

# Chapter 7

## Portfolio Theory

### Road Map

**Part A** Introduction to finance.

**Part B** Valuation of assets, given discount rates.

**Part C** Determination of risk-adjusted discount rates.

- Introduction to return and risk.
- Portfolio theory.
- CAPM and APT.

**Part D** Introduction to derivative securities.

### Main Issues

- Returns of Portfolios
- Diversification
- Diversifiable vs. Non-diversifiable risks
- Optimal Portfolio Selection

# 1 Introduction and Overview

In order to understand risk-return trade-off, we observe:

1. Risks in individual asset returns have two components:
  - Systematic risks—common to most assets
  - Non-systematic risks—specific to individual assets.
2. Systematic risks and non-systematic risks are different:
  - Non-systematic risks are diversifiable
  - Systematic risks are non-diversifiable.
3. Forming portfolios can eliminate non-systematic risks.
4. Investors hold diversified portfolios to reduce risk.
5. Investors care only about portfolio risks—systematic risks.

## 2 Portfolio Returns

A portfolio is simply a collection of assets, characterized by the mean, variances/covariances of their returns,  $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n$ :

- Mean returns:

Asset	1	2	...	n
Mean Return	$\bar{r}_1$	$\bar{r}_2$	...	$\bar{r}_n$

- Variances and covariances:

	$r_1$	$r_2$	...	$r_n$
$r_1$	$\sigma_1^2$	$\sigma_{12}$	...	$\sigma_{1n}$
$r_2$	$\sigma_{21}$	$\sigma_2^2$	...	$\sigma_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$r_n$	$\sigma_{n1}$	$\sigma_{n2}$	...	$\sigma_n^2$

Covariance of an asset with itself is its variance:  $\sigma_{nn} = \sigma_n^2$ .

**Example.** Monthly stock returns on IBM ( $\tilde{r}_1$ ) and Merck ( $\tilde{r}_2$ ):

Mean returns

$\bar{r}_1$	$\bar{r}_2$
0.0149	0.0100

Covariance matrix

	$\tilde{r}_1$	$\tilde{r}_2$
$\tilde{r}_1$	0.007770	0.002095
$\tilde{r}_2$	0.002095	0.003587

Note:  $\sigma_1 = 8.81\%$ ,  $\sigma_2 = 5.99\%$  and  $\rho_{12} = 0.40$ .

## 2.1 Portfolio of Two Assets

A *portfolio* of these two assets is characterized by the value invested in each asset.

Let  $V_1$  and  $V_2$  be the dollar amount invested in asset 1 and 2, respectively. The total value of the portfolio is

$$V = V_1 + V_2.$$

Consider a portfolio in which

- $w_1 = V_1/V$  is the weight on asset 1
- $w_2 = V_2/V$  is the weight on asset 2.

Then,  $w_1 + w_2 = 1$ .

**Example.** You have \$1,000 to invest in IBM and Merck stocks. If you invest \$500 in IBM and \$500 in Merck, then  $w_{\text{IBM}} = w_{\text{Merck}} = 500/1000 = 50\%$ . This is an equally weighted portfolio of the two stocks.

**Example.** If you invest \$1,500 in IBM and  $-\$500$  in Merck (short sell \$500 worth of Merck shares), then  $w_{\text{IBM}} = 1500/1000 = 150\%$  and  $w_{\text{Merck}} = -500/1000 = -50\%$ .

## Return on a portfolio with two assets

The portfolio return is a weighted average of the individual returns:

$$\tilde{r}_p = w_1\tilde{r}_1 + w_2\tilde{r}_2.$$

**Example.** Suppose you invest \$600 in IBM and \$400 in Merck for a month. If the realized return is 2.5% on IBM and 1.5% on Merck over the month, what is the return on your total portfolio?

$$(0.6)(2.5\%) + (0.4)(1.5\%) = 2.1\%.$$

## Expected return on a portfolio with two assets

Expected portfolio return:

$$\bar{r}_p = w_1\bar{r}_1 + w_2\bar{r}_2.$$

Unexpected portfolio return:

$$\tilde{r}_p - \bar{r}_p = w_1(\tilde{r}_1 - \bar{r}_1) + w_2(\tilde{r}_2 - \bar{r}_2).$$

## Variance of return on a portfolio with two assets

The variance of the portfolio return:

$$\begin{aligned}\sigma_p^2 &= \text{Var}[\tilde{r}_p] = \text{E}[(\tilde{r}_p - \bar{r}_p)^2] \\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}.\end{aligned}$$

Variance of the portfolio is the sum of all entries of the following table

	$w_1\tilde{r}_1$	$w_2\tilde{r}_2$
$w_1\tilde{r}_1$	$w_1^2\sigma_1^2$	$w_1w_2\sigma_{12}$
$w_2\tilde{r}_2$	$w_1w_2\sigma_{12}$	$w_2^2\sigma_2^2$

**Example.** Consider again investing in IBM and Merck stocks.

Mean returns

$\bar{r}_1$	$\bar{r}_2$
0.0149	0.0100

Covariance matrix

	$\tilde{r}_1$	$\tilde{r}_2$
$\tilde{r}_1$	0.007770	0.002095
$\tilde{r}_2$	0.002095	0.003587

Consider the equally weighted portfolio:  $w_1 = w_2 = 0.5$ .

