Chapter 10

Forwards and Futures

Road Map

Part A Introduction to finance.

Part B Valuation of assets, given discount rates.

Part C Determination of risk-adjusted discount rate.

Part D Introduction to derivatives.

- Forwards and futures.
- Options.
- Real options.

Main issues

- Forwards and Futures
- Forward and Futures Prices
- Hedging Financial Risks Using Forwards/Futures

1 Forward Contracts

<u>Definition</u>: A forward contract is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.



- The price fixed now for future exchange is the forward price.
- The buyer obtains a "long position" in the asset/commodity.

Features of forward contracts:

- traded over the counter (not on exchanges)
- custom tailored
- no money changes hands until maturity
- non-trivial counter-party risk.

Example. Consider a 3-month forward contract for 10,000 bushels of soybean at a forward price of \$3.5/bushel. The long side is committed to buy 10,000 bushels of soybean from the short side three months from now at the price of \$3.50/bushel.

2 Futures Contracts

Forward contracts have two limitations:

- (a) illiquidity
- (b) counter-party risk.

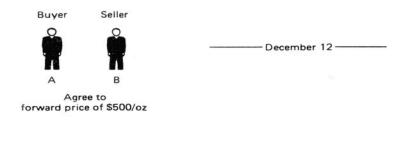
Futures contracts are designed to address these limitations.

<u>Definition</u>: A futures contract is an exchange-traded, standardized, forward-like contract that is marked to the market daily. Futures contract can be used to establish a long (or short) position in the underlying commodity/asset.

Features of futures contracts:

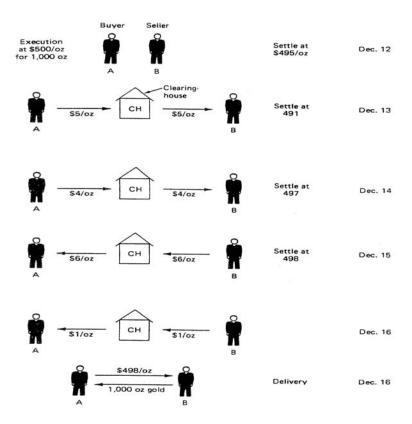
- Standardized contracts:
 - (1) underlying commodity or asset
 - (2) quantity
 - (3) maturity.
- Traded on exchanges.
- Guaranteed by the clearing house little counter-party risk.
- Gains/losses settled daily—marked to market.
- Margin account required as collateral to cover losses.

A Forward Contract





A Futures Contract



Example. Yesterday, you bought 10 December live-cattle contracts at CME, at the closing price of \$0.7455/lb.

- Contract size 40,000 lbs.
- Agreed to buy 400,000 pounds of live cattle in December.
- Value of position yesterday:

$$(0.7455)(10)(40,000) = $298,200.$$

- No money changed hands.
- Initial margin required (5%-20% of contract value).

Today, the futures price closes at 0.7435/lb, 0.20 cents lower. The value of your position is

$$(0.7435)(10)(40,000) = $297,400$$

a loss of \$800.

Forward and futures contracts are derivative securities because

- payoffs determined by prices of the underlying asset
- zero net supply.

3 Forward and Futures Prices

Question: What determines forward and futures prices?

Answer: Forward/futures prices are linked to spot prices.

ContractSpot at
$$t$$
ForwardFuturesPrice S_t F H

Ignoring differences between forwards and futures, we have

$$F \simeq H$$
.

Two ways to buy the underlying for date T:

- 1. Buy forward or futures contract of maturity T.
- 2. Buy the underlying now and store it until T.

Difference between buy-and-store from forward/futures:

- a. Cost of storing (for commodities).
- b. Benefits from storing:
 - Convenience yield (for commodities).
 - Dividends/interests (for financials).

By arbitrage, the costs of these two approaches must equal:

$$F \simeq H = (1 + r_F)S_0 + \text{FV (net cost of storing)}.$$

3.1 Commodities

- 1. Gold.
 - Easy to store—negligible cost of storage.
 - No dividends or benefits.

