ADVANCED ECONOMIC THEORY
Problem Set 2

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1. Assume there are two possible states of the world: $Z = \{G, I\}$, $\Pr(G) = \alpha$ and $\Pr(I) = 1 - \alpha$.

   Each player $1, ..., n$ receives a (conditionally independent) signal in $\{i, g\}$:

$$\begin{array}{ll} \Pr(g \mid G) = p, & \Pr(i \mid G) = 1 - p \\ \Pr(g \mid I) = 1 - p, & \Pr(i \mid I) = p \end{array}.$$

   After observing their signals, players each vote for one of two actions: $V = \{a, c\}$ (no abstention is allowed).

   The final outcome in $X = \{A, C\}$ is determined by a $r-$voting rule:

$$x_r(v_1, ..., v_n) = \begin{cases} A & \#\{i \mid v_i = c\} < r \\ C & \#\{i \mid v_i = c\} \geqslant r \end{cases}.$$

   The utility of each player is given by the common utility $U : X \times Z \to \mathbb{R}$:

$$U(C, G) = U(A, I) = 0,\ U(C, I) = -q,\ U(A, G) = -(1 - q),$$

   where $0 < q < 1$.

   (a) If players vote sincerely, calculate the probability of convicting the innocent (i.e., $\Pr(x_r = C \mid z = I)$) and acquitting the guilty (i.e., $\Pr(x_r = A \mid z = G)$) under majority rule ($r = \lceil \frac{n}{2} \rceil$) and under unanimity ($r = n$).

   (b) Under sincere voting, which rule, majority or unanimity, produces a higher probability of convicting the innocent? What does that imply regarding the preferred voting rule for sufficiently high values of q?

Assume henceforth that $\alpha \geqslant q$.

(c) If players vote sincerely, would there be complete aggregation of information under majority rule? Under unanimity?

(d) Assuming players are fully strategic, characterize the symmetric equilibrium strategies when $r = n$ and when $r = \lceil \frac{n}{2} \rceil$.

(e) When voters are strategic, which rule, majority or unanimity, produces a higher probability of convicting the innocent?

(f) Can information aggregation be complete for any finite n for either voting rule when voters are fully strategic?

2. Consider the following two-person game (Moulin and Vial (1978)):

	L	M	R
T	0,0	1,5	5,1
I	5,1	0,0	1,5
B	1,5	5,1	0,0

(a) Identify the Nash equilibria of this game.

(b) Find the correlated equilibrium that maximizes the sum of players' payoffs (the most efficient correlated equilibrium).

(c) Consider a situation in which after Player 2 chooses his action $c \in \{L, M, R\}$, Player 1 observes a signal $\in$ {"Not L", "Not M", "Not R"}. Let $q(c)$ denote the probability distribution over signals that Player 1 receives when Player 2 chooses action c.

$$
\begin{array}{lll}
q(L)[\text{Not } L] = 0 & q(L)[\text{Not } M] = 1/2 & q(L)[\text{Not } R] = 1/2 \\
q(M)[\text{Not } L] = 1/2 & q(M)[\text{Not } M] = 0 & q(M)[\text{Not } R] = 1/2 \\
q(R)[\text{Not } L] = 1/2 & q(R)[\text{Not } M] = 1/2 & q(R)[\text{Not } R] = 0.
\end{array}
$$

This signaling system allows Player 1 to rule out one of the actions that Player 2 did not choose before choosing her own action. Identify the (Bayesian) equilibria of this game. Is this a special case of a game with communication?

3. **The Revelation Principle**

Consider the game $\Gamma = (N, (C_i)_{i\in N}, (u_i)_{i\in N})$. Using class notation, a communication system is identified by a function $v : \Pi_{i\in N}R_i \to \Pi_{i\in N}M_i$, where R_i and M_i are the sets of reports and messages of player i, respectively. This structure induces a communication game Γ_v (defined in class).

Let $\sigma = (\sigma_i)_{i\in N}$ be any equilibrium in randomized strategies of Γ_v.

(a) Identify the correlated strategy $\mu : C \to [0, 1]$ corresponding to σ.

(b) Show that μ is a correlated equilibrium of Γ.

4. Consider the following three player game:

	L	R
T	$0, 0, 3$	$0, 0, 0$
B	$1, 0, 0$	$0, 0, 0$

A

	L	R
T	$2, 2, 2$	$0, 0, 0$
B	$0, 0, 0$	$2, 2, 2$

B

	L	R
T	$0, 0, 0$	$0, 0, 0$
B	$0, 1, 0$	$0, 0, 3$

C

(Player 1 chooses on of the two rows, Player 2 chooses one of the two columns, and Player 3 chooses one of the three tables).

(a) Show that the pure strategy equilibrium payoffs are $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 0)$.

(b) Show that there is a correlated equilibrium in which Player 3 chooses B and Players 1 and 2 play (T, L) and (B, R) with equal probabilities.

(c) Explain the sense in which Player 3 prefers not to have the information that Players 1 and 2 use to coordinate their actions.