1. Mean of portfolio return:

$$\bar{r}_p = (0.5)(0.0149) + (0.5)(0.0100) = 1.25\%.$$

2. Variance of portfolio return:

	$w_1 \tilde{r}_1$	$w_2 \tilde{r}_2$
$w_1 \tilde{r}_1$	$(0.5)^2(0.007770)$	$(0.5)^2(0.002095)$
$w_2 \tilde{r}_2$	$(0.5)^2(0.002095)$	$(0.5)^2(0.003587)$

$$\begin{aligned}\sigma_p^2 &= (0.5)^2(0.007770) + (0.5)^2(0.003587) + \\ &\quad (2)(0.5)^2(0.002095) \\ &= 0.003888\end{aligned}$$

$$\sigma_p = 6.23\%.$$

## 2.2 Portfolio of Three Assets

Now consider a portfolio of three assets, 1, 2 and 3:

$$w_1 + w_2 + w_3 = 1.$$

- Expected portfolio return:

$$\bar{r}_p = w_1\bar{r}_1 + w_2\bar{r}_2 + w_3\bar{r}_3.$$

- The variance of the portfolio return:

$$\begin{aligned}\sigma_p^2 &= \text{Var}[\tilde{r}_p] = \text{E}[(\tilde{r}_p - \bar{r}_p)^2] \\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + \\ &\quad 2w_1w_2\sigma_{12} + 2w_1w_3\sigma_{13} + 2w_2w_3\sigma_{23}\end{aligned}$$

Again, the portfolio variance is the sum of all table entries

	$w_1\tilde{r}_1$	$w_2\tilde{r}_2$	$w_3\tilde{r}_3$
$w_1\tilde{r}_1$	$w_1^2\sigma_1^2$	$w_1w_2\sigma_{12}$	$w_1w_3\sigma_{13}$
$w_2\tilde{r}_2$	$w_1w_2\sigma_{12}$	$w_2^2\sigma_2^2$	$w_2w_3\sigma_{23}$
$w_3\tilde{r}_3$	$w_1w_3\sigma_{13}$	$w_2w_3\sigma_{23}$	$w_3^2\sigma_3^2$



**Example.** Consider three stocks, IBM, Merck and Intel. Returns on the three stocks have the following covariance matrix:

	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Merck}}$	$\tilde{r}_{\text{Intel}}$
$\tilde{r}_{\text{IBM}}$	0.007770	0.002095	0.001189
$\tilde{r}_{\text{Merck}}$	0.002095	0.003587	0.000229
$\tilde{r}_{\text{Intel}}$	0.001189	0.000229	0.009790

What are the variance and StD of a portfolio with 1/3 invested in each stock (the equally weighted portfolio)?

Since  $w_{\text{IBM}} = w_{\text{Merck}} = w_{\text{Intel}} = 1/3$ , we have

$$\begin{aligned}\sigma_p^2 &= \left(\frac{1}{3}\right)^2 \times (\text{Sum of all elements of covariance matrix}) \\ &= 0.003130\end{aligned}$$

$$\sigma_p = 5.59\%.$$

Note, for each asset individually:

$$\sigma_{\text{IBM}} = 8.81\%$$

$$\sigma_{\text{Merck}} = 5.99\%$$

$$\sigma_{\text{Intel}} = 9.89\%.$$

## 2.3 Portfolio of Multiple Assets

We now consider the portfolio of  $n$  assets.

$$\sum_{i=1}^n w_i = 1.$$

0. The return on the portfolio is:

$$\tilde{r}_p = w_1\tilde{r}_1 + w_2\tilde{r}_2 + \cdots + w_n\tilde{r}_n = \sum_{i=1}^n w_i\tilde{r}_i.$$

1. The expected return on the portfolio is:

$$\bar{r}_p = E[r_p] = w_1\bar{r}_1 + w_2\bar{r}_2 + \cdots + w_n\bar{r}_n = \sum_{i=1}^n w_i\bar{r}_i.$$

2. The variance of portfolio return is:

$$\sigma_p^2 = \text{Var}[\tilde{r}_p] = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

where  $\sigma_{ii} = \sigma_i^2$ .

