x'') - b(x') \leq \int_{x'}^{x''} -(|v-x'| + |x'+v|)g(v)dv. \blacksquare$$

Proof of Proposition 5 (i)+(ii) We start by proving a preliminary Lemma, which characterizes the equilibrium in the second round of the sequential debate.

Lemma A2 *In the second round: (i) The exposed player x must play a mixed strategy, which includes a bid of zero in its support; (ii) the unexposed player places a strictly positive bid if her type is in (-x,x), where bids are strictly increasing in her distance from x, and a zero bid otherwise.*

Proof of Lemma A2: Note first that unexposed players at $[-x, x]$ are the only ones that can potentially bid positive amount as they are the only ones that can win. Suppose that player x is exposed. Let $b_x(b)$ be the distribution over bids that x plays in equilibrium. Let $\underline{b} = \inf b_{b|b_x(b)>0}$.

We first show that $\underline{b} = 0$. Suppose that $\underline{b} > 0$. Consider the unexposed player: None of her types place bids in $(0, \underline{b})$. Moreover, there cannot be an atom of types that bid $\underline{b}$. If there was, these types should bid zero instead unless $b_x(b)$ places an atom on $\underline{b}$. The latter cannot arise in equilibrium as x loses (with a strictly positive probability) when bidding $\underline{b}$, against players in $[-x, x]$ who bid the same amount. Hence the utility of x from bidding some $\underline{b} + \varepsilon$ has to converge to be strictly lower than bidding ε for $\varepsilon \to 0$.

We now show that $b_x(b)$ does not include any atoms. Suppose it places an atom on some b. Then it cannot be that there is an atom of unexposed types who place bid b (as above). Note that some unexposed types must bid below b and some above it. Moreover, the bidding function of the unexposed player is continuous in types and that of the exposed has no gaps (proof similar to the above). Thus an unexposed player who bids b can, for a small enough ε, bid $b + \varepsilon$ and increase her probability of winning by a strictly positive probability. Hence, $b_x(b)$ is a continuous function.

Now consider the best response of an unexposed player $y \in [-x, x]$. Her expected utility from a bid b is (where $\bar{b} = \sup b_{b|b_x(b)>0}$):

$$\int_b^{\bar{b}} -|y - x| b_x(b) db - b$$

and the first order condition is

$$b_x(b(y))|y - x| - 1 \tag{3}$$

Note that whenever the first order condition is satisfied, then $b(y)$ has to be monotone, as the equation (3) cannot be satisfied both for y_1 and y_2 with $b(y_1) = b(y_2)$. As it is continuous in the distance of types from the exposed, and as players who do not satisfy the first order condition will pool at zero, and as the player at $y = x$ or players at $y \to x$ must converge to bid zero, we have that $b(y)$ is monotonically decreasing.

Suppose that some players in $[-x, x]$ bid zero. It can be easily shown that these must be in $[z, x]$ for some z. Consider now the exposed player's expected utility:

$$\int_{-x}^{b^{-1}(b)} -|y - x|g(y)dy - b$$

and her first order condition is

$$b'(y) = -|y - x|g(y)$$

Consider now condition (3). We know that $1 = \int_0^{\bar{b}} b_x(b)db = -\int_{-x}^{z} b_x(b(y))b'(y)dy \leq -\int_{-x}^{x} b_x(b(y))b'(y)dy = -\int_{-x}^{x} \frac{1}{|x-y|}|y - x|g(y)dy < 1$, which implies that there must be an atom in $b_x(b)$, a contradiction as both cannot sides cannot have atoms. Thus, z must equal to x.■

Lemma A2 implies that the expected utility of an exposed player x is her utility from losing to a more moderate player who was not exposed. We now show the implications of this Lemma to the equilibrium in the first round.

LEMMA A3 *In the first round, there is no interval of types in which the costly bidding function (weakly) increases in the distance of a player from the DM.*

PROOF OF LEMMA A3 Consider an equilibrium and some $0 < x' < x''$ such that $b(x)$ increases over $[x', x'']$. Note that switching from a bid $b(x')$ to a bid $b(x'')$ implies that a player can win in the first round against players whose bids are in $[b(x'), b(x'')]$, and that these players are either less extreme than x', both more moderate than x'' and more extreme than x', or more extreme than x''. Note that for all those who are less extreme than x', for both x' and x'', winning against them now gives the same utility as losing to them now, and hence the bids do not affect their utility from these players. Regarding players who are more moderate than x'' but more extreme than x', it is clear that x' benefits more than winning against them now rather than later compared with x'' (the gain for x'' is zero whereas the gain for x' is positive as she will avoid a bidding war in the next stage). Thus, according to these types considered, x' should pay more than x'' to be exposed in the first stage.

