ADVANCED ECONOMIC THEORY

SOCIAL SCIENCE 211C

PROBLEM SET 1 - DUE 4.21.2005

1. <u>Preference for flexibility – completing the proof of Theorem 1</u>. Using class notation,
   a. Show that there exist a real-valued function $a: X \to \Re$ (a negative) such that

   $$x' \succsim x'' \text{ if and only if } \sum_{\{x: x \supseteq x'\}} a(x) \geq \sum_{\{x: x \supseteq x''\}} a(x)$$

   (use an inductive process to define a on equivalence classes of $\succsim$, i.e. sets of the form $\{x \mid x \sim x^*\}$).

   b. Show that if Z is finite, then a binary relation $\succsim$ on X satisfies Axioms 1-3 only if there exists a finite set S and a function $U: Z \times S \to \Re$ such that $v(x) \equiv \sum_{s} [\max_{z \in x} U(z,s)]$ represents $\succsim$.

2. <u>Iterated elimination of weakly dominated strategies</u>.

   a. Define a weakly dominated action in a two-person game.

   b. Consider the following game:

	L	R
T	1, 1	0, 0
M	1, 1	2, 1
B	0, 0	2, 1

   and show that the set of actions that survive a process of iterated elimination of weakly dominated strategies may depend on the order in which actions are eliminated.

3. <u>Iterated elimination of weakly dominated actions in the guessing game</u>. Consider the standard guessing game in which $N = 1$ players have to state simultaneously a number in the interval $[0, 100]$. The winner is the player whose chosen number is closest to the mean of all chosen numbers multiplied by p, $p > 0$, which is common knowledge. The payoff to the winner is a fixed amount $p > 0$, which is divided between the winners upon a tie. Players who do not win receive nothing.
   a. 1. What is the set of strategies that survive the procedure of infinitely repeated simultaneous elimination of weakly dominated strategies when $p < 1$?
      2. What is the set of strategies that survive the procedure of infinitely repeated simultaneous elimination of weakly dominated strategies when $p > 1$? What is the set of pure equilibria of this game?

b. What is the set of pure equilibria of the game described above when $p < 1$, and when $p > 1$?

c. Explain how pre-play communication affects the set of equilibria for $p < 1$ and $p > 1$.

4. <u>Iterated Dominance and Cognitive Sophistication</u>

A population of row and column players engages in the following one-shot game:

	Left	Middle	Right
Top	1,2	1,0	3, 3
Bottom	1,1	1,1	1 - a, 1 – a

where a > 1 and the first and second numbers in each cell denote the row and column players' payoffs, respectively.

a. Identify the actions of both players that survive iterated elimination of weakly dominated actions.
b. Identify the actions of both players that survive iterated elimination of strictly dominated actions.

Assume the population is comprised of three types of players (for both row and column):

Type 0 randomize uniformly across their actions;
Type 1 assume their opponent is type 0 and best respond;
Type 2 assume their opponent is type 0 with probability p and type 1 with probability 1- p.

c. Characterize the choices of type 1 players in the row and column roles.
d. How would type 2 row and column players behave as a function of p and a? When can type 2 behavior be identified from type 1 behavior?
e. An experimenter observes the above game being played one time and tracks the fraction of subjects choosing each of their actions. In particular, she observes a fraction $\hat{b}_T$ and $\hat{b}_B$ of the row players choosing Top and Bottom, respectively. What can she deduce about the percentage of row players of each type?
f. Assume that the distribution of types does not depend on players' roles. Suppose that the experimenter also observes a fraction $\hat{b}_L$, $\hat{b}_M$, and $\hat{b}_R$ of the column players choosing Left, Middle, and Right, respectively. Can she always refine her assessment of the distribution of types from part (e)?