3. The volatility (StD) of portfolio return is:

$$\sigma_p = \sqrt{\text{Var}[\tilde{r}_p]} = \sqrt{\sigma_p^2}.$$

The variance of portfolio return can be computed by summing up all the entries to the following table:

	$w_1 r_1$	$w_2 r_2$	$\cdots$	$w_n r_n$
$w_1 r_1$	$w_1^2 \sigma_1^2$	$w_1 w_2 \sigma_{12}$	$\cdots$	$w_1 w_n \sigma_{1n}$
$w_2 r_2$	$w_2 w_1 \sigma_{21}$	$w_2^2 \sigma_2^2$	$\cdots$	$w_2 w_n \sigma_{2n}$
$\cdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$w_n r_n$	$w_n w_1 \sigma_{n1}$	$w_n w_2 \sigma_{n2}$	$\cdots$	$w_n^2 \sigma_n^2$

The variance of a sum is not just the sum of variances! We also need to account for the covariances.

In order to calculate return variance of a portfolio, we need

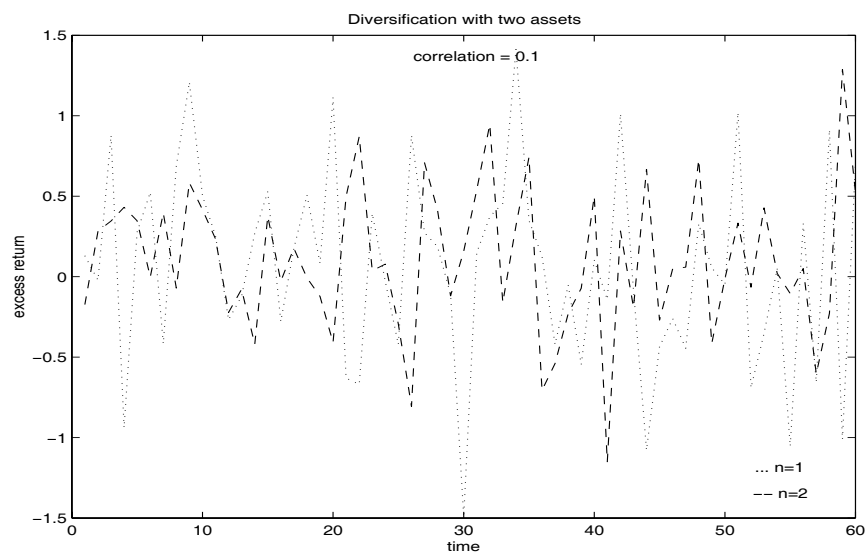
- (a) portfolio weights
- (b) individual variances
- (c) all covariances.

### 3 Diversification

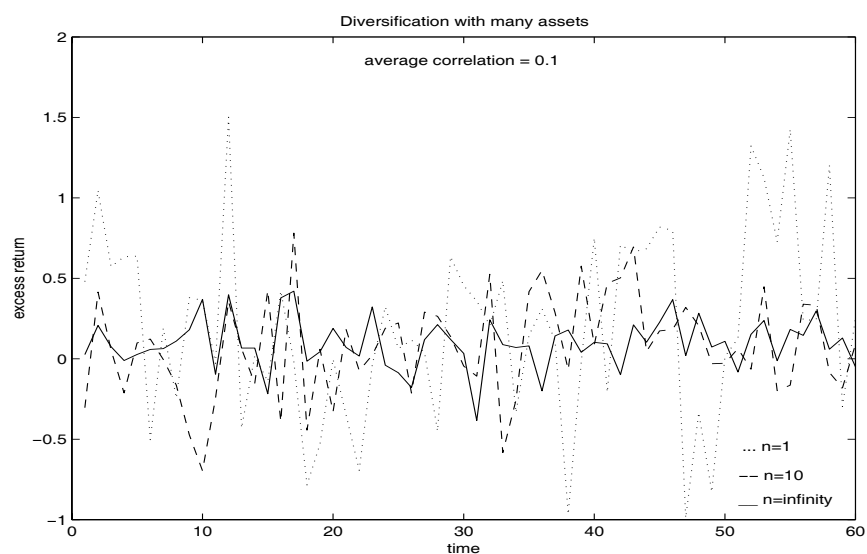
**Lesson 1:** Diversification reduces risk.

