

## Similarity and Polarization in Groups

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**ABSTRACT.** The focus of this paper is the endogenous formation of peer groups. We study a model in which agents choose their peers prior to making decisions on multiple issues. Agents differ in how much they value the decision outcomes on one issue relative to another. While each individual can collect information on at most one issue, all information is shared within the group. Thus, the group's preference composition affects the type of information that gets collected. We characterize *stable groups*, groups that are optimal for all their members. When information is free, stable groups must be sufficiently homogeneous. Furthermore, stability requires more similarity among extremists than among moderate individuals. When information is costly, a free rider problem arises, and makes extreme peers more desirable, as they are more willing to invest in information acquisition. We show that, as information costs grow, polarization appears and becomes increasingly extreme in stable groups.

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## 1. INTRODUCTION

There are many realms in which individuals choose *whom* they interact with, socially and strategically. Individuals choose which clubs to join, internet forums to participate in, neighborhoods to live in, schools to go to, auctions to bid in, and so on. In fact, new technologies such as online social networks, blogs, etc. allow users to choose their peers without any physical or geographical constraint. Interestingly, a vast empirical literature in sociology suggests consistent patterns of group formation. Indeed, agents appear to associate with those similar to them (e.g., in demographics, political opinions, or beliefs).<sup>1</sup> Moreover, in an era in which the Internet penetration rate in North America is 73% (and 87% among teenagers), concerns have been raised about the consequences of unlimited socialization among individuals of similar, and possibly extreme, views.<sup>2</sup>

While, over the decades, the analysis of strategic interactions across domains has received wide attention, theoretically and empirically, the group of agents involved is usually assumed to be determined exogenously.<sup>3</sup> The focus of the current paper is the analysis of an extended game in which, first, agents *choose* their group of peers and, second, a strategic interaction within the group takes place. The goal is to study how the interplay between the group formation stage and the strategic interaction stage determine the properties of the peer groups that arise in equilibrium. In particular, we provide a taxonomy of environments generating groups that consist of similar, heterogenous, or highly polarized agents, in terms of their fundamental characteristics.

We study a model in which agents make individual decisions on two different issues. These issues can be a metaphor for life-decisions on financial investments and education, candidates to vote for in local and general elections, consumption goods to buy for different uses (e.g., food and wine), etc. Individuals' utilities differ in the relative weight they put on each issue. For example, depending on their personal circumstances and demographic characteristics, agents may vary in how much they

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<sup>1</sup>See the literature review for a brief summary of the work on this phenomenon, known as *homophily*.

<sup>2</sup>The leading online social networks contain a large volume of members. As of the beginning of 2009, MySpace had over 125 million members, while Facebook had over 200 million members from all over the world. In 2007, Forrester Research estimated that 80% of teenagers of age 12-17 visited online social networks, and 50% visited these online networks at least weekly. Since then, analysts have reported persistent growth (see Lipsman (2008)). The public sphere has continuously scrutinized the role of the web in enabling extreme and intolerant behavior. For example, the Columbine shooters reportedly learned how to construct sophisticated bombs through their internet activity. In the massacre's aftermath, many advocated federal censorship of the internet (Newsbuster 11/21/2008). In fact, several measures are already taken in U.S. jurisdiction to limit minors' access to online communities (e.g., the Children's Internet Protection Act).

<sup>3</sup>See below for a description of several exceptions.

worry about their cash-on-hand relative to their children’s educational prospects. Similarly, agents may vary in how much they care about choosing a high-quality candidate in local relative to general elections (for instance, because they differ in how much they are affected by the quality of local public services relative to, say, foreign policy); or, depending on their personal tastes and hobbies, they might differ in how much they are concerned with the quality of food they eat relative to the quality of wine they drink. Each agent’s taste can be characterized by a parameter in  $[0, 1]$ , proxying for how much she cares about one issue relative to the other.

We assume that there is some uncertainty about the optimal choice on each of the two issues and that, prior to making decisions, each individual can collect information on, at most, one issue.<sup>4</sup> Initially, we assume that information collection is *free*. Besides gathering information directly, we assume that individuals have access to information through their peers. Indeed, agents have the possibility of forming groups. *What defines a peer group is that the information collected by each member is made public within the group.*

For any fixed group of individuals, we characterize the equilibrium choice of information collection, a mapping from the composition of tastes of the agents in the group into the volume of information collected on each issue.

Given this characterization, we then step back and consider the group of peers as an *object of choice*. Depending on her tastes, and foreseeing the type of information that is collected within each group, each individual prefers certain peer groups to others. We characterize the peer group’s optimal composition for each individual’s taste. We show that, for each individual, there is a large equivalence class of optimal groups, potentially with maximal variance of tastes.

*Stable groups* are ones that satisfy natural equilibrium constraints in the group-formation stage. That is, *a group is stable if it is optimal for all its members.*

Our first main result provides a characterization of stable groups in the setting just described. We show that stable groups of a fixed size  $n$  are identified by a partition of the taste parameter range  $[0, 1]$  into sub-intervals. In particular, a group is stable if and only if there exists an interval in this partition that contains the taste parameters of *all* the group members. This result suggests that if

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<sup>4</sup>This is a simplification capturing the idea that agents may face constraints on the effort they can exert on information gathering.

each member has some leverage in choosing her peers, *stability occurs when tastes are sufficiently close*. Intuitively, a group is stable only when all its members agree on the optimal way to allocate the group's information gathering across the two issues, and this occurs when tastes are sufficiently similar. The intervals identifying stable groups exhibit interesting comparative statics. Specifically, these intervals are wider for moderate tastes and become narrower as tastes become more extreme. This implies that *stability requires more similarity for extreme individuals than for moderate ones*.

Next, we show that as the group size grows large, for non-extreme individuals, stability remains consistent with groups composed of members of *different*, although sufficiently close, tastes. That is, intermediate intervals in the partition *do not* converge to singletons. In fact, the only intervals converging to singletons are the two extreme ones. This suggests that, as groups become larger (due to, say, a new technology that enables easier connections, such as email, sms, online social networks, etc.), the level of similarity within stable groups remains constant for groups composed of moderate individuals, but becomes stronger within groups composed of extremists.

The idea that each agent always collects information and faces only a choice on the type of information to gather is relevant for situations in which some form of information collection is par for the course – e.g., when deciding what major to choose in college once enrolled, which part of the newspaper to read in the morning, and so on. An important complementary case corresponds to environments in which information is costly, and each agent first faces the choice of whether or not to acquire information altogether. If she does, she again needs to choose the issue on which to collect information. This setting is the focus of Section 4. It is relevant to many environments, ranging from farmers who choose whether or not to experiment on a new grain (which is costly in terms of risk), to researchers who decide whether to invest in learning a new software, to teenagers who decide whether to read music magazines.

When information gathering is costly, a free rider problem can arise within groups that are stable in the free-information case. Since more extreme individuals have greater incentives to acquire information on the issue they care about, introducing group polarization has the benefit of mitigating the free rider problem by weakening the incentive constraints in the information-collection stage.

Our second main result characterizes stable groups in the costly-information case. When groups

are very small, the free rider problem is not severe, and stable groups are the same as those we describe in the free-information case. As the group size increases, the free rider problem becomes more acute. Consequently, for any positive information-gathering cost, for sufficiently large groups, stability entails *extreme preference polarization*. Nonetheless, for intermediate group sizes, two types of groups are stable. First, we show that some of the (sufficiently homogeneous) stable groups in the free-information case survive as stable. Second, we characterize stable *mildly polarized groups*. These groups include both some extremists, who collect information on their preferred issue, and some moderates, who are close enough in their preferences to agree on the optimal allocation of the number of signals collected across issues.

Since the driving force behind the appearance of stable heterogeneity is the free rider problem, reducing the cost of information collection plays a similar role to a reduction of group size: they both alleviate the free rider problem. A decrease in the cost of information may occur with the development of a new technology for information gathering (such as the internet, or a search engine). Thus, our results suggest that, *as technology improves, stable groups exhibit more similarity in tastes*. Indeed, when information costs are high, polarized individuals are necessary in a group to gather information on some issue. As information costs decrease, information gathering becomes feasible for moderate individuals also. Thus, agreement within the group on the optimal way to go about collecting information becomes the prominent criterion around which stable groups form. This is consistent with a large body of empirical work studying the effects of new information technologies on social affiliations. For instance, Sproull, and Kiesler (1991) depicted how the introduction of the telephone affected connection between similar peer groups. Recently, Rosenblat and Mobius (2004) studied coauthored papers in top economics journals between the years 1969-1999. They showed how the introduction of the internet in the early 1990's is linked with a 20% decrease in the realization of projects with a dissimilar coauthor.<sup>5</sup>

Our baseline notion of stability is a strong one, in that each individual can potentially deviate to groups composed of agents with *any* taste combination. This is a reasonable assumption when

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<sup>5</sup>See also Lynd and Lynd (1929), which illustrates how the introduction of the automobile, which lessened geographical constraints, increased the prominence of clubs and coincided with an increase in peer connections based on common interests.

the population is very large. However, in small populations, deviations are restricted by the existing partition of agents into groups. We study the case of a small population of agents in Section 5. We call a partition of the population *stable* if no agent prefers to join a group (an element in the partition) different than the one she is in or to remain by herself. Certainly, the grand coalition containing all agents is always stable and is the most efficient stable allocation. Our last main result provides conditions under which full segregation, a partition into groups that contain only agents with *the same* taste, can arise as a stable allocation. We exploit the comparative statics developed in the free-information case to show that, in a similar spirit, *segregation is easier to sustain for individuals of extreme tastes than of moderate ones*.

While our analysis is presented in terms of an information story, it is important to note that its crucial element is that agents have one unit of (possibly costly) effort that they can exert on either issue and that the resulting utilities for all agents exhibit decreasing marginal returns with respect to the total effort exerted on each issue. As a robustness check, in Section 6 we show that our results hold for a large class of production functions of public goods. Section 6 also illustrates the generality of our results with respect to the number of issues at hand, the extent of externalities between agents' actions, and the protocols restricting agents' moves from one group to another.

**Related Literature.** Lazarsfeld and Merton (1954) coined the term *homophily* – literally meaning “love for the same” – capturing the tendency of socially connected individuals to be similar to one another.<sup>6</sup> Since that time, there has been a growing body of work identifying homophily across fields, ranging from economics (see Benhabib, Bisin, and Jackson (2009)), to political science (see Huckfeldt and Sprague (1995)), to sociology (see McPherson, Smith-Lovin, and Cook (2001)).

In general, similarity of connected individuals on malleable traits (such as political affiliation, education, etc.) can be rooted in one of two processes: (i) selection, or assortative matching, in which similarity begets association, the process modeled in this paper; or (ii) socialization or convergence, in which social ties generate similarity. One way to disentangle these processes entails a study of exogenous characteristics, such as height or race (see Goeree, McConnell, Mitchell, Tromp, and Yariv

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<sup>6</sup>The observation that people connect to those similar to them is, in fact, a rather old one. Aristotle remarked in his *Rhetoric* and *Nicomachean Ethics* that people “love those who are like themselves.” Plato commented in *Phaedrus* that “similarity begets friendship.”

(2009), Marmaros and Sacerdote (2006), and Mayer and Puller (2008), who identify significant levels of homophily with respect to these attributes). Another approach is to consider longitudinal data, as in Kandel (1978). She studied adolescent friendships and the extent of similarity across dyadic connections regarding four attributes (frequency of current marijuana use, level of educational aspirations, political orientation, and participation in minor delinquency) at several stages of friendship formation and dissolution. Kandel found that observed homophily was the outcome of a significant combination of both types of processes.

On the theoretical side, two recent papers directly address preferences for similarity. Currarini, Jackson, and Pin (2008) *assume* homophilic behavior and study its consequences in a friendship formation model. Peski (2008) derives a preference for similarity endogenously. He assumes certain properties of preferences over friends (complementarities between direct friends and second-degree friends) and the possibility of confusing people who are similar to one another. The necessity to differentiate friends and enemies as much as possible then leads individuals to form friendships with those who are similar.

The underlying idea that the group of players in a strategic situation is, in itself, endogenous motivates some of the work on club formation (see, e.g., Ellickson, Grodal, Scotchmer, and Zame (1999) and Wooders, Cartwright, and Selten (2006)). The basic model of that literature assumes some form of externality across individuals and studies endogenous group formation (often in a general equilibrium setup) in the presence of these externalities. Our approach differs in that externalities in our setting arise only through the sharing of information (specifically, no goods are traded after groups are formed). Furthermore, we focus on the characteristics of the emergent groups (namely, the distribution of tastes as a function of the environment's fundamentals).

Several elements of our model are reminiscent of work in other areas. First, the idea that agents may choose peer groups that match their preferences is an ongoing theme in the theory of public choice, going back to Tiebout (1956). These models define municipalities by the government services and tax rates that they offer. Individuals choose a community that maximizes their utility. Nonetheless, the strategic interaction that follows and the structure of utilities are very different. Furthermore, much of that work is concerned with the efficiency of such processes, rather than with the similarity or

heterogeneity of equilibrium communities. Second, the idea that agents' preferences may alleviate incentive constraints in collective settings with costly information appears in some recent mechanism design literature.<sup>7</sup> Third, the notion that agents optimally select those with whom they communicate appears also in Calvó-Armengol, De Martí, and Prat (2009).<sup>8</sup> They consider a set of connected agents who differ in the accuracy of their *exogenously* provided private information, and are ex-ante identical otherwise. Furthermore, externalities in their model arise through both (costly) information sharing, as well as through ultimate actions that are taken. In contrast, we characterize the endogenous similarity or heterogeneity within groups in which all agents freely communicate with all others (and the only externalities present are information-based).

Recently, there has been a proliferation of work illustrating the potential explanatory power of social connections and individual outcomes across contexts, covering public goods provision, crime, job search, political alliances, trade, friendships, and information collection.<sup>9</sup> Particularly in view of the vast literature on homophily, an important empirical issue in this literature is that correlations between behavior and outcomes of individuals and their peers may be driven by common unobservables and, therefore, be spurious (see Evans, Oates, and Schwab (1992) and Manski (1993, 2000)). Understanding similarity patterns is potentially important for mitigating such endogeneity problems.

## 2. THE MODEL

There are two issues at stake:  $A$  and  $B$ , each taking a value in  $\{0, 1\}$ . The values of  $A$  and  $B$  are determined independently at the outset of the game. For expositional simplicity, we assume that each issue  $I \in \{A, B\}$  has equal probability of receiving the value 0 or 1.<sup>10</sup> Issues can stand for many problems, ranging from the choices of the best food shops and wine shops, to voting for the highest-quality political candidates running for two different positions.

Each agent needs to make a decision on each issue – that is, pick an action  $v = (v_A, v_B) \in$

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<sup>7</sup>See Che and Kartik (2008) and Gerardi and Yariv (2008).

<sup>8</sup>While in our model this choice amounts to selecting  $n$  agents out of an infinite pool of individuals of all types, in their setting agents choose the intensity of their communication (regarding both transmission and reception) with each other agent separately.

<sup>9</sup>This literature is too immense to survey here. Important work includes Coleman (1966), Conley and Udry (2005), Foster and Rosenzweig (1995), Glaeser, Sacerdote, Scheinkman (1996), Granovetter (1994), Katz and Lazarsfeld (1954), and Topa (2001).

<sup>10</sup>The entire analysis of the paper can be extended directly to asymmetric priors.

$\{0, 1\} \times \{0, 1\}$ . Each agent is characterized by a taste parameter  $t \in [0, 1]$ . The utility of an agent of taste  $t$  from choosing  $v$  when the realized issues are  $A$  and  $B$  is given by:

$$u(t, v; A, B) = t\mathbf{1}_A(v_A) + (1 - t)\mathbf{1}_B(v_B),$$

where  $\mathbf{1}_I(\cdot)$  is an indicator function, receiving the value of 1 whenever the argument coincides with issue  $I$ , and 0 otherwise. Thus, the agent's goal is to match her actions to the realized issues. The taste parameter  $t$  measures how much an agent's utility is affected by making the right decision on each issue.<sup>11</sup> For example, all agents benefit by choosing a superior supermarket and a superior wine shop, but, depending on their consumption patterns, they may differ in how much one affects their utility with respect to the other. Similarly, agents may be affected differently by the selection of an able candidate in, say, general relative to local elections.

Prior to making a decision, each agent selects simultaneously one of two information sources,  $\alpha$  or  $\beta$ . Information source  $\alpha$  provides the realized issue  $A$  with probability  $q_\alpha > 0$ . That is, upon choosing information source  $\alpha$ , the agent observes a signal  $s \in \{0, 1, \emptyset\}$  according to:

$$\Pr(s = A) = q_\alpha, \quad \Pr(s = \emptyset) = 1 - q_\alpha.$$

Similarly, information source  $\beta$  provides the realized issue  $B$  with probability  $q_\beta > 0$ . In what follows, we sometimes refer to a signal collected from sources  $\alpha$  and  $\beta$  as “ $\alpha$ -signal” and “ $\beta$ -signal,” respectively.<sup>12</sup> The idea behind the assumption that each agent can gather at most one signal is that each agent has a limited budget of resources to allocate to information gathering. In Section 4, we generalize the setup to environments in which information is costly and, therefore, each agent has the option to forego information gathering altogether.

A group consists of a set of  $n \geq 2$  agents.<sup>13</sup> What defines a group of peers in our model is information sharing. That is, after all information sources are selected, all signals are realized and

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<sup>11</sup>While in this setting the issues are common value (so, the right decision for all agents is the same), our analysis would follow similar lines if agents had different opinions on what is the right decision conditional on the realized issue.

<sup>12</sup>Information sources related to one issue can be, for example, a specialized magazine, a TV channel, an internet search, and so on.

<sup>13</sup>Assuming that the group size is exogenous captures situations in which agents face a fixed constraint on how much effort they can invest in communication, or social interactions. While the group size is exogenous throughout Sections 3 and 4, it becomes endogenous in the analysis in Section 5.