Two ways to buy gold for T:

- Buy now for S_0 and hold until T.
- ullet Buy forward, pay F and take delivery at T.

No-arbitrage requires that

$$F = S_0(1 + r_F)^T \simeq H.$$

Example. Gold quotes on 2006.08.21 are

- Spot price \$625.70/oz
- 2007 February futures (CMX) \$641.40/oz.

The implied 6-month interest rate is $r_F = 5.08\%$.

2. Oil.

- Costly to store.
- Additional benefits, convenience yield, for holding physical commodity (over holding futures).
- Not held for long-term investment (unlike gold), but mostly held for future use.

Let the percentage holding cost be c and convenience yield be y. We have

$$F = S_0 [1 + r_F - (y - c)]^T$$
$$= S_0 (1 + r_F - \widehat{y})^T$$
$$\simeq H$$

where

$$\widehat{y} = y - c$$

is the net convenience yield.

Example. Prices on 2006.08.21 are

- Spot oil price 72.45/barrel (light sweet)
- Nov 06 oil futures price 74.22/barrel (NYM)
- 3-month interest rate is 5.40% (LIBOR).

Annualized net convenience yield is: $\hat{y} = -4.74\%$.

For commodity futures:

1. Contango means:

- (a) spot prices are lower than futures prices, and/or
- (b) prices for near maturities are lower than for distant.

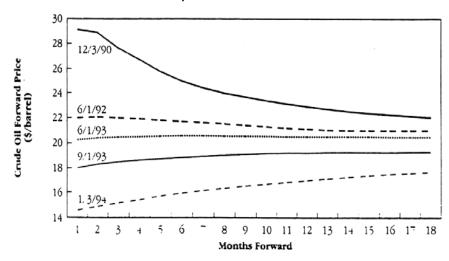
2. Backwardation means:

- (a) spot prices are higher than futures prices, and/or
- (b) prices for near maturities are higher than for distant.

Backwardation occurs if net convenience yield exceeds interest rate:

$$\widehat{y} - r_F = y - c - r_F > 0.$$

Crude oil forward price curves for selected dates



3.2 Financials

For financial futures, the underlying are financial assets. Financials have the following features:

- No cost to store (the underlying asset).
- Dividend or interest on the underlying.
- 1. Stock index futures.
 - Underlying are bundles of stocks S&P, Nikkei, etc.
 - Futures settled in cash (no delivery).

Let the dividend yield be d, then there is the following relation between the forward/futures price and spot price:

$$F = S_0(1 + r_F - d)^T \simeq H.$$

Deviations from this relation triggers index arbitrage.

Example. Prices on 2006.08.21 are

- S&P 500 closed at 1,279.52
- S&P futures maturing in December closed at 1,313.60.
- 4-month interest rate is 5.42%.

The annual dividend yield is: d = 2.78%.

Note:

- Since the underlying asset is a portfolio in the case of index futures, trading in the futures market is easier than trading in cash market.
- Thus, futures prices may react quicker to macro-economic news than the index itself.
- Index futures are very useful to market makers, investment bankers, stock portfolio managers:
 - hedging market risk in block purchases & underwriting
 - creating synthetic index fund
 - portfolio insurance.

Example. You have \$1 million to invest in the stock market and you have decided to invest in a diversified portfolio. S&P seems a good candidate. How would you do this?

- (a) One approach is to buy S&P in the cash market:
 - buy the 500 stocks
 - weights proportional to their market capitalization.
- (b) Another way is to buy S&P futures:
 - Put the money in your margin account
 - Assuming S&P is at 1000 now and the each contract assigns \$250 to each index point (see, e.g., WSJ), the number of contracts to buy is:

$$\frac{1000000}{(250)(1000)} = 4.$$

As the S&P index fluctuates, the future value of your portfolio (in \$M) would look as follows (ignoring interest payments and dividends):

S&P	Portfolio (a)	Portfolio (b)
900	0.90	0.90
1000	1.00	1.00
1100	1.10	1.10

2. Interest rate futures.

• Underlying assets are riskless or high grade bonds.