Portfolio returns vs. individual asset returns:

1. Two assets:



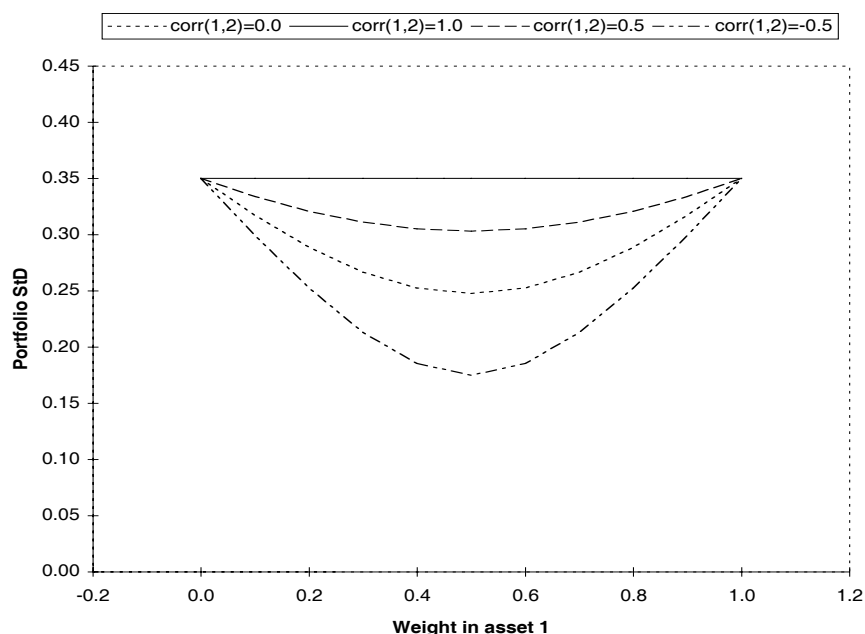
2. Multiple assets:



**Example.** Given two assets with the same annual return StD,  $\sigma_1 = \sigma_2 = 35\%$ , consider a portfolio  $p$  with weight  $w$  in asset 1 and  $1-w$  in asset 2.

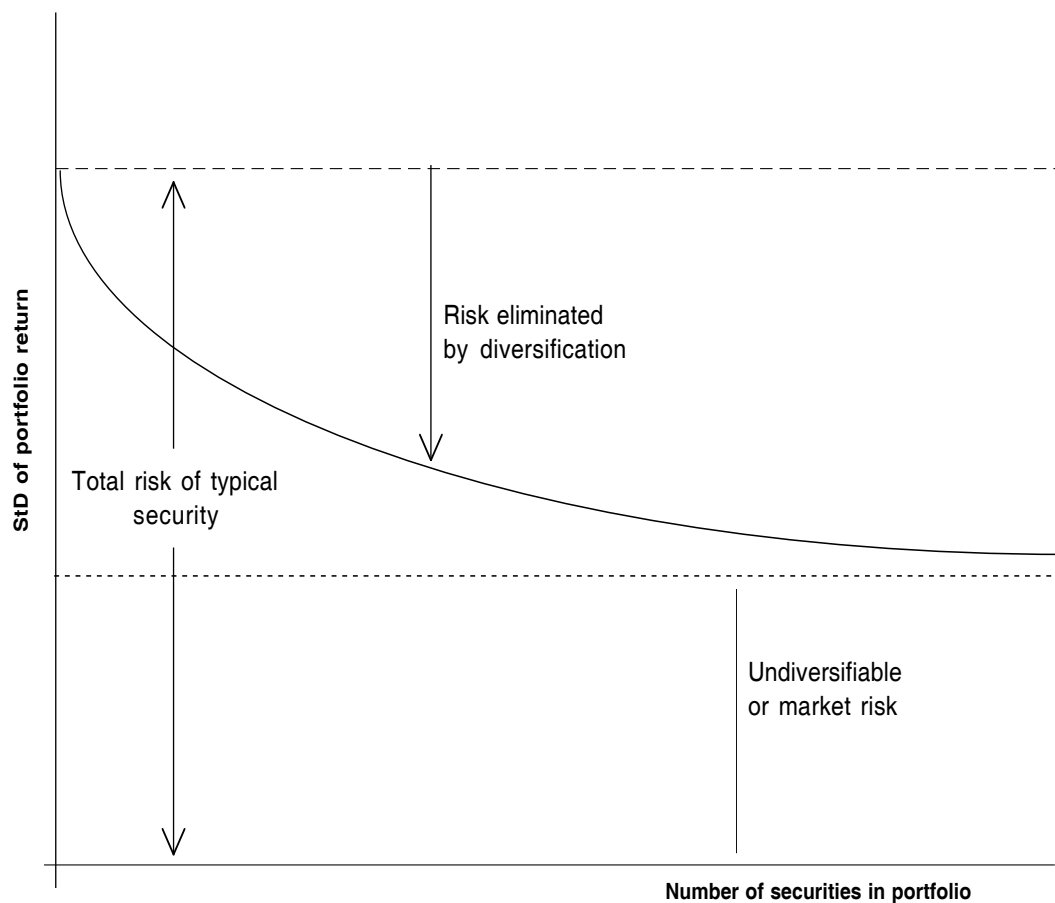
$$\sigma_p = \sqrt{w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_{12}}.$$

From the plot below, the StD of the portfolio return is less than the StD of each individual asset.