*made public within the group.*

This fully defines a game, the *information-collection game*. Note that, once information is collected, agents' best response is to follow the signal whenever the realization of an issue is revealed, and to choose any action if the issue is not revealed (all actions leading to the same expected utility). In that sense, for any group composed of agents with tastes  $(t_1, \dots, t_n)$ , expected payoffs are fully identified by the profile of chosen signals  $(x_1, \dots, x_n)$ , where  $x_i \in \{\alpha, \beta\}$  is the type of signal chosen by agent  $i$ . As a *tie-breaking rule*, we assume that, upon indifference, an agent chooses the information source  $\alpha$  (this will allow us to pin down a unique equilibrium outcome, so simplifies the exposition. Our analysis remains qualitatively intact for any other deterministic tie-breaking rule).

Throughout our analysis, the expected utility of an agent for any given allocation of signals in the group will play a crucial role. Regardless of their action, an uninformed agent makes the right decision on issue  $I$  with probability  $\frac{1}{2}$ . Therefore, when there are  $k^x$  agents in one's group choosing signal  $x$ , the probability that an agent (who best responds to information) makes the right decision on issue  $x$  is  $1 - \frac{1}{2}(1 - q_x)^{k^x}$ ,  $x = \alpha, \beta$ . Thus, any agent of taste  $t$ 's utility when  $k^\alpha$  and  $k^\beta$  are the number of signals chosen from source  $\alpha$  and  $\beta$ , respectively, is given by:

$$U(t, k^\alpha, k^\beta) \equiv t \left[ 1 - \frac{1}{2}(1 - q_\alpha)^{k^\alpha} \right] + (1 - t) \left[ 1 - \frac{1}{2}(1 - q_\beta)^{k^\beta} \right]. \quad (1)$$

It follows that the information-collection game can be identified with a one-shot complete information game in which each agent's actions are given by the available signals  $\{\alpha, \beta\}$  and utilities are specified according to 1. We focus on equilibria in pure strategies of this induced game. As it turns out, given our tie-breaking rule, a pure equilibrium exists, and the number of  $\alpha$  and  $\beta$  equilibrium signals are determined uniquely, as the following lemma guarantees.

**Lemma 1A (Existence and Uniqueness)** *For any group of  $n$  agents with tastes  $t_1 \geq t_2 \geq \dots \geq t_n$ , there exists  $k^* \in \{0, \dots, n\}$  such that all agents  $i \leq k^*$  choosing an  $\alpha$ -signal, and all agents  $i > k^*$  choosing a  $\beta$ -signal, is part of a Nash equilibrium of the information-collection game. Furthermore, all Nash equilibria of the information-collection game entail the same number  $k^*$  of agents choosing an  $\alpha$ -signal.*

We now expand this game and look at an extended game composed of two stages. First, each agent of taste  $t \in [0, 1]$  can choose the tastes of the remaining  $n - 1$  agents in her group.<sup>14</sup> Second, the information-collection game described above is played.

Since Lemma 1A guarantees that the equilibrium number of  $\alpha$ -signals is determined uniquely in the information-collection game, the agent's optimization problem in the first stage of the extended game is well defined. We denote the optimal group chosen by agent  $t$  at the first stage by  $(t_1^*(t), \dots, t_n^*(t))$  (one element of which is  $t$ ).

We define stability in the first stage of the extended game as follows.

**Definition (Stable Group)** *A group  $(t_1, \dots, t_n)$  is stable if it is optimal for all its members – i.e.,*

$$(t_1, \dots, t_n) = (t_1^*(t_i), \dots, t_n^*(t_i)) \text{ for all } i = 1, \dots, n.$$

Stability is, therefore, a natural equilibrium condition for the group-selection stage. Each agent maximizes her expected utility given the tastes of others in the group, foreseeing the equilibrium played in the information-collection game that ensues.

### 3. FREE INFORMATION

We start by spelling out the incentive constraints defining equilibrium information collection in a given group. We then fix the taste parameter of one agent and identify that agent's optimal peer group choice. This allows us to characterize stable groups, the goal of this section.

**3.1. Information Collection.** Given a group of  $n$  agents with tastes  $t_1 \geq t_2 \geq \dots \geq t_n$ , Lemma 1A guarantees the existence and uniqueness of an equilibrium number of  $\alpha$ -signals that is chosen in the information-collection game. Furthermore, Lemma 1A allows us to concentrate on equilibria  $(x_1, \dots, x_n)$  identified by a threshold  $k^* \in \{0, \dots, n\}$  such that  $x_1 = \dots = x_{k^*} = \alpha$  and  $x_{k^*+1} = \dots = x_n = \beta$  (in particular, if  $k^* = 0$  all agents choose a  $\beta$ -signal, and if  $k^* = n$ , all agents choose an  $\alpha$ -signal). In words, we focus on equilibria in which any agent choosing the source  $\alpha$  cares more about the issue  $A$  than does any agent choosing the source  $\beta$ .

Note that if an agent of taste  $t$  prefers getting an  $\alpha$ -signal over a  $\beta$ -signal, so would any agent of taste  $t' > t$ . Similarly, if an agent of taste  $t$  prefers getting a  $\beta$ -signal, so would any agent of taste

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<sup>14</sup>For now, we assume that the pool of potential agents to choose from is infinite and that any combination of tastes is feasible. In Section 5, we restrict the set of agents to a finite population.

$t' < t$ . When the equilibrium threshold is  $k^*$ , the agent with the lowest taste parameter who chooses an  $\alpha$ -signal is the agent with taste  $t_{k^*}$ . From our tie-breaking rule, it follows that the threshold  $k^*$  is determined as the maximal  $k \in \{0, 1, \dots, n\}$  for which agent  $k$  *weakly* prefers an  $\alpha$ -signal over a  $\beta$ -signal. That is, the maximal  $k \in \{0, 1, \dots, n\}$  for which

$$U(t_k, k, n - k) \geq U(t_k, k - 1, n - k + 1). \quad (2)$$

If (2) is not satisfied for any agent in the group (i.e.,  $U(t_1, 0, n) > U(t_1, 1, n - 1)$ ), then  $k^* = 0$ .

**3.2. Optimal Group Composition.** Given a group size  $n$ , we now analyze the optimal group composition from the point of view of an agent with taste parameter  $t \in [0, 1]$ .

Denote by  $n_f^\alpha(t)$  the optimal (and maximal, upon indifference) number of  $\alpha$ -signals the agent with taste parameter  $t$  would choose out of a total of  $n$  available signals. That is, *fixing a taste parameter  $t$  in (2)*,  $n_f^\alpha(t)$  is the maximal integer  $k$  such that (2) is satisfied. If (2) is not satisfied for any  $k$ , we define  $n_f^\alpha(t) = 0$ . Naturally,  $n_f^\alpha(t)$  increases with  $t$  and with the group size  $n$ . Similarly, the optimal number of  $\beta$ -signals for an agent of taste parameter  $t$  is denoted by  $n_f^\beta(t) \equiv n - n_f^\alpha(t)$ .

Any optimal group for the agent with taste parameter  $t$  must be composed so that  $n_f^\alpha(t)$  agents collect an  $\alpha$ -signal, and  $n_f^\beta(t)$  agents collect a  $\beta$ -signal. Groups consisting of all agents sharing the taste parameter  $t$  are, therefore, optimal. Nonetheless, since extreme agents of taste  $t = 1$  (or  $t = 0$ ) always best respond with the choice of  $\alpha$ - (or  $\beta$ -) signals, an optimal group for the agent of taste  $t$  can also be composed of just the right number of extremists on each side, thereby achieving maximal polarization. The following Proposition characterizes all the optimal groups.

**Proposition 1 (Free Information – Optimal Groups)** *Given a group size  $n$ , for any taste  $t \in [0, 1]$ , there exist  $l(t), h(t) \in [0, 1]$ ,  $l(t) \leq t < h(t)$ , such that any group of agents with tastes  $t_1 \geq t_2 \geq \dots \geq t_n$  (one of which is  $t$ ) is optimal if and only if*

$$t_1 \geq t_2 \dots \geq t_{n_f^\alpha(t)} \geq l(t) \quad \text{and} \quad h(t) > t_{n_f^\beta(t)+1} \geq \dots \geq t_n.$$

Intuitively, to achieve her optimal allocation of signals  $(n_f^\alpha(t), n_f^\beta(t))$  across the two sources  $\alpha$  and  $\beta$ , an agent with taste parameter  $t$  has to select  $n_f^\alpha(t)$  agents who care enough about issue  $A$  so

that they choose an  $\alpha$ -signal in the information-collection game, and  $n_f^\beta = n - n_f^\alpha(t)$  agents who care enough about issue  $B$  to choose a  $\beta$ -signal in the information-collection game. Thus, optimal groups must be characterized by two thresholds – one assuring that  $n_f^\alpha(t)$  agents have a sufficiently high taste parameter, and the other assuring that  $n_f^\beta(t)$  agents have a sufficiently low taste parameter.

Proposition 1 suggests that in situations in which one agent has leverage in choosing her group partners, the agent cares only about order-statistic type of attributes of the group's distribution of tastes. In particular, optimal groups can entail maximal polarization, containing extreme agents on both sides of the taste spectrum.

**3.3. Stable Groups.** We now turn to the case in which *all* members of the group have some leverage in choosing their peers. As we saw above, an agent's optimal group entails her optimal allocation of  $n$  signals across the two sources  $\alpha$  and  $\beta$  being selected in the information-collection game. Thus, in a *stable* group in which all agents optimize on their peers' tastes, all agents have to agree on the optimal allocation of signals across the two sources. In particular, a group formed by identical agents is always stable and *stable groups always exist* in this setting. More generally, stable groups are characterized as follows.

**Proposition 2 (Free Information – Stable Groups)** *For any group size  $n$ , there exists a partition  $\{T_k^n\}_{k=0}^n$  of the interval  $[0, 1]$ , where  $T_0^n = [0, t(1))$ ,  $T_k^n = [t(k), t(k+1))$ , for  $k = 1, \dots, n-1$ , and  $T_n^n = [t(n), 1]$  such that:*

1. *A group comprised of agents with tastes  $(t_1, \dots, t_n)$  is stable if and only if there exists  $k = 0, \dots, n$ , such that for all  $i$ ,  $t_i \in T_k^n$ . That is, all taste parameters belong to the same element of the partition.*
2. *The length of the intervals  $\{T_k^n\}_{k=1}^{n-1}$  is increasing for  $k = 1, \dots, \hat{k}$ , where  $\hat{k}$  is such that  $\frac{1}{2} \in T_{\hat{k}}^n$ , and decreasing for  $k = \hat{k}, \dots, n-1$ . Thus, the intervals  $\{T_k^n\}_{k=1}^{n-1}$  are narrower for extreme tastes, and wider for moderate tastes.<sup>15</sup>*

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<sup>15</sup>In the Appendix, we show that, under mild conditions on  $q_\alpha, q_\beta$ , and  $n$ , this result extends to the entire sequence  $\{T_k^n\}_{k=0}^n$ . For example,  $(1 - q_\alpha)(1 - q_\beta) < 1/2$  and  $n$  high enough, are sufficient to guarantee this.

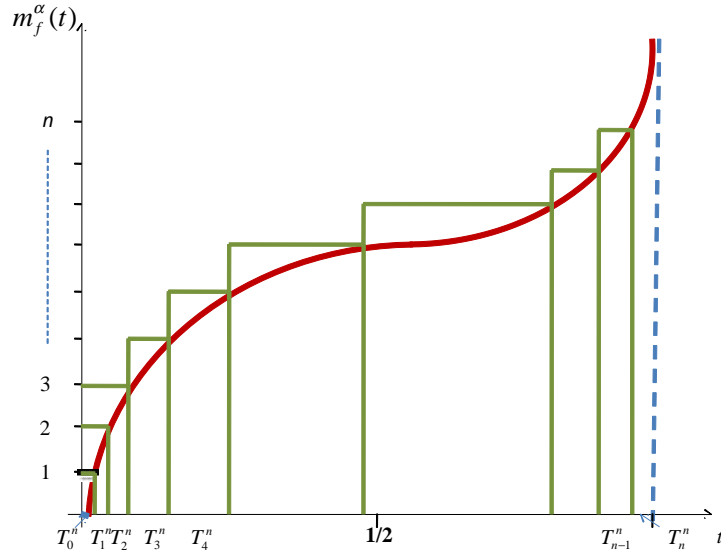


Figure 1: Stable Groups in the Free Information Case

While Proposition 1 shows that, from an individual point of view, optimal groups can entail extreme polarization, the first part of Proposition 2 guarantees that stable groups can be formed only by agents whose tastes are close enough – namely, they lie in one of the intervals  $T_k^n$ . Thus, the length of the intervals  $\{T_k^n\}_{k=0}^n$  provides a proxy for equilibrium homophily: the narrower an interval is, the closer the agents' tastes have to be in order for them to form a stable group. The second part of Proposition 2 addresses how the intervals' lengths are affected by the proximity of the intervals to the extreme tastes. In particular, point (2) of Proposition 2 implies that *stability requires more similarity for extreme individuals than for moderate ones*.

In order to highlight the intuition of this result, it is useful to consider the optimal number of  $\alpha$ -signals  $n_f^\alpha(t)$  that each agent of taste  $t$  would choose. Indeed, denoting by  $m_f^\alpha(t)$  the real number achieving equality within constraint (2), we get that  $n_f^\alpha(t) = \lfloor m_f^\alpha(t) \rfloor$  if  $m_f^\alpha(t) \in [0, n]$ ,  $n_f^\alpha(t) = 0$  if  $m_f^\alpha(t) < 0$ , and  $n_f^\alpha(t) = n$  if  $m_f^\alpha(t) > n$ . Roughly speaking,  $m_f^\alpha(t)$  captures the point at which an agent of taste  $t$  equates the marginal return from an  $\alpha$ -signal with the marginal return from a  $\beta$ -signal. The function  $m_f^\alpha(t)$  is depicted in Figure 1. Note that an agent with taste parameter  $t = \frac{1}{2}$  cares equally about both issues. In particular, by raising  $t$  above  $\frac{1}{2}$ , the relative value of an  $\alpha$ -signal tends

to increase since the agent cares more about issue  $A$  than issue  $B$ . However, a countervailing force is in effect. Indeed, since the optimal number of  $\alpha$ -signals increases with  $t$ , the marginal return of an  $\alpha$ -signal tends to decrease. As it turns out, the first force prevails, and, as a result,  $m_f^\alpha(t)$  is convex to the right of  $t = \frac{1}{2}$  (and, analogously, concave to the left of  $t = \frac{1}{2}$ ).<sup>16</sup>

Each interval in the partition  $\{T_k^n\}_{k=0}^n$  includes all taste parameters of agents who agree on a given optimal allocation of signals. In Figure 1, agreement on the number of  $\alpha$ -signals  $n_f^\alpha(t) = k$  corresponds to the interval of taste parameters that is projected from  $[k, k+1)$  on the  $y$ -axis. That is, the interval of tastes for which  $m_f^\alpha(t) \in [k, k+1)$  and  $n_f^\alpha(t) = k$ . The shape of  $m_f^\alpha(t)$  induces the property described in point (2) of Proposition 2 directly.<sup>17</sup>

**3.4. Group Size.** We now look at how the stable group characterization in Proposition 2 is affected by changes in group size and number of signals that each agent can acquire. For a group of size  $n$ , we denote, as before, by  $\{T_k^n\}_{k=0}^n$  the partition of the unit interval into sub-intervals defining stability. We call the stable groups that choose all signals from the same source (i.e., have taste parameters in either  $T_0^n$  or in  $T_n^n$ ) *extreme stable groups*. We call all other stable groups *non-extreme stable groups*.

**Proposition 3 (Convergence for Large  $n$ )** *Consider two agents of taste parameters  $t, t'$ .*

1. *If they both belong to a non-extreme stable group of size  $n \geq 2$ , then they both belong to a non-extreme stable group of some size  $n' > n$ .*
2. *If they both belong to an extreme stable group of size  $n$ , then they both belong to an extreme stable group of any smaller size  $n' < n$ . Furthermore, extreme stable groups become fully homogeneous (containing only the most extreme agents) as group size becomes infinitely large.*

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<sup>16</sup>As a purely technical intuition, it is easy to think of the case in which  $q \equiv q_\alpha = q_\beta$ . Equating marginal returns from signals then boils down to a formula of the form:  $\frac{t}{1-t} = (1-q)^{n-2m_f^\alpha(t)-1}$ . Since the left-hand side diverges as  $t$  approaches 1, and the right-hand side is bounded, the described balance of forces for high  $t$  follows.

<sup>17</sup>The reason why the extreme intervals can generally follow a different pattern is the following. Note that the extreme intervals collect all the taste parameters  $t$  such that the problem of the optimal allocation of signals across sources  $\alpha$  and  $\beta$  has a corner solution. Then, for example, for a given  $q_\beta$ , if both  $q_\alpha$  and  $n$  are very low, there is a wide range of  $t$ 's for which it is optimal to get  $n$   $\beta$ -signals (the marginal returns from source  $\alpha$  are very low). In order for the pattern to carry through for the extreme intervals, the signal accuracies and the group size need to be sufficiently large.