We now consider the last category of players, those who are more extreme than x''. Recall that the equilibrium is symmetric. The benefit of some $x < x''$ from winning against a pair of players $v, -v$ with $|v| > x''$, vs. losing to these players in the first round is

$$\begin{aligned} \Pr(b^2(x|v^*) &< b^2(v^*))|v-x| + b^2(x|v^*) \\ + \Pr(b^2(x|v^*) &< b^2(-v^*))|x+v| + b^2(x|-v^*) \end{aligned}$$

where the superscript $*$ denotes that the player v is exposed, $b^2(x|v^*)$ is the bid of player x when v is exposed, and $b^2(v^*)$ is the bid of v when she is exposed. Taking a derivative w.r.t. x, and using the envelope theorem, the above expression is

$$-\Pr(b^2(x|v^*) < b^2(v^*)) + \Pr(b^2(x|-v^*) < b^2(-v^*)).$$

This expression is negative, as x bids higher against v than against $-v$ (by Lemma A2) whereas v and $-v$ behave in the same way when they are exposed. This implies that for all pairs of v who are more extreme than x'', the benefit of x' from exposing herself is higher than the benefit of x'', which implies that x'' cannot pay more than x', a contradiction.■

Lemma A4 *In the first round, the costly bidding function cannot strictly decrease in the distance of a player from the DM.*

Proof of Lemma A4: Suppose it is. Consider a deviation of the median player 0 from $b(0)$ to $b(1)$. The other player, who is necessarily exposed, will believe that there is no player who is more moderate than her. This means that the exposed player will exert no resources in the second round. The median can then spend some small ε and win attention in the second round, and win the debate at a cost $b(1) + \varepsilon < b(0)$.■

Lemma A5: *In any equilibrium that involves zero costly bids in the first round, costless bids must strictly decrease in the distance of players from the decision maker.*

Proof of Lemma A5: Suppose instead that the equilibrium has strictly increasing bids at some interval (x', x''). This implies that a type x' loses the first round against more extreme types. However, she is better off winning against these types than engaging in a bidding war and can do so at no costs in the first stage, a contradiction.■

We have shown that the equilibrium must involve zero costly bids, and that the costless bids must be decreasing in the distance of a player from the DM. It is left to show that such an equilibrium exists. The upward constraint is trivially satisfied as a player is indifferent regarding losing now or losing later to more moderate players. On the other hand, the downward constraint is strictly satisfied as the more moderate players prefer to win now than win later and exert costly bids. In terms of costless bids therefore, no player can improve her position. Moreover, given that all other players are using zero costly bids and the described costless bids, no player can improve her position by increasing her costly bid. A player does not want to increase her costly bid as this would mean that she will have the utility of losing in the second round to more moderate types, which is what happens in the current equilibrium without any costs.

This completes the proof of (i)+(ii).

Consider now the simultaneous game. Suppose that the equilibrium is such that the more moderate wins with probability 1. It must be that $b^k(1) = b^k(-1) = 0$ for both slots as these types know they always lose, on equilibrium path, so they should not spend any resources. But continuity and the symmetry of the strategies implies that there an $\underline{x} > 0$ such that $b^k(x)$, $k = 1, 2$, is strictly decreasing (increasing) for $x > \underline{x}$ ($x <$ -$\underline{x}$) or $b^k(x) = 0$ for all $x > \underline{x}$ and all $x <$ -$\underline{x}$. If the former or the latter possibilities hold for both $k = 1$ and $k = 2$, there is either wasteful spending in the former case or a deviation by player 1 who can win against all those in $[-1, -\underline{x}] \cup [\underline{x}\ , 1]$ by bidding a small bid in both slots, guaranteeing a strictly positive utility which contradicts our premise. Consider now the case in which in one slot, say slot 1, $b^1(x) = 0$ for all $x > \underline{x}$ and all $x <$ -$\underline{x}$ and in slot 2, $b^2(x)$ is strictly decreasing (increasing) for $x > \underline{x}$ ($x <$ -$\underline{x}$). In this case the debate for these types is determined in slot 2. Consider two types, x and x' such that $x > x' > \underline{x}$. Under our assumption, $b^2(x') > b^2(x)$ but this is not possible as x is more willing to mimic x' than x' is to mimic x; such deviations only affect the competition against types in $[-x, -x'] \cup [x', x]$, types which x is more willing to bid against.

This concludes the proof of (iii) and of Proposition 5.■