**Lesson 2:** Certain risks cannot be diversified away.

## Impact of Diversification On Portfolio Risk

 $\rho = 1.0$  and  $\rho = 0.5$ 

Risk comes in two types:

- Diversifiable risk.
- Non-diversifiable risk.

**Example.** An equally-weighted portfolio of  $n$  assets:

	$w_1 r_1$	$\cdots$	$w_n r_n$
$w_1 r_1$	$w_1^2 \sigma_1^2$	$\cdots$	$w_1 w_n \sigma_{1n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$w_n r_n$	$w_n w_1 \sigma_{n1}$	$\cdots$	$w_n^2 \sigma_n^2$

- A typical variance term:  $\left(\frac{1}{n}\right)^2 \sigma_{ii}$ .
  - Total number of variance terms:  $n$ .
- A typical covariance term:  $\left(\frac{1}{n}\right)^2 \sigma_{ij}$  ( $i \neq j$ ).
  - Total number of covariance terms:  $n^2 - n$ .

Add all the terms:

$$\begin{aligned}
 \sigma_p^2 &= \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} = \sum_{i=1}^n \left(\frac{1}{n}\right)^2 \sigma_{ii} + \sum_{i=1}^n \sum_{j \neq i}^n \left(\frac{1}{n}\right)^2 \sigma_{ij} \\
 &= \left(\frac{1}{n}\right) \left(\frac{1}{n} \sum_{i=1}^n \sigma_i^2\right) + \left(\frac{n^2 - n}{n^2}\right) \left(\frac{1}{n^2 - n} \sum_{i=1}^n \sum_{j \neq i}^n \sigma_{ij}\right) \\
 &= \left(\frac{1}{n}\right) (\text{average variance}) + \left(\frac{n^2 - n}{n^2}\right) (\text{average covariance}).
 \end{aligned}$$

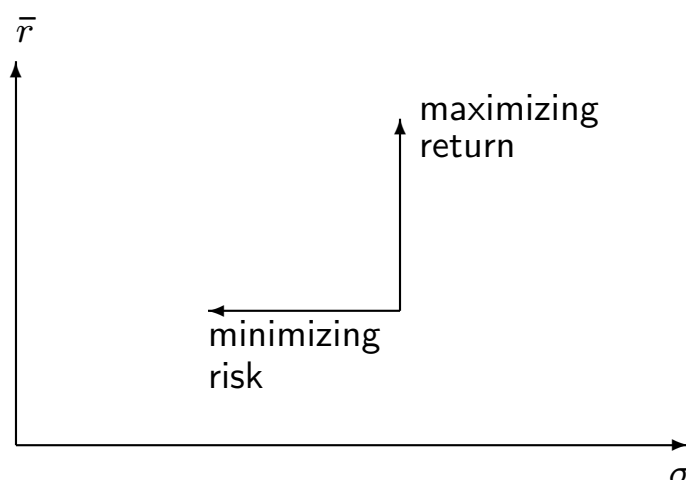
As  $n$  becomes very large:

- Contribution of variance terms goes to zero.
- Contribution of covariance terms goes to “average covariance”.

## 4 Optimal Portfolio Selection

How to choose a portfolio:

- Minimize risk for a given expected return? or
- Maximize expected return for a given risk?



Formally, we need to solve the following problem:

$$(P) : \quad \text{Minimize} \quad \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \\ \{w_1, \dots, w_n\}$$