Since all agents with taste parameters belonging to the same interval  $T_k^n$  have to agree on how to optimally allocate  $n$  signals across the  $\alpha$  and  $\beta$  sources, one may conjecture that, as  $n$  grows arbitrarily large, these intervals converge to singletons, larger groups displaying greater homophily. Proposition 3 illustrates that this is the case only for extreme stable groups. Nonetheless, for non-extreme stable groups, Proposition 3 shows that this conjecture does not hold, and the same degree of similarity found for small group sizes tends to persist for larger sizes as well. In particular, the first part of Proposition 3 guarantees that, as the group size increases, the *non-extreme* intervals  $\{T_k^n\}_{k=1}^{n-1}$  do not converge to singletons. It is sufficient for two different agents to agree on an optimal source allocation for a given group size  $n \geq 2$  for them to keep on agreeing for larger and larger groups.<sup>18</sup>

The intuition for this result is the following. For simplicity, assume that  $q \equiv q_\alpha = q_\beta$  and that two agents of differing tastes agree on the optimal way to allocate the number of signals  $n$ . This implies that, for each of these two agents, the marginal utilities of the two signals  $\alpha$  and  $\beta$  generated by this allocation are roughly the same. Suppose, now, that we give these agents the possibility to allocate two more signals (that is,  $n' = n + 2$ ). The best way to keep on equalizing the marginal utilities of these signals must be to allocate one signal to source  $\alpha$  and the other to  $\beta$ . Thus, these agents still share the same optimal allocation for  $n + 2$  signals. More formally, notice that when  $k$  of  $n$  signals are  $\alpha$ -signals, the marginal return for an agent of taste  $t$  from an  $\alpha$ -signal is proportional to  $t(1 - q)^k$ , while their marginal return from a  $\beta$ -signal is proportional to  $(1 - t)(1 - q)^{n-k}$ . The ratio of these marginals remains the same if we increase  $k$  by  $l$  and  $n$  by  $2l$ . In other words, if  $k$   $\alpha$ -signals and  $n - k$   $\beta$ -signals is an optimal allocation for an agent of taste  $t$  of  $n$  signals,  $k + l$   $\alpha$ -signals and  $n + l - k$   $\beta$ -signals would be an optimal allocation of  $n + 2l$  signals for that agent. In particular, the intermediate intervals characterizing stable groups remain the same for even (or odd) group sizes (their number increases with  $n$ , however).<sup>19,20</sup>

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<sup>18</sup>Moreover, in the proof of Proposition 3, we show that any such two agents can disagree on *at most* one signal for any larger group size (that is, their taste parameters must belong to either a unique interval or to two contiguous intervals of the series  $\{T_k^{n'}\}_{k=1}^{n'-1}$  for any  $n' > n$ ).

<sup>19</sup>For extreme stable groups, the argument is slightly different. Indeed, extreme stable sets contain agents for which the optimal allocation of  $n$  signals is a corner solution, which does not equalize the marginal utility of signals. As the number of signals  $n$  increases, more agents whose tastes are not at the extremes of the interval  $[0, 1]$  tend to reach interior solutions.

<sup>20</sup>Note that the comparative statics with respect to group size  $n$  are fully isomorphic to a comparative statics exercise in which the group size is fixed, but we *increase the number of signals that each agent acquires from 1 to any  $h \geq 1$* . In this case, group stability requires all agents in the group to agree on how to allocate  $n \times h$  signals. Thus, the same

It has been empirically observed that larger groups tend to be characterized by an increased degree of similarity.<sup>21</sup> In light of this evidence, it is important to read Proposition 3 correctly. Since it is, indeed, the case that, as  $n$  increases, the extreme intervals tend to break down into an increasing number of smaller intervals, our result does not necessarily contradict this empirical evidence. Instead, it qualifies it. Proposition 3 highlights the fact that the location within the taste spectrum may play an important role in identifying this sort of comparative statics: *The tendency of larger groups to display more similarity should be stronger for extreme taste parameters than for moderate ones.*

#### 4. COSTLY INFORMATION

We now turn to situations in which access to information is costly: we assume that observing a signal of either source comes at a cost of  $c > 0$ . This corresponds to any context in which expertise requires effort (e.g., following the media, learning a new software, etc.). As before, our analysis follows three steps. First, for any given group, we analyze the equilibria in the information-collection game. Second, we characterize the optimal groups for any agent in the population. Third, we characterize the stable groups.

The following notation is useful for the analysis that follows. First, for  $x = \alpha, \beta$ , let  $n_c^x(t)$  denote the maximal number of  $x$ -signals acquired in a group for which an agent of taste  $t$  is willing to acquire an  $x$ -signal rather than no signal at all. Formally,  $n_c^\alpha(t)$  is the maximal integer  $h$  such that, no matter how many  $w$   $\beta$ -signals are acquired,

$$U(t, h, w) - U(t, h - 1, w) = \frac{t}{2} (1 - q^\alpha)^{h-1} q^\alpha \geq c. \quad (3)$$

Similarly,  $n_c^\beta(t)$  is the maximal integer  $h$  such that, no matter how many  $w$   $\alpha$ -signals are acquired,

$$U(t, w, h) - U(t, w, h - 1) = \frac{1-t}{2} (1 - q^\beta)^{h-1} q^\beta \geq c. \quad (4)$$

Obviously,  $n_c^\alpha(t)$  is increasing in  $t$  and  $n_c^\beta(t)$  is decreasing in  $t$ .

Second, let  $n_{\max}^\alpha$  be the maximal number of  $\alpha$ -signals that agents with the most extreme taste parameter  $t = 1$  are willing to acquire. That is,  $n_{\max}^\alpha \equiv n_c^\alpha(1)$ . Analogously, for  $\beta$ -signals, let  $n_{\max}^\beta \equiv$

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considerations made for changes in group size apply to changes in  $h$  as well.

<sup>21</sup>See, for instance, Currarini, Jackson, and Pin (2008) and references therein.

$n_c^\beta(0)$ . Note that  $n_{\max}^\alpha$  and  $n_{\max}^\beta$  are the maximal number of  $\alpha$ - and  $\beta$ -signals acquired, respectively, in any equilibrium of the information-collection game.

Finally, let  $\underline{t}^\alpha = \min \{t \mid n_c^\alpha(t) = n_{\max}^\alpha\}$ , so that the interval  $[\underline{t}^\alpha, 1]$  represents the set of all taste parameters with which an agent is willing to acquire the  $n_{\max}^\alpha$ 'th  $\alpha$ -signal. Likewise, let  $\bar{t}^\beta = \max \{t \mid n_c^\beta(t) = n_{\max}^\beta\}$ , so that  $[0, \bar{t}^\beta]$  represents the set of all taste parameters with which an agent is willing to acquire the  $n_{\max}^\beta$ 'th  $\beta$ -signal. These intervals correspond to agents with ‘‘sufficiently extreme’’ tastes to be willing to invest in information when the maximal possible number of signals is being acquired.

**4.1. Information Collection.** When information is costly, each agent’s decision is comprised of two layers in the information-collection game. Each agent decides, first, whether or not to collect information. Then, upon deciding to collect information, she decides the source of that information. In analogy with the free-information case, we assume that an agent who is indifferent between acquiring and not acquiring a signal invests in information, and an agent who is indifferent between an  $\alpha$ - and a  $\beta$ -signal chooses an  $\alpha$ -signal. We focus our analysis on pure strategy equilibria (pertaining to both layers). Therefore, in equilibrium, the agent knows the number of other agents in the group who acquire information, and the analysis of the second layer boils down to that performed for the free-information case. However, it is important to note that some information collection equilibria may involve agents not collecting information.

Given a group of agents of tastes  $t_1 \geq \dots \geq t_n$ , an information-collection equilibrium is characterized by the profile of chosen sources  $(x_1, \dots, x_n)$ , where  $x_i \in \{\alpha, \beta, \emptyset\}$  is the source chosen by agent  $i$ , and  $\emptyset$  stands for agent  $i$  not acquiring information. In Lemma 1B in the Appendix, which is an analogue of Lemma 1A, we show that equilibria outcomes are unique. This allows us to concentrate on equilibria in which the agents who acquire  $\alpha$ -signals ( $\beta$ -signals) care more than all other agents about issue  $A$  (issue  $B$ ), i.e., equilibria in which agents  $i = 1, \dots, k^\alpha$  acquire an  $\alpha$ -signal, and agents  $i = k^\beta, \dots, n$  acquire a  $\beta$ -signal, with  $k^\beta \geq k^\alpha + 1$ .

The fact that information is costly introduces a *free rider problem*. Indeed, in any equilibrium in which  $k^\beta > k^\alpha + 1$ , the agents  $i$ ,  $k^\alpha < i < k^\beta$ , do not have enough incentives to collect information on either issue. However, in the initial group selection, an agent can manipulate the free rider problem.

This is the focus of the next section.

**4.2. Optimal Group Composition.** Let us now address the problem of an agent of taste  $t$  who is given the opportunity to choose the  $n - 1$  other members of her group.

Recall from Section 3 that, from the point of view of an individual agent of taste  $t$ , if  $c = 0$ , the optimal group consists of  $n_f^\alpha(t)$  agents who gather an  $\alpha$ -signal and  $n_f^\beta(t)$  agents who gather a  $\beta$ -signal. From now on, we refer to such allocation of signals as the *unconstrained optimal allocation* for an agent of taste  $t$ . From Proposition 1, the group composition that achieves agent  $t$ 's unconstrained optimal allocation involves  $n_f^\alpha(t)$  agents who care enough about issue  $A$  (with taste parameter  $t \geq l(t)$ ) and  $n_f^\beta(t)$  agents who care enough about issue  $B$  (with taste parameter  $t < h(t)$ ). However, in choosing a group, an individual now has to consider constraints in addition to those pertaining to the free-information case. In particular, the number of  $x$ -signals cannot exceed  $n_{\max}^x$  for any source  $x = \alpha, \beta$ . Therefore, the agent of taste parameter  $t$  has hope of achieving her unconstrained optimum only if  $n_f^x(t) \leq n_{\max}^x$  for  $x = \alpha, \beta$ . Proposition 4 describes the outcome of the optimal group selection of an agent of taste  $t$ .

**Proposition 4 (Costly Signals – Optimal Groups)** *Consider an agent of taste parameter  $t$ . In the optimal group composition for  $c > 0$ , the agent*

1. *achieves the unconstrained optimal allocation  $(n_f^\alpha(t), n_f^\beta(t))$  if and only if it is feasible, i.e.,  $n_f^x(t) \leq n_{\max}^x$  for  $x = \alpha, \beta$ , and it induces the agent to invest in information, i.e.,  $n_f^x(t) \leq n_c^x(t)$  for at least one  $x \in \{\alpha, \beta\}$ ;*
2. *implements, at most,  $n_{\max}^x$  signals  $x$  for each  $x = \alpha, \beta$  for which  $n_f^x(t) > n_{\max}^x$ .*

Note that the presence of the free rider problem induces an agent to select peers that are in a *more polarized* subset of the optimal group set characterized in Proposition 1 for the free-information case.

**4.3. Stable Groups.** We now turn to the characterization of stable groups in the costly-information case. First of all, note that *stable groups always exist*. Indeed, having  $n$  agents of taste  $t = 1$  and  $\min\{n, n_{\max}^\alpha\}$  all acquiring an  $\alpha$ -signal, or analogously, having  $n$  agents of taste  $t = 0$  and  $\min\{n, n_{\max}^\beta\}$

all acquiring a  $\beta$ -signal, both constitute stable groups.<sup>22</sup> Our goal is to characterize all stable groups in the presence of information costs. We show that the structure of stable groups depends crucially on the group's size  $n$ .

While we present the analysis for different group sizes, our results have a natural analogue in terms of information costs. Indeed, increasing the size of the group is reminiscent of increasing costs in that both make the free rider problem more severe. Formally, note that by increasing  $c$ , we reduce the number of affordable signals  $n_c^x(t)$  for all  $t$ ,  $x = \alpha, \beta$  (in particular, we reduce  $n_{\max}^\alpha$  and  $n_{\max}^\beta$ ). Thus, our results shed light on the effects of new technologies that decrease costs of information gathering (e.g., the introduction of the automobile, telephone, internet, search engines, etc.).

**Large Groups.** Recall that the maximal number of  $x$ -signals that can be acquired in equilibrium is  $n_{\max}^x$ . Thus, the maximal number of agents who can conceivably acquire information in equilibrium is  $n_{\max}^\alpha + n_{\max}^\beta$ . In that respect, we refer to groups containing more than  $n_{\max}^\alpha + n_{\max}^\beta$  members as *large*. The following proposition provides the characterization of large stable groups.

**Proposition 5A (Stability – Large  $n$ )** *When  $n > n_{\max}^\alpha + n_{\max}^\beta$ , stable groups take one of the following forms:*

1.  $n_{\max}^\alpha$  agents whose taste falls in  $[\underline{t}^\alpha, 1]$  and  $n_{\max}^\beta$  agents whose taste falls in  $[0, \bar{t}^\beta]$ ;
2.  $n$  agents of taste  $t = 1$ , or  $n$  agents of taste  $t = 0$ .

The first part of Proposition 5A describes stable groups in which the number of signals gathered in the information-collection phase is maximized. When groups are very large, any agent with  $t \in (0, 1)$  would desire a group in which the maximal amount of information on both issues is acquired. The only way to achieve this volume of information is to have a group in which  $n_{\max}^\alpha$  agents are in the interval  $[\underline{t}^\alpha, 1]$  and  $n_{\max}^\beta$  agents are in the interval  $[0, \bar{t}^\beta]$ . Note that, while in the free-information case stability always entailed some degree of similarity, the first part of Proposition 5A suggests that for large group sizes, stability is consistent with *extreme group polarization*.<sup>23</sup>

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<sup>22</sup>We avoid issues of equilibrium selection in that we assume that when a subgroup of agents of identical tastes acquires information, they cannot guarantee lower investment in information by shifting to a different group in which a different agent acquires their specific signals.

<sup>23</sup>We refer to groups that contain at least one agent in the interval  $[0, \bar{t}^\beta]$  and another agent in the interval  $[\underline{t}^\alpha, 1]$  as

In the second part of Proposition 5A, we describe a group, which is *always stable*, in which all agents have the same, extreme, taste parameter. This is a knife-edge case in which all agents get *no* utility from signals on their less-cared-for issue, and the only signals collected are  $n_{\max}^x$  signals on the more-cared-for issue  $x$ .

Going back to the analogy between large group size  $n$  and high information cost  $c$ , as noted, a higher information cost  $c$  lowers both  $n_{\max}^\alpha$  and  $n_{\max}^\beta$ . Thus, Proposition 5A suggests that polarization is extreme when information costs are high.

**Small Groups.** When groups are small, the optimal composition of signals for each agent entails few signals from each source. This suggests a weaker free rider problem, implying the possibility of the unconstrained optimal allocation being consistent with stability.

The following Lemma will be useful in linking stable groups with costly information to those identified in the free-information case through Proposition 2. For any  $t \in [0, 1]$ , let us define  $n_c(t) \equiv n_c^\alpha(t) + n_c^\beta(t)$ . It is the maximal number of signals that a group of agents of the same taste  $t$  would be willing to get in equilibrium.

**Lemma 2 (Individual Incentives and Group Size)** *Whenever  $n \leq n_c(t)$ ,  $n_f^x(t) \leq n_c^x(t)$  for  $x = \alpha, \beta$ .*

Lemma 2 links the size of the group to personal incentives to acquire information. As long as group size is sufficiently small (in terms of its size relative to  $n_c(t)$ ), the agent engages in information acquisition when her unconstrained optimal allocation of signals occurs. To see why this is the case, suppose that  $n_f^\alpha(t) + n_f^\beta(t) = n \leq n_c(t)$  but, for instance,  $n_f^\alpha(t) > n_c^\alpha(t)$ . Then, it must be the case that  $n_f^\beta(t) < n_c^\beta(t)$ . Since  $n_f^\alpha(t)$  and  $n_f^\beta(t)$  represent the unconstrained optimal allocation, they are selected in a way that (almost) equates the marginal returns from signals on either source. However, since  $n_f^\beta(t) < n_c^\beta(t)$ , the marginal benefit of the  $n_f^\beta(t)$ -th  $\beta$ -signal is greater than  $c$ . Thus, the marginal benefit of the  $n_f^\alpha(t)$ -th  $\alpha$ -signal should be greater than  $c$  as well, in contradiction to  $n_f^\alpha(t) > n_c^\alpha(t)$ .

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*extremely polarized.* In fact, Proposition 5A suggests that as  $n$  increases (or  $c$  decreases) there will be a greater volume of agents on each side of the taste spectrum. Note that these groups are never stable in the free-information case.

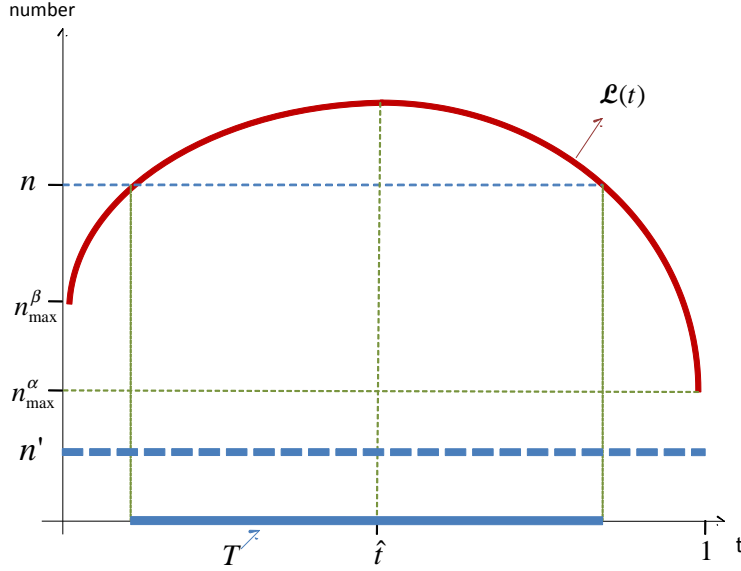


Figure 2: Small Group Size (Low Costs)

We now turn to the characterization of stable groups when  $n < n_{\max}^{\alpha} + n_{\max}^{\beta}$ . First, some of the homogeneous groups described in Proposition 2 are still stable when information is costly. Denote by  $m_c^{\alpha}(t)$  and  $m_c^{\beta}(t)$  the real numbers achieving equality within the constraints (3) and (4), respectively. We get that, for  $x = \alpha, \beta$ ,  $n_c^x(t) = \lfloor m_c^x(t) \rfloor$  if  $m_c^x(t) \geq 0$  and  $n_c^x(t) = 0$  if  $m_c^x(t) < 0$  (note that, by construction,  $m_c^x(t) < n_{\max}^x + 1$  for  $x = \alpha, \beta$ ). The number  $m_c^x(t)$  captures the point at which an agent of taste  $t$  equates the marginal return from an  $x$ -signal to the cost  $c$ . The function  $n_c(t)$  is then approximated by the function  $\mathcal{L}(t) = \max\{m_c^{\alpha}(t), 0\} + \max\{m_c^{\beta}(t), 0\}$  in Figure 2.<sup>24</sup> It is easy to see that  $\mathcal{L}(0) = n_{\max}^{\beta}$ ,  $\mathcal{L}(1) = n_{\max}^{\alpha}$ , and, as we show in the Appendix,  $\mathcal{L}(t)$  is piece-wise concave and reaches a maximum at  $\hat{t} \in (0, 1)$  when it is concave and positive.<sup>25</sup> Looking at Figure 2, when  $\min\{n_{\max}^{\alpha}, n_{\max}^{\beta}\} < n \leq n_c(\hat{t})$ , there is always a set of taste parameters  $T$  such that for any  $t \in T$ ,  $n \leq n_c(t)$ . By Lemma 2, this guarantees that groups formed by agents sharing the same unconstrained optimal allocation (corresponding to stable groups in the free-information case) and with tastes within  $T$ , are able to achieve their optimal allocation in the costly-information case as

<sup>24</sup>The shape depicted in Figure 2 corresponds to sufficiently low costs. See discussion below regarding comparative statics with respect to  $c$  for a description of the dependence of  $\mathcal{L}(t)$  on  $c$ .

