ADVANCED ECONOMIC THEORY
Problem Set 3

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1. **Stationary cardinal utility - Axiomatic foundations for the discounted utility functional form.**

   Using class notation,

   (a) Give examples illustrating that Axiom 2 (stationarity) $\nRightarrow$ Axiom 3 (intertemporal independence) and Axiom 3 $\nRightarrow$ Axiom 2.

   (b) Show that Axioms 1-4 are equivalent to the existence of the presentation discussed in class where u and v are *continuous*.

   (c) Show that if $y_c \succ_Y y_{c'}$ then $(c, c', y) \succ_Y (c', c, y) \quad \forall y \in Y$. Assume u and v of the presentation theorem are differentiable and define

   $$1 + \rho(c) \equiv \frac{\partial U/\partial c_0}{\partial U/\partial c_1}(y_c).$$

   What is $\rho(c)$ in terms of u and v? What is the economic interpretation of ρ?

2. **A Cue Theory of Consumption - Filling the gaps.** Using class notation,

   (a) Show that the threshold value $\hat{x}$ corresponding to the No-cue Bellman equation is increasing with the discount factor δ, the outside option ξ, and the compensatory process weighting factor $1 - \alpha$.

   (b) Show that indeed

   $$V(x^R, x^G) = \frac{\mu^R}{1 - \delta\mu^G} W(x^R : \tilde{\delta} = \frac{\delta\mu^R}{1 - \delta\mu^G}) + \frac{\mu^G}{1 - \delta\mu^R} W(x^G : \tilde{\delta} = \frac{\delta\mu^G}{1 - \delta\mu^R}).$$

3. **Doing it now or later - welfare in a quasi-hyperbolic world.**

Using class notation, denote a person's *long-run utility* of performing the activity at time t by: $U^0(t) \equiv v_t - c_t$. We'll use that for welfare analysis. We will restrict our attention to the set $\Sigma^M = \{(v,c) : v_t, c_t \leq M\}$ for some positive real M.

a. Suppose costs are immediate. Show that:

1. $\lim_{b \to 1} (\sup_{\Sigma^M} [U^0(t_{TC}) - U^0(t_S)]) = 0;$

2. For any $b < 1$, $\sup_{\Sigma^M} [U^0(t_{TC}) - U^0(t_N)] = 2M.$

Are **all** TC period selves necessarily weakly better off than **all** N and S period selves (global Pareto criterion)?

b. Suppose rewards are immediate. Show that:

1. $\lim_{b \to 1} (\sup_{\Sigma^M} [U^0(t_{TC}) - U^0(t_N)]) = 0;$

2. For any $b < 1$, $\sup_{\Sigma^M} [U^0(t_{TC}) - U^0(t_S)] = 2M.$

Are **all** TC period selves necessarily weakly better off than **all** N and S period selves?

4. **The Social Discount Rate**

Assume the utility of an agent at time t =0,1,2,…, T is given by:

$$U_t(c_0, c_1, ...c_T) = \sum_{m=1}^{t} d^m u(c_{t-m}) + u(c_t) + \sum_{n=1}^{\infty} b^n u(c_{t+n})$$

where 0 ≤ β, δ ≤ 1, and u is increasing and concave.

Assume policy choices $c^T = (c_0, c_1, ..., c_T)$ are subject to a budget constraint:

$\sum_{t=0}^{T} \frac{c_t}{(1+r)^t} \leq W$ where r is the rate of return on savings and W is the initial endowment.

The social planner considers a weighted average of the different selves' utilities:

$$S(U_0, U_1, ..., U_T) = \sum_{t=0}^{T} a_t U_t \text{ where } a_t \geq 0 \text{ for all } t.$$

a. Show that maximizing S under the budget constraint is equivalent to maximizing

$\sum_{t=0}^{T} q_t u(c_t)$ where $q_t = \sum_{s=0}^{t-1} a_s b^{t-s} + a_t + \sum_{s=t+1}^{T} a_s d^{s-t}$.

b. Show that if $d \leq b^{-1}$ then for all $t \in \{0,1,\ldots,T-1\}$, $\frac{q_{t+1}}{q_t} \geq b$. When is the inequality strict?

c. Let $s_t = \frac{u'(c_{t+1})}{u'(c_t)}$. Show that for the social planner's policy, if $d < b^{-1}$, $\frac{ds_s}{da_t} \leq 0$ for $s < t$ and $\frac{ds_s}{da_t} \geq 0$ for $s \geq t$. What's the intuition for this observation?