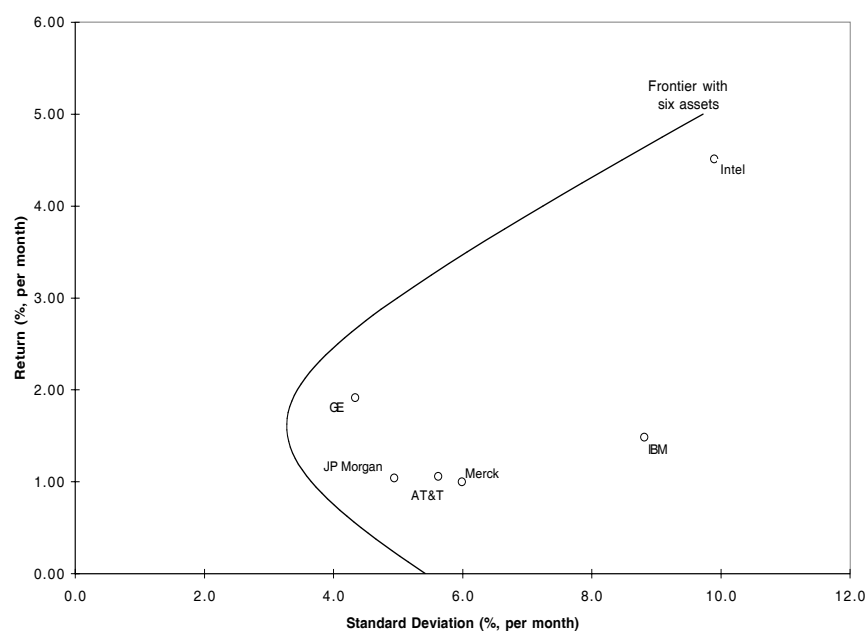
$$\text{subject to} \quad (1) \quad \sum_{i=1}^n w_i = 1$$

$$(2) \quad \sum_{i=1}^n w_i \bar{r}_i = \bar{r}_p.$$



## 4.1 Solving optimal portfolios “graphically”

Portfolio Frontier from Stocks of  
IBM, Merck, Intel, AT&T, JP Morgan and GE



Definition: Given an expected return, the portfolio that minimizes risk (measured by StD) is a *mean-StD frontier portfolio*.

Definition: The locus of all frontier portfolios in the mean-StD plane is called *portfolio frontier*. The upper part of the portfolio frontier gives *efficient* frontier portfolios.

## 4.2 Portfolio Frontier with Two Assets

Assume  $\bar{r}_1 > \bar{r}_2$  and let  $w_1 = w$  and  $w_2 = 1 - w$ .

Then

$$\bar{r}_p = w\bar{r}_1 + (1 - w)\bar{r}_2$$

$$\sigma_p^2 = w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\sigma_{12}.$$

For a given  $\bar{r}_p$ , there is a unique  $w$  that determines the portfolio with expected return  $\bar{r}_p$ :

$$w = \frac{\bar{r}_p - \bar{r}_2}{\bar{r}_1 - \bar{r}_2}.$$

The risk of the portfolio is

$$\sigma_p = \sqrt{\left(\frac{\bar{r}_p - \bar{r}_2}{\bar{r}_1 - \bar{r}_2}\right)^2 \sigma_1^2 + \left(\frac{\bar{r}_p - \bar{r}_1}{\bar{r}_2 - \bar{r}_1}\right)^2 \sigma_2^2 + 2\left(\frac{\bar{r}_p - \bar{r}_2}{\bar{r}_1 - \bar{r}_2}\right)\left(\frac{\bar{r}_p - \bar{r}_1}{\bar{r}_2 - \bar{r}_1}\right)\sigma_{12}}.$$

## Without Short Sales

When short sales are not allowed,  $w_1 \geq 0$ ,  $w_2 \geq 0$  and

$$\bar{r}_1 \geq \bar{r}_p \geq \bar{r}_2.$$

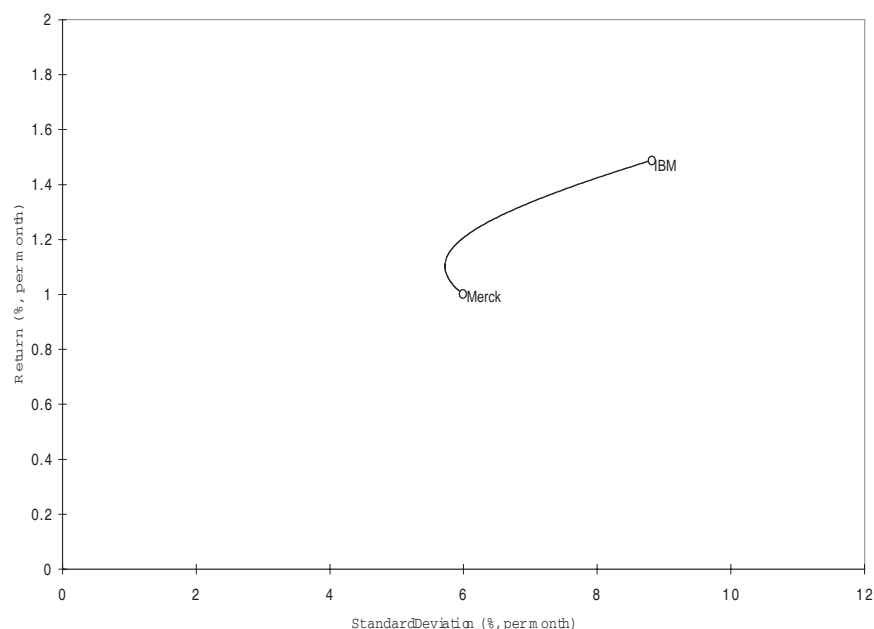
**Example.** A universe of only IBM and Merck stocks:

Covariances	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Merck}}$
$\tilde{r}_{\text{IBM}}$	0.007770	0.002095
$\tilde{r}_{\text{Merck}}$	0.002095	0.003587
Mean (%)	1.49	1.00
StD (%)	8.81	5.99

Portfolios of IBM and Merck

Weight in IBM (%)	0	10	20	30	40	50	60	70	80	90	100
Mean return (%)	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.34	1.39	1.44	1.49
StD (%)	5.99	5.80	5.72	5.78	5.95	6.23	6.62	7.08	7.61	8.19	8.81

## Portfolio Frontier when Short Sales Are Not Allowed



## With Short Sales

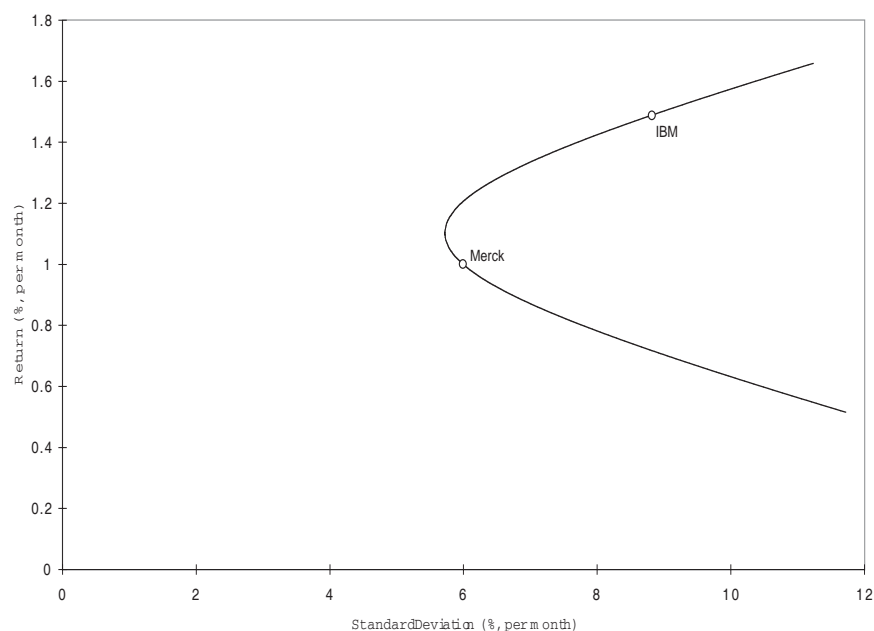
When short sales are allowed, portfolio weights are unrestricted.

**Example.** (Continued.)

Covariances	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Merck}}$
$\tilde{r}_{\text{IBM}}$	0.007770	0.002095
$\tilde{r}_{\text{Merck}}$	0.002095	0.003587
Mean	1.49%	1.00%
StD	8.81%	5.99%

Portfolios of IBM and Merck										
Weight in IBM (%)	-40	-20	0	20	40	60	80	100	120	140
Mean return (%)	0.80	0.90	1.00	1.10	1.20	1.29	1.39	1.49	1.59	1.69
StD (%)	7.70	6.69	5.99	5.72	5.95	6.62	7.61	8.81	10.16	11.60

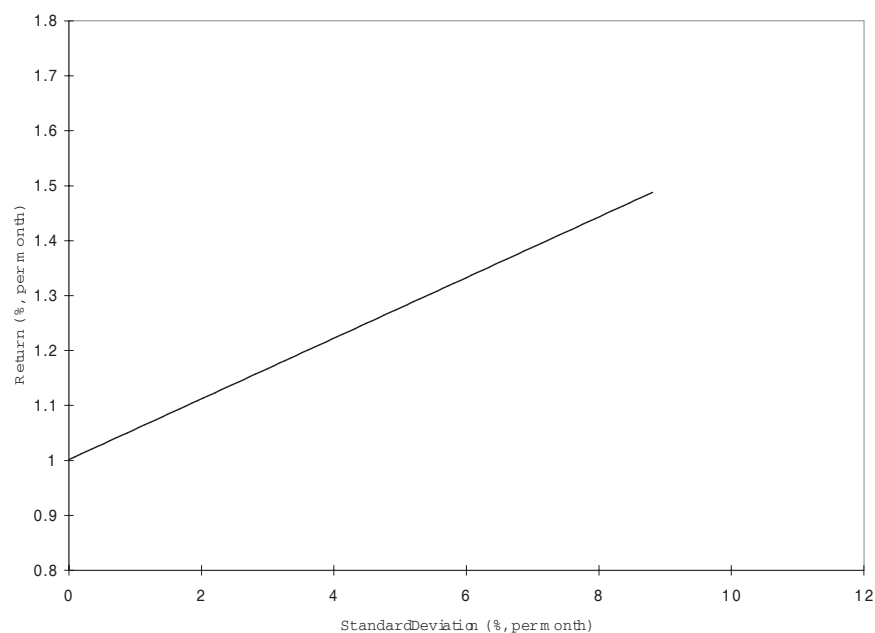
### Portfolio Frontier when Short Sales Are Allowed