<sup>25</sup>The concavity of  $\mathcal{L}(t)$  is a direct consequence of the decreasing marginal returns from signals.

well. That is, if stable groups of size  $n$  in the free-information case are characterized by the partition  $\{T_k^n\}_{k=0}^n$ , when information comes at a cost of  $c$ , for all the elements in the partition such that  $T_k^n \cap T \neq \emptyset$ , any group of  $n$  agents in  $T_k^n \cap T$  is stable.

We now turn our attention to polarized groups. The highly polarized groups appearing for large groups (Proposition 5A) are not stable for smaller group sizes.<sup>26</sup> Nonetheless, for intermediate group sizes, ones for which  $n > \min\{n_{\max}^\alpha, n_{\max}^\beta\}$ , *mildly polarized* groups in which some agents have extreme tastes, while others have moderate (and similar) tastes, can be stable. Indeed, suppose that  $n > n_{\max}^\alpha$ . We now illustrate that a stable group can be comprised of a sub-group of extremists who care a lot about issue  $A$  and get  $n_{\max}^\alpha$   $\alpha$ -signals, and a sub-group of moderates, who all acquire  $\beta$ -signals. Formally, assume that  $n_{\max}^\alpha$  agents care sufficiently about issue  $A$  so that: (i) they have strong enough incentives to collect  $n_{\max}^\alpha$   $\alpha$ -signals; and (ii) their unconstrained optimal allocation involves at least  $n_{\max}^\alpha$   $\alpha$ -signals (so that they do not prefer groups with greater  $\beta$ -signal acquisition). To capture these restrictions we define for  $x = \alpha, \beta$ :

$$W^x = \{t \mid n_c^x(t) = n_{\max}^x \text{ and } n_f^x(t) \geq n_{\max}^x\}.$$

It is easy to see that  $W^\alpha$  and  $W^\beta$  are intervals of the form  $W^\alpha = [\underline{t}, 1]$  and  $W^\beta = [0, \bar{t}]$ . Suppose  $n_{\max}^\alpha$  of the  $n$  agents have taste parameters in the interval  $W^\alpha$ . By construction, these agents are part of an optimal group whenever the number of  $\alpha$ - and  $\beta$ -signals collected is  $n_{\max}^\alpha$  and  $n - n_{\max}^\alpha$ , respectively.

We choose the remaining  $n - n_{\max}^\alpha$  agents to satisfy two conditions: (i) they care enough about issue  $B$  to have sufficient incentives to collect  $n - n_{\max}^\alpha$   $\beta$ -signals; and (ii) they care enough about issue  $A$  to assure that they would not prefer a group in which more than  $n - n_{\max}^\alpha$   $\beta$ -signals are collected. Formally, for  $x, y = \alpha, \beta$ ,  $x \neq y$ , we define:

$$Z^x = \{t \mid n_c^x(t) \geq n_f^x(t) \text{ and } n_f^x(t) = n - n_{\max}^y\}.$$

Thus, if we select the remaining  $n - n_{\max}^\alpha$  agents to have taste parameters within  $Z^\beta$ , by construction, these agents will be in an optimal group. In particular, combined with the  $n_{\max}^\alpha$  agents with tastes in

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<sup>26</sup>To see why, consider a polarized group in which some agents care very much about issue  $A$  (and their optimal groups entail  $n_{\max}^\alpha$   $\alpha$ -signals from source  $\alpha$ ), and some agents care very much about issue  $B$  (and their optimal groups entail  $n_{\max}^\beta$   $\beta$ -signals). Clearly, when  $n < n_{\max}^\alpha + n_{\max}^\beta$ , not all agents can be in their optimal group.

$W^\alpha$ , they form a stable group. Note that for any taste parameter  $t$  in  $Z^\beta$ ,  $n_f^\beta(t) = n - n_{\max}^\alpha < n_{\max}^\beta$ . Therefore,  $Z^\beta$  does not contain the extreme taste parameter  $t = 0$ . This implies that the stable group we just constructed involves a *milder degree of polarization* with respect to the large-group-size case.

The following proposition summarizes our discussion and provides the full characterization of stable groups for small group sizes (note that we continue using the notation of  $\{T_k^n\}_{k=0}^n$  for the partition corresponding to stable groups in the free-information case).<sup>27</sup>

**Proposition 5B (Stability – Small  $n$ )** *When  $n < n_{\max}^\alpha + n_{\max}^\beta$ , stable groups take one of the following forms:*

1. *If there is an interval  $T$  such that  $n \leq n_c(t)$  for all  $t \in T$ , then, for any  $k = 0, \dots, n$  for which  $T \cap T_k^n \neq \emptyset$ , any  $n$  agents of tastes in  $T \cap T_k^n$  (**homogeneous groups**).*
2. *If  $n > n_{\max}^x$ ,  $n_{\max}^x$  agents with tastes in  $W^x$ , and  $n - n_{\max}^x$  agents with tastes in  $Z^y$  (**mildly polarized groups**).*
3.  *$n$  agents of taste  $t = 1$ , or  $n$  agents of taste  $t = 0$ .*

Regarding the case  $n = n_{\max}^\alpha + n_{\max}^\beta$ , whenever  $n_{\max}^x > 1$  for  $x = \alpha, \beta$ , the groups described in Proposition 5A are the only stable ones. However, if  $n_{\max}^x = 1$  for some  $x$ , say  $\alpha$ , the full characterization of the stable groups includes additional types of groups consisting of  $n_{\max}^\beta$  agents on the  $t = 0$  extreme and one moderate agent. Since the full characterization involves some minor technical subtleties without adding qualitative novelties, we refer the interested reader to Proposition 5C presented in the Appendix.

Note that a consequence of our results is that there is a set of moderate taste parameters such that individuals with those tastes can only be part of stable groups that are either small (emulating the free-information environment) or very large (in which the moderate agents free ride on the extremists in the group, who collect all the information).<sup>28</sup>

<sup>27</sup>Note that whenever  $n < \min\{n_{\max}^\alpha, n_{\max}^\beta\}$  part (3) of Proposition 5B is subsumed in part (1).

<sup>28</sup>It is interesting to consider the consequences of side payments. When groups are small, so that stability entails the acquisition of the same signal of the free-information case, side payments have no consequence. For large groups, side payments allow agents to share the cost of information and invest (albeit indirectly) in more than one signal. In that case, the availability of side payments generates more information acquisition. In particular, for large group sizes, the

The analogy between small group size  $n$  and small information cost  $c$  suggests that, as  $c$  decreases, some polarization can still persist in stable groups (point (2) of Proposition 5B), but it is milder than the extreme polarization emerging for high  $c$ . Eventually, as  $c$  keeps on decreasing, polarized groups tend to disappear. Moreover, as  $c$  decreases, homogeneous groups identical to those identified as stable in the free-information case start emerging as stable (point (1) of Proposition 5B). In particular, stable groups composed of moderate individuals arise first, and eventually, for lower information costs, groups formed by extreme individuals emerge as well (see  $n'$  in Figure 2).<sup>29</sup>

A message that comes out of our analysis is that when *information gathering becomes cheaper (as a result, for instance, of better information technologies)*, *stable groups tend to become more homogeneous*. As mentioned in the Introduction, this insight is backed up by a large body of empirical work. Indeed, the introduction of the telephone made social affiliations depend far more on shared interests (e.g., Sproull and Kiesler (1991)). Similarly, the introduction of the internet is associated with a significant increase in the similarity of academic coauthors (see Rosenblat and Mobius (2004) and references therein).

**The Effects of Costs on Homogeneous Groups.** Information costs affect the shape of the function  $\mathcal{L}(t)$  and then, via point (1) of Proposition 5B, the interplay between group size and the selection of stable homogeneous groups from the free-information case. When costs are low, any agent is willing to acquire at least one signal on either issue (i.e., for all  $t$ ,  $n_c^\alpha(t), n_c^\beta(t) \geq 1$ ). In that case,  $\mathcal{L}(t)$  takes the form depicted in Figure 2. In terms of the stable homogeneous groups, as long as  $n \leq \min \{n_{\max}^\alpha, n_{\max}^\beta\}$  (for instance,  $n'$  in the figure), any stable group in the free-information environment is also stable when information comes at a cost  $c$ . As  $n$  increases, the set  $T$  shrinks and contains less extreme taste parameters. Thus, when costs are low, as group size increases, *the homogeneous stable groups that survive are those composed of members with more moderate tastes*.

As costs increase, extreme agents are willing to acquire information only on the issue they care

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introduction of side payment will tend to generate more similarity in stable groups. To see why, consider a polarized group (i.e., a group formed *only* by extremists on both issues). In such a group, an extremist on issue  $A$  will have an incentive to pay an extremist on issue  $B$  to acquire an  $\alpha$ -signal rather than a  $\beta$ -signal. However, the same goal could have been achieved at a lower cost by selecting another extremist on issue  $A$  as a peer in the first place. This suggests that extreme polarization still arises in groups that are sufficiently large (the lower bound on group size for extreme polarization to arise is higher than in our setting).

<sup>29</sup>Note that this is the case for a sufficiently large  $n$ . When  $n$  is relatively low, the pattern with which homogeneous stable groups emerge is slightly different and described below.

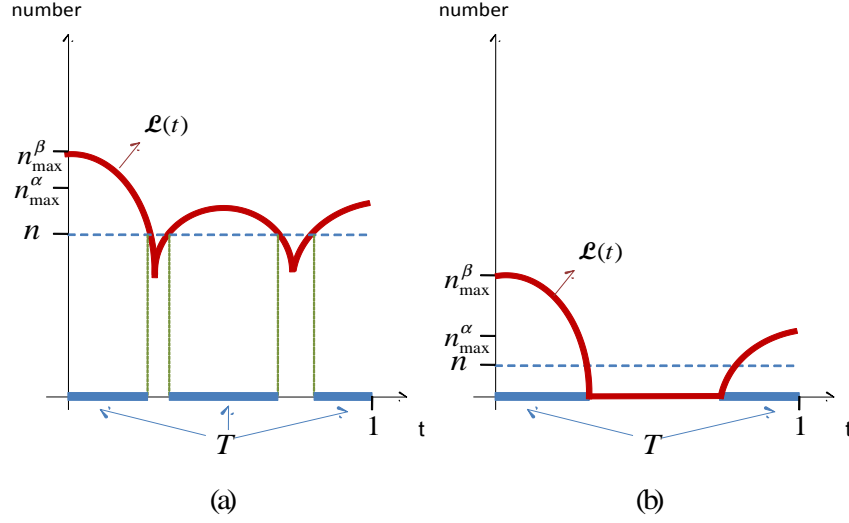


Figure 3: Stable Groups as Costs Increase

most about, and  $\mathcal{L}(t)$  becomes piece-wise concave (see panel (a) of Figure 3). For such situations, *homogenous stable groups are composed of members with either moderate or extreme tastes.*

As costs increase even more, there may be a range of moderate types who are not willing to acquire information on either issue and  $\mathcal{L}(t)$  has a dip for a range of moderate tastes (see panel (b) of Figure 3). In that case, whenever  $n \leq \mathcal{L}(t)$  for some  $t$ , either  $n_c^\alpha(t) = 0$  or  $n_c^\beta(t) = 0$ . Suppose  $n_c^\alpha(t) = 0$ . By construction,  $n_f^\beta(t) \leq n_c^\beta(t)$ . From Lemma 2, it must then be the case that  $n_f^\alpha(t) \leq n_c^\alpha(t) = 0$ . In particular,  $t$  belongs to the extreme interval  $T_0^n$ . An analogous analysis pertains to the case in which  $n_c^\beta(t) = 0$ . We therefore deduce that when costs are high, and  $n$  sufficiently low, *the homogenous stable groups correspond to extreme stable groups in the free-information case.*

## 5. STABILITY IN A FINITE POPULATION

Thus far, our notion of stability has imposed no restrictions on the groups available for the agents to join. Indeed, agents contemplate all possible combinations of tastes when choosing their optimal peer group. This is a good description for very large (strictly speaking, infinite) populations and allows us to derive a clean characterization of stable groups in different contexts. However, when a finite population of agents is partitioned into groups, there is a restricted set of groups that is conceivably

available to an agent. In this section, we study partitions of agents into groups that are “endogenously” stable. That is, we look for partitions of the population into groups such that no feasible deviation of an agent to a different existing group (or to a singleton), is profitable.

Suppose, then, that there is a finite set of agents  $N = \{1, \dots, l\}$ . Let  $N_i \subseteq N$  be the set of agents with taste  $t_i$  and let  $|N_i| = m_i$ . Thus, we can write  $N = \cup_{i=1}^r N_i$ . Without loss of generality, we assume  $t_1 > \dots > t_r$ .

In analogy with our baseline setup, the extended game that the agents in the set  $N$  play consists of two stages. First, the population  $N$  is partitioned into groups. Let  $\mathcal{G} = \{G_1, \dots, G_s\}$  be the resulting partition of  $N$ .<sup>30</sup> Importantly, here we do *not* exogenously fix the size of the groups composing the partition  $\mathcal{G}$ .

The second stage of the game coincides with the information-collection game described in Section 2. In order to extract the effects of finiteness on stability, and not confound them with free rider problems, we focus our discussion on the free-information case. This implies that, within each group  $G_i$ , an information-collection equilibrium as described in Section 3.1 arises in the second stage. Thus, to complete the analysis of this game, we can focus on the group-formation stage.

We now define stability in this setting as follows.<sup>31</sup>

**Definition (Stable Partition)** *A partition  $\mathcal{G} = \{G_1, \dots, G_s\}$  is stable if there exist no  $G_i, G_j \in \mathcal{G}$ , and  $a \in G_i$  such that agent  $a$  prefers either the group  $G_j \cup \{a\}$  or the singleton  $\{a\}$  to  $G_i$ .*

Since adding a member to a group is costless, and a new group member provides more information to the other members in the information-collection stage, groups always benefit from adding more members. Thus, we do not have to attend to any issues pertaining to the willingness of a group to accept a new member.<sup>32</sup>

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<sup>30</sup>So that  $G_i \cap G_j = \emptyset$  for all  $i \neq j$  and  $\bigcup_{i=1}^s G_i = N$ .

<sup>31</sup>The idea of a stable partition is reminiscent of the notion of the core, which also requires a type of group stability. Nonetheless, there are several important distinctions. First, the setup is different – cooperative games normally specify group values, rather than individual values *within* groups that are *derived endogenously from a strategic interaction*. Furthermore, cooperative solutions (e.g., the core) are more restrictive in that they allow for arbitrary group deviations, not only unilateral ones (thus, *the set of stable groups we look at includes the core*). Second, the core identifies stable allocations of resources, rather than characterizing the emerging partitions themselves.

<sup>32</sup>If communication from agent to agent is costly, a group may be unwilling to accept new members, or prefer particular new members to others. We elaborate on this in Section 6.

Certainly,  $\mathcal{G} = \{N\}$  is always a stable partition and, in fact, it is the welfare-maximizing partition. In what follows, we explore whether inefficient solutions can arise, i.e., we study the stability of other partitions.

The first step in characterizing stable partitions is the observation that agents of the same taste must be contained in the same group.

**Lemma 3 (Minimal Groups)** *In any stable partition  $\mathcal{G}$ , all agents with the same taste must be contained in the same group. That is, for each  $t_i$ , there exists a unique  $G_k \in \mathcal{G}$  such that  $N_i \subseteq G_k$ .*

Since Lemma 3 implies that any group  $G_k$  of a stable partition  $\mathcal{G}$  is such that  $N_i \subseteq G_k$  for at least one  $i$ , the maximal number of groups contained in a stable partition has to equal the total number  $r$  of different taste parameters in the population. The intuition for Lemma 3 is the following. Suppose that two agents  $x$  and  $y$  with the same taste parameter  $t_i$  belong to two groups  $G_h$  and  $G_k$ , respectively, such that  $G_h \neq G_k$ . Consider the agent  $x$  of taste parameter  $t_i$  in  $G_h$ . Since  $\mathcal{G}$  is stable, this agent must prefer to stay in  $G_h$  rather than being in  $G_k \cup \{x\}$ . However, since  $x$  and  $y$  have the same tastes, this implies that agent  $y$  must prefer being in  $G_h \cup \{y\}$  rather than in  $G_k$ , which contradicts the stability of  $\mathcal{G}$ .

We define the *fully segregated partition* to be the partition in which each set is formed only by agents of the same type, the partition  $\mathcal{G} = \{N_1, N_2, \dots, N_r\}$ .

Since Lemma 3 guarantees that agents of the same type cannot be divided across different groups in any stable partition, we can conclude that the fully segregated partition is the least efficient partition that could be stable. In what follows, we aim to identify the conditions under which this partition is, indeed, stable.