Forward prices are simply determined by forward interest rates.

Example. Consider a T-bond with annul coupon rate of 7% (with semi-annual coupon payments) that is selling at par. Suppose that the current short rate is 5%. What should be the 6-month forward price of the T-bond?

Consider the following strategy and its cash flow

Time	0	1	2	3	• • •
Buy T-bond	-100.0	3.5	3.5	3.5	
Sell T-bill	100.0	-102.5	0	0	
Net	0	- 99.0	3.5	3.5	• • •

This strategy allows one to lock in a purchase of the 7% T-bond 6-month later for \$99. No arbitrage requires the current forward price to be \$99.

In general, a bond's forward price is

$$F = S(1 + r - y)$$

where

- \bullet S is its spot price
- r is the spot interest rate
- y = C/S is its coupon yield.

4 Hedging with Forwards and Futures

4.1 Hedging with Forwards

Hedging with forward contracts is simple, because one can tailor the contract to match maturity and size of position to be hedged.

Example. Suppose that you, the manager of an oil exploration firm, have just struck oil. You expect that in 5 months time you will have 1 million barrels of oil. You are unsure of the future price of oil and would like to hedge your position.

Using a forward contract, you could hedge your position by selling forward 1 million barrels of oil. Let S_t be the spot oil price at t (in months). Then,

Position	Value in 5 months (per barrel)
Long position in oil	S_5
Short forward position	$F-S_5$
Net payoff	F

Thus, in this case you know today exactly what you will receive 5 months from now. That is, the hedge is perfect.

4.2 Hedging with Futures

One problem with using forwards to hedge is that they are illiquid.

Thus, if after 1 month you discover that there is no oil, then you no longer need the forward contract. In fact, holding just the forward contract you are now exposed to the risk of oil-price changes.

In this case, you would want to unwind your position by buying back the contract. Given the illiquidity of forward contracts, this may be difficult and expensive.

To avoid problems with illiquid forward markets, one may prefer to use futures contracts.

Example. In the above example, you can sell 1 million barrels worth of futures. Suppose that the size of each futures contract is 1,000 barrels. The number of contract you want to short is

$$\frac{1,000,000}{1,000} = 1,000.$$

Example. We have \$10 million invested in government bonds and are concerned with highly volatile interest rate over the next six months. We want to use the 6-month T-bond futures to protect the value of the portfolio.

- Duration of the bond portfolio is 6.80 years.
- Current futures price is $93\frac{2}{32}$ (for face value of \$100).
 - The T-bond to be delivered has a duration of 9.20 years.
 - Each contract delivers \$100,000 face value of bonds.
 - Futures price for the total contract is \$93,062.50.
 - 6-month interest rate is 4%.

We should short the futures: If interest rate goes up, bond prices go down but a gain is made on the short position of futures.

How many contracts to short?

Duration of the futures contract: $H = B(1+y)^{1/2}$.

$$MD_H = MD_B + (1+y)^{-1}(1/2) = 9.20 + (1.04)^{-1}(0.5) = 9.68.$$

Match duration:

$$(\# \text{ of contracts})(93,062.50)(9.68) = (10,000,000)(6.80).$$

Thus:

$$(\# \text{ of contracts}) = \left(\frac{10,000,000}{93,062.50}\right) \left(\frac{6.80}{9.68}\right) = 75.48.$$

Since futures contracts are standardized, they may not perfectly match your hedging need. The following mismatches may arise when hedging with futures:

- Maturity
- Contract size
- Underlying asset.

Thus, a perfect hedge is available only when

- 1. the maturity of futures matches that of the cash flow
- 2. the contract has the same size as the position to be hedged
- 3. the cash flow being hedged is linearly related to the futures'.

In the event of a mismatch between the position to be hedged and the futures contract, the hedge may not be perfect.

5 Homework

Readings:

- BKM Chapters 22, 23, 27.
- BMA Chapter 27.

Assignments:

• Problem Set 6