Consider a fully segregated partition, and consider an agent  $x$  of type  $t_i$  belonging to group  $N_i$  who is considering deviating to the group  $N_j \cup \{x\}$ . By staying in  $N_i$ , agent  $x$  is able to implement her optimal allocation of  $m_i$  signals across the sources  $\alpha$  and  $\beta$  in the information-collection stage. Deviating to  $N_j \cup \{x\}$  cannot be strictly profitable if  $m_j < m_i$ . On the other hand, if  $m_j \geq m_i$ , the group  $N_j \cup \{x\}$  is bigger than  $N_i$ , which implies more information gathered in the information-

collection stage. Thus, a deviation to  $N_j \cup \{x\}$  tends to be less beneficial if: (i) the taste parameters  $t_i$  and  $t_j$  are far from one another (as agent  $x$  finds herself in a group in which other agents' optimal choice of signal sources is very different than hers); and (ii) the size of the groups  $m_i$  and  $m_j$  are close to one another (as agent  $x$  benefits less from an increase in group size). To focus on the first issue (identifying taste distributions that allow for segregation) and ease our exposition, from now on we assume that  $|N_i| = m$  for all  $i$ . We are now ready to identify necessary and sufficient conditions for the fully segregated partition to be stable. This characterization directly exploits the characterization of the stable groups via the sequence  $\{T_k^m\}$  developed in Proposition 2. In order to ease the description of our results, here we assume that the sufficient conditions for the *full* sequence  $\{T_k^m\}_{k=0}^m$  to follow the pattern described in point (2) of Proposition 2 are met.<sup>33</sup>

**Proposition 6 (Stable Segregation)** *For any  $\{t_1, \dots, t_r\}$ , there is a sequence of intervals  $\{\mathcal{T}_i\}_{i=1}^r$  such that:*

1. *Full segregation is a stable partition if and only if for any  $i \neq j$ ,  $t_j \notin \mathcal{T}_i$ .*
2.  *$t_i \in \mathcal{T}_i$  for all  $i \in \{1, \dots, n\}$ , and for all  $i \in \{1, \dots, r\}$ ,  $\mathcal{T}_i$  is the union of contiguous intervals of the sequence  $\{T_k^m\}_{k=0}^m$ .*
3. *The sequence  $\{\mathcal{T}_i\}_{i=1}^r$  is increasing for  $t_i = t_1, \dots, t_k$ , for some  $k = 1, \dots, r - 1$ , and decreasing for  $t_i = t_{k+1}, \dots, t_r$ . That is, the intervals  $\{\mathcal{T}_i\}_{i=1}^r$  are narrower for extreme tastes, and wider for moderate tastes.*

As discussed above, full segregation is easier to sustain if the taste parameters  $\{t_1, \dots, t_r\}$  in the support of  $N$  are sufficiently far from one another. In particular, point (1) of Proposition 6 states that for any  $t_i \in \{t_1, \dots, t_r\}$ , we can identify an interval  $\mathcal{T}_i$  such that full segregation is stable if and only if all the other taste parameters in  $\{t_1, \dots, t_r\}$  lie *outside*  $\mathcal{T}_i$ . Specifically, consider an agent  $x$  with taste parameter  $t_i \in \{t_1, \dots, t_r\}$ . In the fully segregated partition, such an agent belongs to the group  $N_i$ . To show stability, we need to consider all possible deviations of agent  $x$  to any set  $N_j \cup \{x\}$ ,  $j \neq i$ . Since  $|N_i| = m$  for all  $i = 1, \dots, r$ , checking that deviations to the groups closest in tastes,  $N_{i-1} \cup \{x\}$  and

<sup>33</sup>These conditions are briefly discussed in Footnote 15 and formally derived in the Appendix. Recall that they amount to  $q_\alpha, q_\beta$  and  $m$  being high enough.

$N_{i+1} \cup \{x\}$ , are not profitable is sufficient.<sup>34</sup> Thus, requiring a deviation from  $N_i$  to  $N_{i+1} \cup \{x\}$  not to be profitable allows us to identify an upper bound for  $t_{i+1}$ . Similarly, requiring a deviation from  $N_i$  to  $N_{i-1} \cup \{x\}$  not to be profitable allows us to identify a lower bound for  $t_{i-1}$ . These two bounds together define the interval  $\mathcal{T}_i$ .

Point (2) of Proposition 6 states that the intervals  $\{\mathcal{T}_i\}_{i=1}^r$  are unions of intervals of the sequence  $\{T_k^m\}_{k=0}^m$  described in Proposition 2. To see why this is the case, recall that all agents with taste parameters in the same interval  $T_k^m$  agree on the optimal allocation of  $m$  signals across the two sources  $\alpha$  and  $\beta$  in the information-collection stage. The idea that agents in other groups have sufficiently different tastes is then captured by them agreeing (among their cohesive groups of  $m$  agents) on very different allocation of signals.

Point (3) of Proposition 6 shows the robustness of the basic intuitions obtained in Section 3 to the case where populations are finite. In particular, as is the case for the intervals  $\{T_k^m\}_{k=0}^m$ , the intervals  $\{\mathcal{T}_i\}_{i=1}^r$  are narrower when they contain more-extreme taste parameters. Intuitively, suppose that agent  $x$  has taste parameter  $t_i$ , and consider a deviation from the group  $N_i$  to  $N_{i+1} \cup \{x\}$ . If  $t_{i+1}$  is sufficiently far from  $t_i$ , such a deviation is not profitable. The benefit for agent  $x$  from joining  $N_{i+1}$  is to be in a group formed by  $m + 1$  agents rather than  $m$  agents. On the other hand, the cost of joining  $N_{i+1}$  comes from the expected sub-optimal allocation of  $m + 1$  signals across the sources  $\alpha$  and  $\beta$ . From Proposition 2, we know that the intervals  $\{T_k^m\}_{k=0}^m$ , each containing all the  $t$ 's that correspond to the same optimal allocation of  $m$  signals, are narrower for extreme taste parameters and wider for moderate taste parameters. Thus, for a given distance between  $t_i$  and  $t_{i+1}$ , the disagreement between agent  $x$  and the agents in  $N_{i+1}$  is stronger if  $t_i$  and  $t_{i+1}$  are taste parameters closer to the extremes than if they are moderate. Figure 3 illustrates graphically the structure of the intervals  $\mathcal{T}_i$  and  $\mathcal{T}_{i+1}$  as unions of sets of the original sequence  $\{T_k^m\}_{k=0}^m$ , and the resulting comparative statics we have discussed.

To conclude, one of the main insights from our baseline analysis was that similarity in stable groups is stronger for extreme than for moderate tastes. In the same spirit, point (3) of Proposition 6 shows that *segregation is easier to achieve for extreme types than for moderate ones*.

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<sup>34</sup>We have to consider just one potential deviation for agents with extreme tastes  $t_1$  and  $t_r$ .

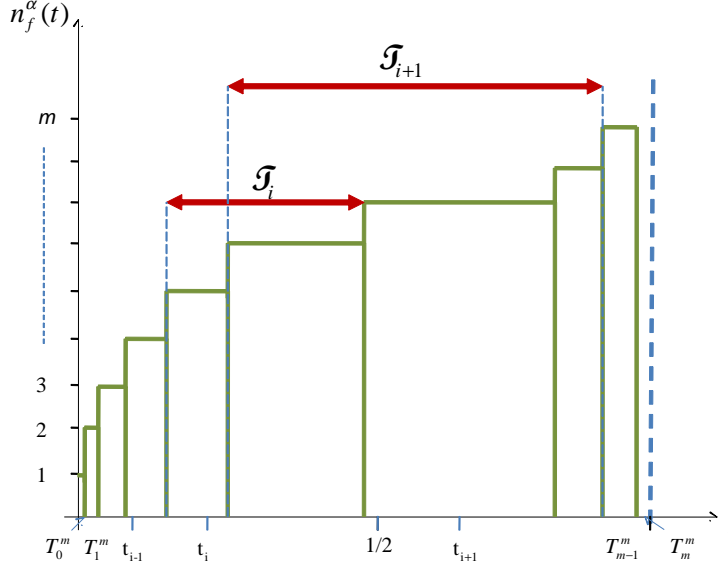


Figure 4: Segregation Intervals

## 6. CONCLUSION AND EXTENSIONS

The model developed in this paper addresses the properties of peer groups that arise in equilibrium when individuals have different tastes and share the information they gather with members of their group. If information gathering is free, stable groups are composed of individuals that are sufficiently similar in tastes. This similarity is more pronounced for extreme agents than for moderate ones. If information gathering is costly, stable groups can display taste polarization, which becomes more extreme as the group size increases. Finally, when the population is small, it is natural to consider stable partitions of the population into groups such that agents' deviations are restricted to joining one of the other groups in the population, or creating a group by themselves. In this setup, we showed that full segregation is easier to achieve for agents with extreme tastes than for agents with moderate ones. In what follows, we discuss some natural extensions of our model.

**6.1. General Production Functions.** A crucial element of our analysis is that agents' utilities exhibit decreasing marginal returns with respect to the total number of signals collected from each source. The more signals an agent receives (directly or indirectly) on a particular issue, the lower the value of an additional signal on that issue. Our information-based setup pinned down precisely

the shape of agents' utilities corresponding to any allocation of signals collected within a group. Nonetheless, the idea that utilities exhibit decreasing marginal returns to effort is a recurring theme in many models, where effort can have many meanings, ranging from information collection, as in the current paper, to investment levels in projects, to physical exertion on the job, etc.

The main assumption in our model is the way units of effort on each dimension (in our case, signals on different issues) are transformed into utiles (throughout the paper, captured by (1)). In a more general setup, agents have to invest effort on two dimensions  $\alpha$  and  $\beta$  that correspond to two different public projects. Agents differ in how much they care about either project. There are two production functions that map effort in the group into utilities derived from either project. Suppose that each agent can invest a unit of effort on either dimension, and that, ultimately, production functions  $f_\alpha$  and  $f_\beta$  represent the resulting outcome on the two dimensions. Suppose that  $f_\alpha, f_\beta$  are increasing and concave.<sup>35</sup> Thus, the general utility for an agent of taste  $t \in [0, 1]$  who is in a group of  $n$  agents,  $k^\alpha$  of whom invest their unit of effort in dimension  $\alpha$  and  $k^\beta = n - k^\alpha$  of whom invest their unit of effort in dimension  $\beta$ , is given by:

$$\tilde{U}(t, k^\alpha, k^\beta) \equiv tf_\alpha(k^\alpha) + (1-t)f_\beta(k^\beta).$$

This generalizes (1).<sup>36</sup>

For an agent of taste  $t$ , the optimal number of  $\alpha$  units out of  $n$  is given by  $n^\alpha(t) = \lfloor m^\alpha(t) \rfloor$ , where  $m^\alpha(t)$  solves

$$tf_\alpha(m^\alpha) + (1-t)f_\beta(n - m^\alpha) = tf_\alpha(m^\alpha - 1) + (1-t)f_\beta(n - m^\alpha + 1) \quad (5)$$

Note that the techniques introduced above to characterize group stability can still be used. In particular, stable groups in the free-information case are still characterized, as in point (1) of Proposition 2, by a partition of the interval  $[0, 1]$  into subintervals of types that agree on the optimal allocation of  $n$  signals across the two dimensions  $A$  and  $B$ . Nonetheless, the comparative statics pertaining to the potential dispersion within stable groups, i.e., point (2) of Proposition 2, depends on the shape of the production functions. Indeed, for  $x = \alpha, \beta$ , define  $\phi_x(k) \equiv f_x(k+1) - f_x(k)$  and

<sup>35</sup>Note that in terms of our information story, this allows for signals within a dimension to be correlated.

<sup>36</sup>In our analysis in the previous sections,  $f_\alpha(k^\alpha) = 1 - \frac{1}{2}(1 - q_\alpha)^{k^\alpha}$  and  $f_\beta(k^\beta) = 1 - \frac{1}{2}(1 - q_\beta)^{k^\beta}$ .

$\phi_x^{(1)}(k) = f'_x(k+1) - f'_x(k)$ . Applying the Implicit Function Theorem to condition (5), we get

$$\frac{dm^\alpha(t)}{dt} = \frac{-\frac{1}{t(1-t)}}{\frac{\phi_\alpha^{(1)}(m^\alpha-1)}{\phi_\alpha(m^\alpha-1)} + \frac{\phi_\beta^{(1)}(n-m^\alpha)}{\phi_\beta(n-m^\alpha)}},$$

which is positive.<sup>37</sup> Recall that, in our previous setting, the sign of  $\frac{d^2m^\alpha(t)}{dt^2}$  is positive for  $t > \frac{1}{2}$  and negative for  $t < \frac{1}{2}$ , which determines the interval pattern described in point (2) of Proposition 2. In this more general case, differentiating again, and rearranging terms, it is easy to show that the sign of  $\frac{d^2m^\alpha(t)}{dt^2}$  depends on the third derivative of the production functions.<sup>38</sup> In general, the third derivative of the production functions plays an important role in affecting the curvature of  $m^\alpha$  and, consequently, the relative lengths of the intervals that characterize the stable groups. Intuitively, since the optimal number of  $\alpha$ -signals increases with  $t$ , the marginal return of an  $\alpha$ -signal decreases with  $t$ . For  $m_f^\alpha(t)$  to be convex to the right of  $t = \frac{1}{2}$  (and, analogously, concave to the left of  $t = \frac{1}{2}$ ), the first force must prevail. In particular, the marginal return of an  $\alpha$ -signal (or  $\beta$ -signal) cannot decrease too rapidly. This, in turn, translates into bounds on the magnitude of the third derivatives of the production functions.

To shed some light on the link between the production functions and the structure of the stable groups, assume that  $f \equiv f_\alpha = f_\beta$ . It is easy to show that

$$\text{sign} \left[ f^{(l)}(k) \right] = (-1)^{l+1} \quad \text{for } l \leq 3 \quad (6)$$

is a sufficient condition for the function  $m^\alpha(t)$  to be convex for sufficiently large  $t$  and concave for sufficiently low  $t$ . Thus, in those ranges of  $t$ , the characterization of Section 3 still holds. That is, stability requires more similarity for more-extreme tastes.

<sup>37</sup>Our assumptions on the production functions guarantee that  $\phi_x(k) > 0$  while  $\phi_x^{(1)}(k) < 0$  for  $x = \alpha, \beta$ .

<sup>38</sup>Formally, for  $x = \alpha, \beta$  denoting  $\phi_x^{(2)}(k) = f''_x(k+1) - f''_x(k)$ ,  $\text{sign} \left( \frac{d^2m^\alpha(t)}{dt^2} \right)$  coincides with the sign of the following:

$$\begin{aligned} & \frac{dm^\alpha(t)}{dt} \left[ \phi_\alpha^{(1)}(m^\alpha-1) - \phi_\beta^{(1)}(n-m^\alpha) \right] \times \\ & \left\{ \frac{dm^\alpha(t)}{dt} \left[ t\phi_\alpha^{(1)}(m^\alpha-1) + (1-t)\phi_\beta^{(1)}(n-m^\alpha) \right] - [\phi_\alpha(m^\alpha-1) + \phi_\beta(n-m^\alpha)] \right\} + \\ & \frac{dm^\alpha(t)}{dt} \left[ t\phi_\alpha^{(2)}(m^\alpha-1) - (1-t)\phi_\beta^{(2)}(n-m^\alpha) \right] \times [\phi_\alpha(m^\alpha-1) + \phi_\beta(n-m^\alpha)]. \end{aligned}$$

Two examples of commonly used production functions that satisfy condition (6) are the following:

1.  $f(k) = k^\gamma$ , where  $\gamma \in (0, 1)$ . Note that

$$f^{(l)}(k) = \left[ \prod_{j=0}^{l-1} (\gamma - j) \right] k^{\gamma-l} \quad \text{for all } l > 0,$$

which satisfies (6).

2.  $f(k) = 1 - e^{-k\delta}$ , where  $\delta > 0$ . Here,

$$f^{(l)}(k) = (-1)^{l+1} \delta^l e^{-k\delta} \quad \text{for all } l > 0,$$

so that, again, (6) is satisfied.

Finally, in the same spirit of Proposition 3, it is possible to show that, under conditions that bound  $\frac{dm^\alpha(t)}{dt}$  above uniformly, the subintervals of types that agree on the optimal allocation of  $n$  signals across the two dimensions do not converge to singletons as the group size diverges.<sup>39</sup>

**6.2. Multiple Issues.** Our results extend to a multi-dimensional setting with  $h > 2$  issues. In this case, the type space is  $T = \{(t^1, \dots, t^h) \geq 0 \mid \sum_{l=1}^h t^l = 1\}$ . Each  $t^l$  represents the weight an individual assigns to issue  $l = 1, \dots, h$ , and the accuracy of a signal on issue  $l$  is given by  $q_l$ . Following similar analysis to that in Section 3, the characterization of stable groups of size  $n$  in the free-information case is identified by a partition  $\{T_{(j_1, j_2, \dots, j_h)}^n\}_{j_m \geq 0, \sum j_m = n}$  of  $T$ . Each element in the partition  $T_{(j_1, \dots, j_h)}$  corresponds to an  $(h-1)$ -dimensional rectangle consisting of types whose optimal allocation of  $n$  signals calls for  $j_m$  signals on each dimension  $m$ . Moreover, an analogue of point (2) of Proposition 2 extends to the multi-dimensional case as well. Indeed, controlling for the allocation of signals across all issues but  $m, m' \in \{1, \dots, h\}$ , we get a subset of the elements of the partition  $\{T_{(j_1, j_2, \dots, j_h)}^n\}_{j_m \geq 0, \sum j_m = n}$  that exhibits similar comparative statics those of Proposition 2. In particular, stability requires more similarity for more extreme values of  $t^m$  and  $t^{m'}$ .

The analysis in Section 4 can also be directly extended to the multi-dimensional setting. Indeed, agents that are extremist on one dimension are still the ones with the highest incentives to collect

<sup>39</sup>Namely, we need to require that for any  $t \in (0, 1)$  there exists  $v$  such that for all  $n$ ,  $\frac{dm^\alpha(t)}{dt} < v$ .

information on that dimension. Thus, for any dimension  $l = 1, \dots, h$ , we can identify the maximal number of signals that can be collected in equilibrium on that dimension as  $n_{\max}^l$ . Whenever  $n \geq \sum_{l=1}^h n_{\max}^l$ , extremely polarized groups are stable, as in Proposition 5A. Finally, if the group size is small enough, stability entails the acquisition of a signal profile no different than that acquired in the free-information case, and the characterization of stable groups is similar to the one in Proposition 5B.

**6.3. Adding New Members.** In the setup presented in Sections 2, 3, and 4, the group size is fixed. This assumption is relaxed in Section 5, where individuals can opt to leave their group and join another one. However, in that analysis, the other group members always benefit (at least weakly) from the addition of a member of any taste. Nonetheless, if adding new members is limited or costly, a group could prefer certain additional members over others, or stop accepting new members altogether. As a natural extension, then, we now consider the problem of a group that is given the opportunity of adding one new member and has to select the member's taste parameter. Note that this extension can be seen as the first step toward an analysis of the dynamics of group formation.

Consider a group of agents with tastes  $t_1 > \dots > t_n$  and let  $k^*$  be the number of  $\alpha$ -signals collected in equilibrium in this group. Suppose that an agent of taste  $t^P$  chooses an additional agent for the group.<sup>40</sup>

The following proposition illustrates the optimal range of tastes for an additional member that an agent with taste parameter  $t^P$  would choose.

**Proposition 8 (Optimal Additional Members)** *There exists  $\tilde{t} \in [0, 1]$  such that*

1. *If  $t_{k^*+1} < \tilde{t} < t^P$ , then any agent with taste  $t \geq \tilde{t}$  would be optimal.*
2. *If  $t^P < \tilde{t} < t_{k^*+1}$ , then any agent with taste  $t < \tilde{t}$  would be optimal.*
3. *In all other cases, any agent would be optimal.*

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<sup>40</sup>The agent of taste  $t^P$  can be a principal who contemplates adding an expert to an already existing team, a club that considers a new member, a group of friends adding new people to their circle, etc. Alternatively, the agent of taste  $t^P$  can be thought of as the pivotal member in a group using an election to determine the tastes of an additional member in any of the above contexts. In general, the agent who makes the choice may or may not be part of the original group of agents.

The proof is straightforward, and, therefore, omitted. Intuitively, if, in the original group, precisely  $k^*$  agents chose source  $\alpha$ , then adding a member can shift the number of agents choosing source  $\alpha$  to  $\hat{k} \in \{k^*, k^* + 1\}$ . Thus, when the selecting agent cares sufficiently about the realization of issue  $A$ , she would like  $k^* + 1$   $\alpha$ -signals to be collected in the new group. Then, two cases are possible: if there are already  $k^* + 1$  agents who are willing to choose  $\alpha$ -signals in a group of  $n + 1$  agents (i.e., they agree with the selecting agent on the allocation of the new available signal), then the additional member's tastes are inconsequential. Otherwise, a new member who agrees with the selecting agent on the allocation of the new available signal has to be introduced.<sup>41</sup>

**6.4. Externalities.** In our setup, once information is collected, all agents in the group agree on which is the more likely realization of each issue. Therefore, adding positive action externalities (say, by requiring all agents in a group to make the same collective decision, determined by a threshold vote), would not alter our results. A natural extension of our setup would allow for non-trivial externalities in actions. In our model, this would require adding another dimension of heterogeneity, one pertaining to reactions to information. That is, agents may differ in their inclinations to choose the action 1 on either issue.<sup>42</sup> Externalities in actions would then take the form of agents caring positively about the number of agents in their group who make similar decisions to theirs.<sup>43</sup> In such a model, there is an interplay between both types of heterogeneity (that regarding issue weights, and that regarding actions within each issue). Indeed, action externalities may affect agents' incentives to acquire information on either dimension.<sup>44</sup> In our baseline analysis, such a setup would, if anything, generate stronger similarity with respect to issue weights. It would also lead to limited heterogeneity in tastes over choices within each issue.

In reality, there are many examples in which there are no action externalities (e.g., internet forums,

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<sup>41</sup>It follows that  $\tilde{t}$  is the taste parameter that identifies an additional agent who would be indifferent between  $k^*$  and  $k^* + 1$   $\alpha$ -signals (in the new group of  $n + 1$  agents).

<sup>42</sup>For our information structure, as long as each agent wants to match the realization of each issue to some extent, this type of heterogeneity would play a role only when the realization of one issue does not get revealed in the information collection stage.

<sup>43</sup>For example, if  $A$  stands for food and  $B$  stands for wine, the two realizations within each issue may be thought of as two restaurants and two wine bars, respectively, and agents care about how many of their peers join them in each. Similarly, if  $A$  captures issues regarding local elections and  $B$  issues regarding general elections, agents ultimately vote on decisions in both types of elections and may care about the number of agents who would vote with them.

<sup>44</sup>Note that in such a setup, an agent needs to worry about the effect of an additional signal on either issue not only on their own action, but also on others'.

long-distance friendships, online social networks, etc.). Conceptually, one of the contributions of our analysis is to show that, *even in the absence of action externalities*, homophily can still arise through pure information-based externalities. Moreover, information-based homophily follows some interesting comparative statics patterns, analyzed in this paper, and supported by evidence, that an externalities-based model would not be able to deliver endogenously.

## 7. REFERENCES

- Benhabib, J., A. Bisin, and M. O. Jackson** (editors) (2009), *The Handbook of Social Economics*, Elsevier.
- Calvó-Armengol A., J. De Martí and A. Prat** (2009), “Endogenous Communication in Complex Organizations,” mimeo.
- Che, Y-K. and N. Kartik** (2008), “Opinions as Incentives,” mimeo.
- Coleman, J.** (1966), *Medical Innovation: A Diffusion Study*, Second Edition, Bobbs-Merrill. New York.
- Conley, T., and C. Udry** (2005), “Learning About a New Technology: Pineapple in Ghana,” mimeo, Yale University.
- Currarini, S., M. O. Jackson, and P. Pin** (2008), “An Economic Model of Friendship: Homophily, Minorities and Segregation,” *Econometrica*, forthcoming.
- Ellickson, B., B. Grodal, S. Scotchmer, and W. R. Zame** (1999), “Clubs and the Market,” *Econometrica*, **67**, 1185-1218.
- Evans, W. N., W. E. Oates, and R. M. Schwab** (1992), “Measuring Peer Group Effects: A Study of Teenage Behavior,” *The Journal of Political Economy*, **100(5)**, 966-991.
- Foster, A. D. and M. R. Rosenzweig** (1995), “Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture,” *The Journal of Political Economy*, **103(6)**, 1176-1209.
- Gerardi, D. and L. Yariv** (2008), “Costly Expertise,” *American Economic Review, Papers and Proceedings*, **98(2)**, 187-193.
- Glaeser, E. L., B. Sacerdote and J. A. Scheinkman** (1996), “Crime and Social Interaction,”

*The Quarterly Journal of Economics*, **111(2)**, 507-548.

**Goeree, J., M. A. McConnell, T. Mitchell, T. Tromp, and L. Yariv** (2009), "The 1/d Law of Giving," *American Economic Journal: Microeconomics*, forthcoming.

**Granovetter, M.** (1994), *Getting a Job: A Study of Contacts and Careers*, Northwestern University Press. Evanston.

**Huckfeldt, R. R. and J. Sprague** (1995), *Citizens, Politics and Social Communication*, Cambridge Studies in Public Opinion and Political Psychology.

**Kandell, D. B.** (1978), "Homophily, Selection and Socialization in Adolescent Friendships," *The American Journal of Sociology*, **84(2)**, 427-436.

**Katz, E. and P. F. Lazarsfeld** (1955), *Personal influence: The part played by people in the flow of mass communication*, Glencoe, IL: Free Press.

**Lazarsfeld, P. F. and R. K. Merton** (1954), "Friendship as a social process: a substantive and methodological analysis," in *Freedom and Control in Modern Society*, ed. M Berger, 18-66. New York: Van Nostrand.

**Lipsman A.** (2008), "Social Networking Explodes Worldwide as Sites Increase Their Focus on Cultural Relevance," comScore, 12/3/2008.

**Lynd, R. S., and H. M. Lynd** (1929), *Middletown: A Study in Contemporary American Culture*. New York, Harcourt, Brace and Company, New York, NY.

**Manski, C. F.** (1993), "Identification of Endogenous Social Effects: The Reflection Problem," *The Review of Economic Studies*, **60(3)**, 531-542.

**Manski, C. F.** (2000), "Economic Analysis of Social Interactions," *The Journal of Economic Perspectives*, **14(3)**, 115-136.

**Marmaros, D. and B. Sacerdote** (2006), "How do Friendships Form?," *The Quarterly Journal of Economics*, **121(1)**, 79-119.

**Mayer, A. and S. L. Puller** (2008), "The Old Boy (and Girl) Network: Social Network Formation on University Campuses," *Journal of Public Economics*, **92(1-2)**, 329-347.

**McPherson, M., L. Smith-Lovin, and J. Cook** (2001), "Birds of a Feather: Homophily in Social Networks," *Annual Review of Sociology*, **27**, 415-444.

**Peski, M.** (2008), “Complementarities, Group Formation and Preferences for Similarity,” mimeo.

**Rosenblat, T. S. and M. M. Mobius** (2004), “Getting Closer or Drifting Apart,” *Quarterly Journal of Economics*, **119(3)**, 971-1009.

**Sproull, L. and S. Kiesler** (1991), *Connections – New Ways of Working in the Networked Organization*, MIT Press, Cambridge, MA.

**Tiebout, C. M.** (1956), “A Theory of Local Expenditures,” *The Journal of Political Economy*, **64(5)**, 416-424.

**Topa, G.** (2001), “Social Interactions, Local Spillovers and Unemployment,” *The Review of Economic Studies*, **68(2)**, 261-295.

**Wooders, M., E. Cartwright, and R. Selten** (2006), “Behavioral Conformity in Games with Many Players,” *Games and Economic Behavior*, **57**, 347-360.

## 8. APPENDIX

**Proof of Lemma 1A.** Existence follows directly from the construction in Section 3. Indeed, choosing  $k^* = 0$  if (2) is not satisfied for any positive integer and  $k^*$  as the maximal integer between 0 and  $n$  satisfying (2) otherwise clearly defines a monotonic equilibrium as prescribed. In order to show uniqueness of the number of  $\alpha$ -signals chosen in equilibrium, we show that if  $(x_1, \dots, x_n)$  is a pure equilibrium such that for some  $i > j$ ,  $x_i = \alpha$  and  $x_j = \beta$ , then  $(y_1, \dots, y_n) \in \{\alpha, \beta\}^n$ , where  $y_l = x_l$  for all  $l \neq i, j$ ,  $y_i = \alpha$ , and  $y_j = \beta$  constitutes a pure equilibrium as well. Indeed, assume that

$$n^\alpha = |\{x_l = \alpha, l \neq i, j\}| \quad \text{and} \quad n^\beta = |\{x_l = \beta, l \neq i, j\}|.$$

$(x_1, \dots, x_n)$  consisting an equilibrium requires that agent  $i$  best responds. In particular,  $U(t_i, n^\alpha + 1, n^\beta + 1) \geq U(t_i, n^\alpha + 2, n^\beta)$ . That is,

$$\begin{aligned} & t_i \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{n^\alpha + 1} \right] + (1 - t_i) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{n^\beta + 1} \right] \\ \geq & t_i \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{n^\alpha + 2} \right] + (1 - t_i) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{n^\beta} \right]. \\ \Leftrightarrow & \frac{1 - t_i}{t_i} \geq \frac{(1 - q_\alpha)^{n^\alpha + 1} q_\alpha}{(1 - q_\beta)^{n^\beta} q_\beta}. \end{aligned}$$

Similarly, agent  $j$  best responding requires that  $U(t_j, n^\alpha + 1, n^\beta + 1) \geq U(t_j, n^\alpha, n^\beta + 2)$ , translating to

$$\frac{t_j}{1 - t_j} \geq \frac{(1 - q_\beta)^{n^\beta + 1} q_\beta}{(1 - q_\alpha)^{n^\alpha} q_\alpha}.$$

Since  $\frac{1-t_j}{t_j} \geq \frac{1-t_i}{t_i}$ , it follows that the above best response restrictions hold for agents  $i$  and  $j$  under the profile  $(y_1, \dots, y_n)$  as well, while all other players' best responses remain unchanged. The claim follows.  $\blacksquare$

**Proof of Proposition 1.** For any taste  $t$ , an optimal choice for a group entails choosing  $n_f^\alpha(t)$  agents who choose an  $\alpha$ -signal and  $n_f^\beta = n - n_f^\alpha(t)$  agents who choose a  $\beta$ -signal. From Lemma 1, this is tantamount to choosing a group of agents with tastes  $t_1 \geq t_2 \geq \dots \geq t_n$  (of which agent  $t$  is a member) such that condition (2) holds only for  $i = 1, \dots, n_f^\alpha(t)$ . Thus,

1. For  $i = 1, \dots, n_f^\alpha(t)$ ,

$$U(t_i, n_f^\alpha(t), n_f^\beta(t)) \geq U(t_i, n_f^\alpha(t) - 1, n_f^\beta(t) + 1).$$

Define  $l(t)$  so that

$$U(l(t), n_f^\alpha(t), n_f^\beta(t)) = U(l(t), n_f^\alpha(t) - 1, n_f^\beta(t) + 1).$$

Thus,  $l(t)$  is the preference parameter of an agent who is precisely indifferent between allocations prescribing  $n_f^\alpha(t)$  or  $n_f^\alpha(t) - 1$   $\alpha$ -signals. The condition then translates to  $t_1 \geq \dots \geq t_{n_f^\alpha(t)} \geq l(t)$ .

2. For  $i = n_f^\alpha(t) + 1, \dots, n$ ,

$$U(t_i, n_f^\alpha(t), n_f^\beta(t)) \geq U(t_i, n_f^\alpha(t) + 1, n_f^\beta(t) - 1).$$

In analogy to the above, define  $h(t)$  so that

$$U(h(t), n_f^\alpha(t), n_f^\beta(t)) = U(h(t), n_f^\alpha(t) + 1, n_f^\beta(t) - 1).$$

That is,  $h(t)$  is the preference parameter of an agent who is precisely indifferent between allocations prescribing  $n_f^\alpha(t)$  or  $n_f^\alpha(t) + 1$   $\alpha$ -signals. The condition then translates to  $h(t) > t_{n_f^\alpha(t)+1} \geq \dots \geq t_n$ .  $\blacksquare$

**Proof of Proposition 2.**

1. For any  $k = 1, \dots, n$ , denote by  $t(k)$  the taste parameter with which an agent is indifferent between  $k - 1$  and  $k$   $\alpha$ -signals. That is,

$$U(t(k), k - 1, n - k + 1) = U(t(k), k, n - k).$$

It follows that an agent with taste parameter  $t$  would like no  $\alpha$ -signals whenever  $t \in [0, t(1))$ , would like  $n$   $\alpha$ -signals whenever  $t \in [t(n), 1]$ , and would like  $n_f^\alpha(t) = k \in \{1, \dots, n - 1\}$   $\alpha$ -signals if and only if  $t \in [t(k), t(k + 1))$ . Since, from Proposition 1, each agent can achieve her optimal allocation of signals with some group, stability boils down to all of the group members agreeing on the ideal number of signals of each source that are to be collected. The claim follows.

2. From the definition of  $n_f^\alpha(t)$ , we have  $n_f^\alpha(t) = \lfloor m_f^\alpha(t) \rfloor$  if  $m_f^\alpha(t) \in [0, n]$ ,  $n_f^\alpha(t) = 0$  if  $m_f^\alpha(t) < 0$ , and  $n_f^\alpha(t) = n$  if  $m_f^\alpha(t) > n$ , where  $m_f^\alpha(t)$  is the number achieving equality in condition (2). Simple algebraic manipulation yields:

$$m_f^\alpha(t) = \frac{\ln\left(\frac{1}{t} - 1\right) + n \ln(1 - q_\beta) + \ln(1 - q_\alpha) + \ln\frac{q_\beta}{q_\alpha}}{\ln(1 - q_\alpha) + \ln(1 - q_\beta)}.$$

Differentiating  $m_f^\alpha(t)$  we get:

$$\begin{aligned} \frac{dm_f^\alpha(t)}{dt} &= -\frac{1}{t(1-t)\ln[(1-q_\alpha)(1-q_\beta)]} > 0; \\ \frac{d^2m_f^\alpha(t)}{dt^2} &= \frac{1-2t}{[t(1-t)]^2\ln[(1-q_\alpha)(1-q_\beta)]}. \end{aligned}$$

Therefore,  $m_f^\alpha(t)$  is an increasing function that is concave up to  $t = \frac{1}{2}$  and convex thereafter. Since for any  $k = 1, \dots, n - 1$ ,  $T_k^n = \left(m_f^\alpha\right)^{-1}([k, k + 1))$ , this implies that the sequence of intervals  $\{T_k^n\}_{k=1}^{n-1}$  is such that the intervals are increasing in length until the interval  $T_{\frac{n}{2}}^n$  such that  $1/2 \in T_{\frac{n}{2}}^n$  and decreasing thereafter.

Let us now address the extreme intervals  $T_0^n$  and  $T_n^n$ . We will now show that these intervals follow the same pattern of  $\{T_k^n\}_{k=1}^{n-1}$  if either  $q_\alpha, q_\beta$  are high enough, or  $n$  is high enough. Recall that  $T_0^n = [0, t(1))$  and  $T_n^n = [t(n), 1]$ . From the definition of  $t(k)$  above, we get

$$t(k) = \frac{(1 - q_\beta)^{n-k} q_\beta}{(1 - q_\alpha)^{k-1} q_\alpha + (1 - q_\beta)^{n-k} q_\beta}.$$

We have that  $t(1) \leq t(2) - t(1)$  (i.e., interval  $T_0^n$  is shorter than  $T_1^n$ ) if and only if

$$\frac{2(1 - q_\beta)^{n-1} q_\beta}{q_\alpha + (1 - q_\beta)^{n-1} q_\beta} \leq \frac{(1 - q_\beta)^{n-2} q_\beta}{(1 - q_\alpha) q_\alpha + (1 - q_\beta)^{n-2} q_\beta}.$$

Rearranging, the above condition is satisfied if and only if

$$2q_\alpha(1 - q_\alpha)(1 - q_\beta) + (1 - q_\beta)^{n-1} q_\beta \leq q_\alpha. \quad (7)$$

Condition (7) is a necessary and sufficient condition on  $q_\alpha, q_\beta$ , and  $n$  such that the interval  $T_0^n$  follows the same pattern of the sequence  $\{T_k^n\}_{k=1}^{n-1}$ . Note that, since  $(1 - x)x$  is maximized in  $[\frac{1}{2}, 1]$  at  $x = 1/2$ ,  $q_\alpha, q_\beta \geq \frac{1}{2}$  is a sufficient condition to guarantee (7), since we have

$$2(1 - q_\alpha)q_\alpha(1 - q_\beta) + (1 - q_\beta)^{n-1} q_\beta \leq (1 - q_\alpha)q_\alpha + \frac{1}{4} \leq \frac{1}{2} \leq q_\alpha.$$

Moreover, if  $(1 - q_\alpha)(1 - q_\beta) < 1/2$ , condition (7) holds for  $n$  large enough.

The interval  $T_n^n$  is shorter than  $T_{n-1}^n$  if and only if  $1 - t(n) \leq t(n) - t(n-1)$ . After rearranging, this is equivalent to

$$2q_\beta(1 - q_\alpha)(1 - q_\beta) + (1 - q_\alpha)^{n-1} q_\alpha \leq q_\beta. \quad (8)$$

As before, it is easy to see that if  $(1 - q_\alpha)(1 - q_\beta) < 1/2$ , condition (8) is satisfied for large enough  $n$ , and that  $q_\alpha, q_\beta \geq \frac{1}{2}$  is a sufficient condition for (8) to be satisfied for any  $n$ . ■

### Proof of Proposition 3.

1. To stress the comparative statics on  $n$ , let  $x_f^\alpha(n, t) \equiv m_f^\alpha(t)$  with  $m_f^\alpha(t)$ , as defined before, for group size  $n$ . Then, for any  $t \in [0, 1]$ , we have

$$\begin{aligned} x_f^\alpha(n, t) &= \frac{\ln\left(\frac{1}{t} - 1\right) + n \ln(1 - q_\beta) + \ln(1 - q_\alpha) + \ln\frac{q_\beta}{q_\alpha}}{\ln(1 - q_\alpha) + \ln(1 - q_\beta)} \\ &= A + Bn + C \ln\left(\frac{1}{t} - 1\right), \end{aligned}$$

where  $A = \frac{\ln(1-q_\alpha) + \ln \frac{q_\beta}{q_\alpha}}{\ln(1-q_\alpha) + \ln(1-q_\beta)}$ ,  $B = \frac{\ln(1-q_\beta)}{\ln(1-q_\alpha) + \ln(1-q_\beta)}$ , and  $C = (\ln(1-q_\alpha) + \ln(1-q_\beta))^{-1}$ . Consider  $t'$ ,  $t''$ , and  $n$  such that  $t', t'' \in T_k^n$  for some  $k = 1, \dots, n-1$ . From the definition of  $\{T_k^n\}_{k=1}^{n-1}$ , this implies that  $\lfloor x_f^\alpha(n, t') \rfloor = \lfloor x_f^\alpha(n, t'') \rfloor$ , so that  $t', t'' \in T_{\lfloor x_f^\alpha(n, t') \rfloor}^n$ .

First, we show that there exists  $\hat{n} > n$  such that  $t', t'' \in T_h^{\hat{n}}$  for some  $h = 1, \dots, \hat{n}-1$ . Let  $\Delta = \max\{x(t', n) - \lfloor x(t', n) \rfloor, x(t'', n) - \lfloor x(t'', n) \rfloor\}$ . That is,  $\Delta$  is the maximal distance between  $x(t', n)$  and  $x(t'', n)$  and their rounding down to an integer. Let  $r$  be an integer such that  $rB - \lfloor rB \rfloor < 1 - \Delta$ . Therefore, for the integer  $d \equiv \lfloor rB \rfloor$ , we have  $B \in [\frac{d}{r}, \frac{d+1-\Delta}{r})$ . Define  $\hat{n} = n + r$ . Notice that

$$x(t', \hat{n}) \in [x(t', n) + d, x(t', n) + d + 1 - \Delta);$$

$$x(t'', \hat{n}) \in [x(t'', n) + d, x(t'', n) + d + 1 - \Delta).$$

From the definition of  $\Delta$ , it follows that  $\lfloor x(t', \hat{n}) \rfloor = \lfloor x(t'', \hat{n}) \rfloor = \lfloor x(t', n) \rfloor + d$ . Thus,  $t', t'' \in T_{\lfloor x_f^\alpha(n, t') \rfloor + d}^{\hat{n}}$  which proves our claim.

Moreover, note that, if  $t' > t''$ ,  $x_f^\alpha(n, t') - x_f^\alpha(n, t'') = C [\ln(\frac{1}{t'} - 1) - \ln(\frac{1}{t''} - 1)] < 1$ . Thus, for any  $n'$ ,  $x_f^\alpha(n', t') - x_f^\alpha(n', t'') = x_f^\alpha(n, t') - x_f^\alpha(n, t'') < 1$ . This implies that for any  $n'$ , either  $t', t'' \in T_h^{n'}$ , or  $t' \in T_{h+1}^{n'}$  and  $t'' \in T_h^{n'}$  for some  $h$ . That is, if  $t', t''$  are two taste parameters for which there is agreement on the optimal number of  $\alpha$ -signals in  $\{1, \dots, n-1\}$  for a group of size  $n$ , then even if they disagree for some  $n' > n$ , such disagreement can pertain to the allocation of at most *one* signal.

**2.** Observe that the interval  $T_0^n$  contains all  $t$  such that  $t < \frac{(1-q_\beta)^{n-1} q_\beta}{q_\alpha + (1-q_\beta)^{n-1} q_\beta} = t^n(1)$ . For any  $q_\alpha, q_\beta \in (0, 1)$ ,  $t^n(1) \searrow_{n \rightarrow \infty} 0$ . It follows that  $T_0^{n'} \subsetneq T_0^n$  for any  $n' > n$ . Furthermore, the interval  $T_0^n$  shrinks to a singleton as the size of the group becomes infinitely large. Similarly,  $T_n^n$  contains all  $t \geq \frac{q_\beta}{(1-q_\alpha)^{n-1} q_\alpha + q_\beta} = t^n(n)$ . For any  $q_\alpha, q_\beta \in (0, 1)$ ,  $t^n(n) \nearrow_{n \rightarrow \infty} 1$ , and so  $T_n^{n'} \subsetneq T_n^n$  for any  $n' > n$  and the interval  $T_n^n$  shrinks to a singleton as the size of the group becomes infinitely large. ■

**Lemma 1B (Costly Information – Existence and Uniqueness)** *For any group of  $n$  agents with*

*tastes  $t_1 \geq t_2 \geq \dots \geq t_n$ , there exists  $k^\alpha \in \{0, \dots, n\}$  and  $k^\beta \in \{1, \dots, n+1\}$ ,  $k^\beta > k^\alpha$ , such that all agents  $i \leq k^\alpha$  acquiring the  $\alpha$ -signal, all agents  $i \geq k^\beta$  acquiring the  $\beta$ -signal, and all other agents not acquiring information, constitutes a Nash equilibrium of the information-collection game. Furthermore, all Nash equilibria of the information-collection game entail the same num-*

ber of agents acquiring the  $\alpha$ - and  $\beta$ -signals.

**Proof of Lemma 1B.** Let  $t_1 \geq \dots \geq t_n$ . Each agent has to decide, first, whether to acquire a signal or forgo information gathering and, second, upon deciding to acquire a signal, whether to acquire an  $\alpha$ -signal or a  $\beta$ -signal.

To construct an equilibrium in the information-collection game, let  $\mu^\alpha$  be the maximal integer  $h$  such that (3) holds with taste  $t_h$ . That is, the maximal integer  $h$  for which  $\frac{t_h}{2} (1 - q^\alpha)^{h-1} q^\alpha \geq c$ .

Similarly, let  $\mu^\beta$  be the minimal integer  $h$  such that (4) holds with taste parameter  $t_h$ . So  $h$  is the minimal integer for which  $\frac{(1-t_h)}{2} (1 - q^\beta)^{h-1} q^\beta \geq c$ .

If  $\mu^\alpha \geq \mu^\beta$ , then all agents can be induced to acquire information, and the construction of an equilibrium can be replicated from the proof of Lemma 1A.

If  $\mu^\alpha < \mu^\beta$ , define  $k^\alpha \equiv \mu^\alpha$  and  $k^\beta \equiv \mu^\beta$ . From our definitions of  $\mu^\alpha$  and  $\mu^\beta$ , in order to illustrate that the suggested profile constitutes an equilibrium, all that remains to be shown is that an agent acquiring a signal  $x$ , does not prefer to acquire a signal  $y \neq x$  when all other agents follow the profile. Indeed, suppose that  $i \leq k^\alpha < k^\beta$  and observe that

$$U(t_i, k^\alpha, k^\beta) - c \geq U(t_i, k^\alpha - 1, k^\beta) > U(t_i, k^\alpha - 1, k^\beta + 1) - c,$$

where the first inequality follows from (3) holding for  $i$ , and the second from the fact that  $\mu^\alpha < \mu^\beta$ . Thus, an agent of taste  $t_i$  does not profit from deviating to a choice of a  $\beta$ -signal instead of an  $\alpha$ -signal. An analogous argument holds for  $i \geq \mu^\beta > \mu^\alpha$ .

In order to show uniqueness of the number of  $\alpha$ -signals and  $\beta$ -signals chosen in equilibrium, we show that if  $(x_1, \dots, x_n)$  is a pure equilibrium such that for some  $i > j$ ,  $x_i = \alpha$  and  $x_j = \emptyset$ , then  $(y_1, \dots, y_n) \in \{\alpha, \beta, \emptyset\}^n$ , where  $y_l = x_l$  for all  $l \neq i, j$ ,  $y_i = \emptyset$ , and  $y_j = \alpha$  constitutes a pure equilibrium as well. Indeed, assume that  $n^\alpha = |\{x_l = \alpha, l \neq i, j\}|$  and  $n^\beta = |\{x_l = \beta, l \neq i, j\}|$ . The profile  $(x_1, \dots, x_n)$  constituting an equilibrium requires that

$$\begin{aligned} U(t_i, n^\alpha, n^\beta) - U(t_i, n^\alpha - 1, n^\beta) &= \frac{t_i}{2} (1 - q^\alpha)^{n^\alpha - 1} q^\alpha \geq c; \\ U(t_i, n^\alpha, n^\beta) - U(t_i, n^\alpha - 1, n^\beta + 1) &\geq 0; \\ U(t_j, n^\alpha + 1, n^\beta) - U(t_j, n^\alpha, n^\beta) &= \frac{t_j}{2} (1 - q^\alpha)^{n^\alpha} q^\alpha < c; \\ U(t_j, n^\alpha, n^\beta + 1) - U(t_j, n^\alpha, n^\beta) &= \frac{(1-t_j)}{2} (1 - q^\beta)^{n^\beta} q^\beta < c, \end{aligned}$$

where the first two inequalities correspond to agent  $i$ 's willingness to acquire the  $\alpha$ -signal (rather than avoiding investment in information, or investing in the  $\beta$ -signal), and the latter two inequalities assure that agent  $j$  is willing not to acquire a signal.

However, since  $t_j \geq t_i$ , it must be the case that

$$\begin{aligned} U(t_j, n^\alpha, n^\beta) - U(t_j, n^\alpha - 1, n^\beta) &= \frac{t_j}{2} (1 - q^\alpha)^{n^\alpha - 1} q^\alpha \geq c; \\ U(t_i, n^\alpha + 1, n^\beta) - U(t_i, n^\alpha, n^\beta) &= \frac{t_i}{2} (1 - q^\alpha)^{n^\alpha} q^\alpha < c. \end{aligned}$$

Furthermore, from the proof of Lemma 1A, it follows that

$$U(t_j, n^\alpha, n^\beta) - U(t_j, n^\alpha - 1, n^\beta + 1) \geq 0.$$

Suppose now that

$$U(t_i, n^\alpha, n^\beta + 1) - U(t_i, n^\alpha, n^\beta) = \frac{(1 - t_i)}{2} (1 - q^\beta)^{n^\beta} q^\beta \geq c.$$

Then,

$$U(t_i, n^\alpha, n^\beta) - U(t_i, n^\alpha - 1, n^\beta + 1) \leq U(t_i, n^\alpha, n^\beta) - U(t_i, n^\alpha, n^\beta + 1) \leq -c < 0,$$

in contradiction to agent  $i$  preferring to acquire the  $\alpha$ -signal over the  $\beta$ -signal in the original equilibrium (the second constraint above). Since all other equilibrium constraints remain the same, it follows that  $(y_1, \dots, y_n) \in \{\alpha, \beta, \emptyset\}^n$  is, indeed, an equilibrium. Similarly, it is easy to see that if for some  $i > j$ ,  $x_i = \emptyset$  and  $x_j = \beta$ , then  $(y_1, \dots, y_n) \in \{\alpha, \beta, \emptyset\}^n$ , where  $y_l = x_l$  for all  $l \neq i, j$ ,  $y_i = \beta$ , and  $y_j = \emptyset$  constitutes a pure equilibrium as well. The case in which an equilibrium profile  $(x_1, \dots, x_n)$  entails  $x_i = \alpha$  and  $x_j = \beta$  for  $i > j$  follows the proof of Lemma 1A. ■

**Proof of Proposition 4.** In order to show that the conditions in (1) are sufficient, we construct an optimal group as follows. If an agent is selected to be part of the group and is to collect, say, an  $\alpha$ -signal, then she must have a taste parameter  $t'$  such that (i) she prefers to gather an  $\alpha$ -signal rather than a  $\beta$ -signal, that is,  $t' \geq l(t)$ , as in the  $c = 0$  case, and (ii) has enough incentives to gather an  $\alpha$ -signal rather than no signal, that is  $n_c^x(t') \geq n_f^x(t)$ . Thus,  $n_f^x(t) \leq n_{\max}^x$  is a necessary condition for

the unconstrained optimal allocation  $(n_f^\alpha(t), n_f^\beta(t))$  to be achievable. Moreover, to achieve her optimal allocation  $(n_f^\alpha(t), n_f^\beta(t))$ , the agent of taste parameter  $t$  has to have incentives *herself* to acquire a signal from at least one source. That is,  $n_f^x(t) \leq n_c^x(t)$  for at least one  $x \in \{\alpha, \beta\}$ . Note that if such incentives cannot be provided, meaning  $n_f^x(t) > n_c^x(t)$  for  $x = \alpha, \beta$ , an optimal group would entail the optimal allocation short of one signal.

To show (2), suppose, for example, that  $n_f^\alpha(t) > n_{\max}^\alpha$  and  $n_f^\beta(t) \leq n_{\max}^\beta$ . Then, there is no selection of group members that allows our agent to achieve  $n_f^\alpha(t)$  signals from source  $\alpha$ . Thus, after choosing  $n_{\max}^\alpha$  agents that collect  $\alpha$ -signals (i.e., chosen in the interval  $[\underline{t}^\alpha, 1]$ ), the agent is better off choosing the remaining agents to collect as many  $\beta$ -signals as possible (this can be achieved, for instance, by choosing them in the interval  $[0, \bar{t}^\beta]$ ). This could lead to more  $\beta$ -signals than in the unconstrained solution.

If both  $n_f^\alpha(t) > n_{\max}^\alpha$  and  $n_f^\beta(t) > n_{\max}^\beta$ , the agent chooses a group in which  $n_{\max}^\alpha$  and  $n_{\max}^\beta$  signals from sources  $\alpha$  and  $\beta$  are collected, respectively. This can be achieved by selecting  $n_{\max}^\alpha$  agents in the interval  $[\underline{t}^\alpha, 1]$  and  $n_{\max}^\beta$  in the interval  $[0, \bar{t}^\beta]$ . ■

**Proof of Proposition 5A.** Suppose  $n > n_{\max}^\alpha + n_{\max}^\beta$ . Any agent of taste  $t = 0$  is in an optimal group as long as there are  $n_{\max}^\alpha$  agents who are acquiring an  $\alpha$ -signal. Similarly, any agent of taste  $t = 1$  is in an optimal group as long as there are  $n_{\max}^\beta$  agents who are acquiring a  $\beta$ -signal. Any agent with  $t \in (0, 1)$  is in an optimal group as long as there are  $n_{\max}^\alpha$  and  $n_{\max}^\beta$  agents acquiring an  $\alpha$ - and  $\beta$ -signal, respectively (indeed, she can contemplate a group with  $n_{\max}^\alpha$  and  $n_{\max}^\beta$  agents of taste  $t = 1$  and  $t = 0$ , respectively). Therefore, stable groups take one of the forms (1) or (2). ■

**Proof of Lemma 2.** Suppose that  $n_f^\alpha(t) > n_c^\alpha(t)$ . Then, it must be the case that  $n_f^\beta(t) < n_c^\beta(t)$  (otherwise,  $n = n_f^\alpha(t) + n_f^\beta(t) > n_c^\alpha(t) + n_c^\beta(t) = n_c(t)$ , contrary to our assumption). That is,  $n_c^\beta(t) \geq n_f^\beta(t) + 1$ . In particular,

$$\frac{1-t}{2} \left[ 1 - (1-q_\beta)^{n_f^\beta(t)+1} \right] - \frac{1-t}{2} \left[ 1 - (1-q_\beta)^{n_f^\beta(t)} \right] \geq c.$$

From the definition of  $n_f^\alpha(t), n_f^\beta(t)$ ,

$$\begin{aligned} & \frac{t}{2} \left[ 1 - (1 - q_\alpha)^{n_f^\alpha(t)} \right] + \frac{1-t}{2} \left[ 1 - (1 - q_\beta)^{n_f^\beta(t)} \right] \\ & \geq \frac{t}{2} \left[ 1 - (1 - q_\alpha)^{n_f^\alpha(t)-1} \right] + \frac{1-t}{2} \left[ 1 - (1 - q_\beta)^{n_f^\beta(t)+1} \right], \end{aligned}$$

which is equivalent to

$$\begin{aligned} & t \left[ 1 - (1 - q_\alpha)^{n_f^\alpha(t)} \right] - \left[ 1 - (1 - q_\alpha)^{n_f^\alpha(t)-1} \right] \\ & \geq (1-t) \left[ 1 - (1 - q_\beta)^{n_f^\beta(t)+1} \right] - (1-t) \left[ 1 - (1 - q_\beta)^{n_f^\beta(t)} \right] \geq c \end{aligned}$$

and  $n_c^\alpha(t) \geq n_f^\alpha(t)$ , which contradicts our hypothesis. Identical arguments follow if  $n_f^\beta(t) > n_c^\beta(t)$ . ■

**Analysis of  $\mathcal{L}(t)$ .** The shape of  $n_c(t)$  plays a crucial role in our discussion. As described in the text, we define  $m_c^\alpha$  and  $m_c^\beta$  as the real numbers achieving equality in (3) and (4), respectively, so that

$$\begin{aligned} m_c^\alpha(t) & \equiv \frac{\ln c - \ln t - \ln q_\alpha - \ln 2}{\ln(1 - q_\alpha)} + 1 \\ m_c^\beta(t) & \equiv \frac{\ln c - \ln(1-t) - \ln q_\beta - \ln 2}{\ln(1 - q_\beta)} + 1 \end{aligned}$$

and  $n_c^x(t) = \lfloor m_c^x(t) \rfloor$  if  $m_c^x(t) \geq 0$  and  $n_c^x(t) = 0$  if  $m_c^x(t) < 0$  for  $x = \alpha, \beta$ .

Assume  $c$  is sufficiently small so that  $\frac{c(1-q^\alpha)}{tq^\alpha} \leq 1$  and  $\frac{c(1-q^\beta)}{(1-t)q^\beta} \leq 1$ , i.e.,  $n_c^\alpha(t), n_c^\beta(t) > 0$ , for all  $t$ .

Ignoring rounding, the relevant function for studying  $n_c(t)$  is:

$$\begin{aligned} \mathcal{L}(t) & = \max\{m_c^\alpha(t), 0\} + \max\{m_c^\beta(t), 0\} = \\ & = \frac{\ln c - \ln t - \ln q^\alpha - \ln 2}{\ln(1 - q^\alpha)} + \frac{\ln c - \ln(1-t) - \ln q^\beta - \ln 2}{\ln(1 - q^\beta)} + 2. \end{aligned}$$

The unconstrained maximizing  $\hat{t}$  is given by:

$$\frac{1-\hat{t}}{\hat{t}} = \frac{\ln(1-q^\alpha)}{\ln(1-q^\beta)} \Leftrightarrow \hat{t} = \frac{\ln(1-q^\beta)}{\ln(1-q^\alpha) + \ln(1-q^\beta)}.$$

$\hat{t}$  is the taste parameter maximizing  $\mathcal{L}(t)$ .

Consider the derivative of  $\mathcal{L}(t)$  :

$$\mathcal{L}'(t) = \frac{1}{(1-t)} \frac{1}{\ln(1-q^\beta)} - \frac{1}{t} \frac{1}{\ln(1-q^\alpha)}.$$

Notice that it is positive up to  $\hat{t}$  and negative afterwards. Indeed,  $\mathcal{L}(t)$  is concave.

When there exists a  $t$  for which  $\frac{c(1-q^\alpha)}{tq^\alpha} \leq 1$  and  $\frac{c(1-q^\beta)}{(1-t)q^\beta} > 1$ , these conditions hold within  $[0, \hat{t}]$ , and  $\mathcal{L}(t)$  takes the form

$$\mathcal{L}(t) = m_c^\alpha(t) = \frac{\ln c - \ln t - \ln q^\alpha - \ln 2}{\ln(1-q^\alpha)} + 1,$$

which is decreasing and concave. Similar analysis pertains to ranges for which  $\frac{c(1-q^\alpha)}{tq^\alpha} > 1$  and  $\frac{c(1-q^\beta)}{(1-t)q^\beta} \leq 1$ .

**Proposition 5C** *If  $n = n_{\max}^\alpha + n_{\max}^\beta$ , stable groups take one of the following forms:*

1.  $n_{\max}^\alpha$  agents whose taste falls in  $[\underline{t}^\alpha, 1]$  and  $n_{\max}^\beta$  agents whose taste falls in  $[0, \bar{t}^\beta]$ ;
2.  $n$  agents of taste  $t = 1$ , or  $n$  agents of taste  $t = 0$ ;
3. If  $n_{\max}^\alpha = 1$ , then  $n_{\max}^\beta$  agents of taste 0, and one agent of taste  $t \in (0, 1)$ , where  $t$  satisfies one of the following:
  - (a)  $n_c^\alpha(t) = 0$  and  $U(t, 0, n_{\max}^\beta) \geq U(t, 1, n_{\max}^\beta - 1)$ ; or
  - (b)  $n_f^\alpha(t) = n_c^\alpha(t) = 1$ .

Similarly if  $n_{\max}^\beta = 1$ .

**Proof of Proposition 5C.** The analysis in Proposition 5A carries through as long as  $n_{\max}^\alpha, n_{\max}^\beta > 1$ , and the classes of group compositions in points (1) and (2) constitute all of the stable allocations.

Regarding (3), suppose that  $n_{\max}^\alpha = 1$ . (a) Assume that  $n_c^\alpha(t) = 0$ . First, consider  $t$  for which  $n_f^\alpha(t) = n_c^\alpha(t) = 0$ . The group consisting of an agent of type  $t$  and  $n_{\max}^\beta$  agents of taste 0 is stable, since the agent with non-extreme taste parameter  $t$  does not have enough incentives to get information on one issue, even when she is the first to acquire a signal relevant to it. Moreover, the remaining agents have an extreme taste parameter, so have no incentive to acquire a signal other than the one

pertaining to the issue they care most about. In this case, we have

$$U(t, 0, n_{\max}^{\beta}) - U(t, 1, n_{\max}^{\beta} - 1) \geq U(t, 0, n_{\max}^{\beta} + 1) - U(t, 1, n_{\max}^{\beta}) \geq 0,$$

where the last inequality follows from  $n_f^{\alpha}(t) = 0$ .

Second, consider the case in which  $n_c^{\alpha}(t) = 0$  and  $n_f^{\alpha}(t) = 1$ . The group is stable since the condition  $U(t, 0, n_{\max}^{\beta}) \geq U(t, 1, n_{\max}^{\beta} - 1)$  assures that the utility the agent gets from the  $n_{\max}^{\beta}$ -th  $\beta$ -signal is higher (or equal) to the utility she would get from the first  $\alpha$ -signal.

(b) Suppose  $n_f^{\alpha}(t) = n_c^{\alpha}(t) = 1$ . In this case, the group formed by  $n_{\max}^{\beta}$  agents of taste  $t = 0$  and one agent of type  $t$  is stable as the agent of taste  $t$  implements her unconstrained optimal allocation.

Analogous constructions can be performed when  $n_{\max}^{\beta} = 1$ . ■

**Proof of Lemma 4.** Recall that  $\{t_1, \dots, t_r\}$  is the set of taste parameters  $t_i$  such that there is at least one agent in  $N$  with taste  $t_i$ . Let any group of agents  $G \subseteq N$  be identified by a vector  $(z_1, \dots, z_r)$ , where  $z_l \leq m_l$  is the number of agents of taste  $t_l$  in group  $G$ . If one group identified by  $(x_1, \dots, x_r)$  is (weakly) preferred by an agent of taste  $t_i$  to the group identified by  $(x'_1, \dots, x'_r)$ , we write  $(x_1, \dots, x_r) \succsim_{t_i} (x'_1, \dots, x'_r)$ . Now, suppose that  $\mathcal{G} = \{G_1, \dots, G_s\}$  is a stable allocation, and suppose that  $G_x, G_y, G_x \neq G_y$  both contain at least one agent of taste  $t_i$ . Assume that  $G_x$  is identified by  $(x_1, \dots, x_i, \dots, x_r)$  and that  $G_y$  is identified by  $(y_1, \dots, y_i, \dots, y_r)$ . For an agent of taste  $t_i$ , a deviation from  $G_x$  to  $G_y$  is unprofitable if

$$(x_1, \dots, x_i, \dots, x_r) \succsim_{t_i} (y_1, \dots, y_i + 1, \dots, y_r).$$

Similarly, for an agent of taste  $t'_i$ , a deviation from  $G_y$  to  $G_x$  is unprofitable if

$$(y_1, \dots, y_i, \dots, y_r) \succsim_{t'_i} (x_1, \dots, x_i + 1, \dots, x_r).$$

However, any agent of taste  $t_i$  strictly benefits from having her group augmented by one more member of her own type. That is,  $(z_1, \dots, z_i + 1, \dots, z_r) \succ_{t_i} (z_1, \dots, z_i, \dots, z_r)$ , for any  $(z_1, \dots, z_i, \dots, z_r)$  and  $t_i$  and we get a contradiction. ■

**Proof of Proposition 6.** For  $x = \alpha, \beta$ , denote by  $z^x(t, h) \equiv n_f^x(t)$  the optimal number of  $x$ -signals out of a total of  $h$  signals for an agent of taste  $t$ . Moreover, for any  $t_1, t_2 \in [0, 1]$ , let  $w^x(t_1, t_2, h_1, h_2)$

denote the equilibrium number of  $x$ -signals collected in a group that is composed of  $h_1$  agents of taste  $t_1$  and  $h_2$  agents of taste  $t_2$  (well-defined from Lemma 1A).

**1. and 2.** Observe that if  $z^\alpha(t_1, N) > z^\alpha(t_2, N)$  then either  $w^\alpha(t_1, t_2, 1, N) = z^\alpha(t_2, N) + 1$  (if  $z^\alpha(t_2, N + 1) = z^\alpha(t_2, N) + 1$ ), or  $w^\alpha(t_1, t_2, 1, N) = z^\alpha(t_2, N)$  (if  $z^\alpha(t_2, N + 1) = z^\alpha(t_2, N)$ ).

Consider a fully segregated partition and suppose that agent  $a \in N_i$  has taste parameter  $t_i$  for  $i \in \{1, \dots, r\}$ . Since  $|N_i| = m$  for all  $i$ , checking that such an agent does not have a profitable deviation by joining  $N_{i+1} \cup \{a\}$  and  $N_{i-1} \cup \{a\}$  is enough to guarantee that this agent does not have profitable deviations (note that for  $i = 1, r$ , there is only one constraint to check). Consider a deviation of agent  $a$  from  $N_i$  to  $N_{i+1} \cup \{a\}$ . Since  $t_i \geq t_{i+1}$ ,  $z^\alpha(t_i, n) \geq z^\alpha(t_{i+1}, n)$  for all  $n \geq 1$ . A necessary condition for the deviation not to be strictly beneficial is that  $z^\alpha(t_i, m) > z^\alpha(t_{i+1}, m)$ . Suppose first that  $w^\alpha(t_i, t_{i+1}, 1, m) = z^\alpha(t_{i+1}, m) + 1$ . It follows that the deviation is not profitable whenever

$$\begin{aligned} & t_i \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{z^\alpha(t_i, m)} \right] + (1 - t_i) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{m - z^\alpha(t_i, m)} \right] \\ & \geq t_i \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{z^\alpha(t_{i+1}, m) + 1} \right] + (1 - t_i) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{m - z^\alpha(t_{i+1}, m)} \right], \end{aligned} \quad (9)$$

or, rearranging terms,

$$t_i \left[ (1 - q_\alpha)^{z^\alpha(t_{i+1}, m) + 1} - (1 - q_\alpha)^{z^\alpha(t_i, m)} \right] \geq (1 - t_i) \left[ (1 - q_\beta)^{m - z^\alpha(t_i, m)} - (1 - q_\beta)^{m - z^\alpha(t_{i+1}, m)} \right]. \quad (10)$$

For  $x = \alpha, \beta$ , define  $v^x(k) \equiv (1 - q_x)^k - (1 - q_x)^{k+1}$ , so that  $v^x(k)$  is the marginal contribution of the  $(k + 1)$ 'th  $x$ -signal for  $x = \alpha, \beta$  (up to a factor of  $\frac{1}{2}$ ). Note that, for  $x = \alpha, \beta$ ,  $v^x(k)$  is decreasing in  $k$ . Thus, substituting and rearranging terms, we can rewrite (10) as follows:

$$\frac{t_i}{1 - t_i} \geq \frac{\sum_{k=m - z^\alpha(t_i, m)}^{m - z^\alpha(t_{i+1}, m) - 1} v^\beta(k)}{\sum_{k=z^\alpha(t_{i+1}, m) + 1}^{z^\alpha(t_i, m) - 1} v^\alpha(k)}, \quad (11)$$

where we use the convention that  $\sum_{k=w}^{w-1} v^\alpha(k) \equiv 0$  for any  $w$ . Condition (11) implicitly defines a condition on  $t_i$  and  $t_{i+1}$  for a deviation from  $N_i$  to  $N_{i+1}$  to be unprofitable.

Observe that, if  $w^\alpha(t_i, t_{i+1}, 1, m) = z^\alpha(t_{i+1}, m)$ , a deviation of agent  $a$  of taste parameter  $t_i$  to a group of  $m$  agents of taste parameter  $t_{i+1}$  is less profitable than a deviation to a group in which  $z^\alpha(t_{i+1}, m) + 1$  out of  $m + 1$  agents collect the  $\alpha$ -signal, and therefore, condition (11) is sufficient to guarantee that such deviation is not profitable.

If  $t_i$  is fixed, condition (11) is weaker the lower is  $t_{i+1}$  (that is, the further apart  $t_i$  and  $t_{i+1}$  are). This guarantees that there exists  $\underline{t}(t_i)$  such that a deviation of agent  $a$  in  $N_i$  to  $N_{i+1}$  is unprofitable if and only if  $t_{i+1} < \underline{t}(t_i)$ . If  $t_{i+1} \in T_0^m$  and condition (11) is not satisfied, then  $\underline{t}(t_i) = 0$ . Set  $\underline{t}(t_i) = 0$  if  $z^\alpha(t_i, m) = z^\alpha(t_{i+1}, m) = 0$ .

We can follow a similar procedure by considering a deviation from  $N_i$  to  $N_{i-1} \cup \{a\}$  and defining a taste  $\bar{t}(t_i)$  such that a deviation from  $N_i$  to  $N_{i-1} \cup \{a\}$  is unprofitable if and only if  $t_{i-1} > \bar{t}(t_i)$ . The intervals  $\{\mathcal{T}_i\}_{i=1}^r$  are obtained by setting for any  $i \in \{1, \dots, r\}$ ,  $\mathcal{T}_i \equiv [\underline{t}(t_i), \bar{t}(t_i))$  whenever  $\bar{t}(t_i) < 1$  and  $\mathcal{T}_i \equiv [\underline{t}(t_i), 1]$  whenever  $\bar{t}(t_i) = 1$ . To see that  $\mathcal{T}_i$  are unions of contiguous intervals of the sequence  $\{T_k^m\}_{k=1}^m$ , observe that all  $t_{i+1}$  in the same interval  $T_k^m$  share the same optimal allocation  $(z^\alpha(t_{i+1}, m), z^\beta(t_{i+1}, m))$ . Thus, for a given  $t_i$ , if condition (11) is satisfied for  $t_{i+1} \in T_k^m$ , then it is satisfied for any  $t'_{i+1} \in T_k^m$ .

**3.** Consider  $t^1, t^2, t^1 > t^2$ . For any  $\varepsilon > 0$ , denote by  $t_{-\varepsilon}^l$  and  $t_{+\varepsilon}^l$  the maximal parameters  $t_{i+1}$  for which (11) holds with  $t_i = t^l - \varepsilon$  and  $t_i = t^l + \varepsilon$ , respectively, for  $l = 1, 2$ . Assume that  $t^1$  and  $t^2$  are such that for any  $\varepsilon > 0$ ,  $z^\alpha(t_{-\varepsilon}^l, m) \neq z^\alpha(t_{+\varepsilon}^l, m)$ ,  $l = 1, 2$ . Note that for any sufficiently small  $\varepsilon > 0$ , either  $z^\alpha(t^l - \varepsilon, m) = z^\alpha(t^l + \varepsilon, m)$ ,  $l = 1, 2$  (that is, generically, the case), or one of  $t^1, t^2$  is on the cusp of our original intervals  $\{T_k^m\}_{k=0}^m$ , in which case,

$$\begin{aligned} & t^l \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{z^\alpha(t^l, m)} \right] + (1 - t^l) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{m - z^\alpha(t^l, m)} \right] \\ &= t^l \left[ 1 - \frac{1}{2} (1 - q_\alpha)^{z^\alpha(t^l, m) + 1} \right] + (1 - t^l) \left[ 1 - \frac{1}{2} (1 - q_\beta)^{m - z^\alpha(t^l, m) - 1} \right]. \end{aligned}$$

It follows that for sufficiently small  $\varepsilon > 0$ ,  $z^\alpha(t_{+\varepsilon}^l, m) = z^\alpha(t_{-\varepsilon}^l, m) + 1$ . The consequent gaps pertaining to  $l = 1, 2$  then follow from the comparative statics on the original intervals described in Proposition 2. Note that it is, in fact, sufficient to look at taste parameters on the cusp since an increase within an interval does not change the right-hand side of condition (11), but increases the left-hand side, so makes it easier to satisfy. The other constraints follow similarly. ■