## Several special situations (without short sales)

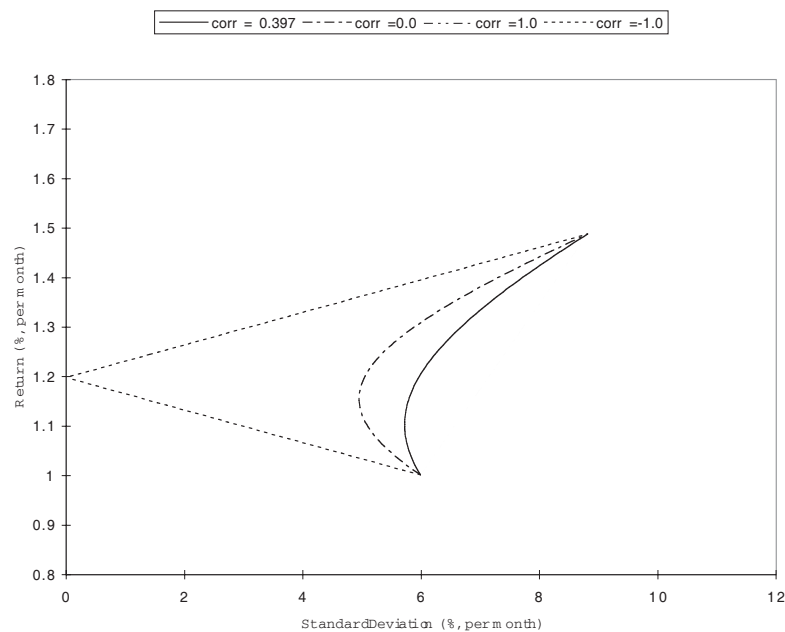
1.  $\sigma_1 = 0$ : Asset 1 is risk-free.

Portfolio Frontier with A Risk-Free Asset



2.  $\rho_{12} = \pm 1$ : Perfect correlation between two assets.

Portfolio Frontiers with Special Return Correlation

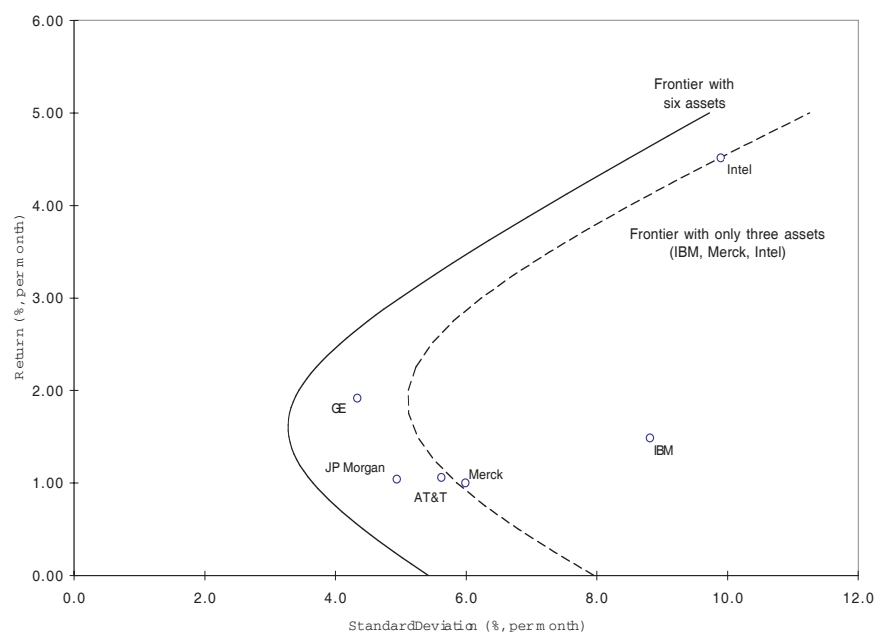


## 4.3 Portfolio Frontier with Multiple Assets

With more than two assets, we need to solve the constrained optimization problem (P).

- Use Excel Solver to solve numerically.
- Details are given in the appendix.

Portfolio Frontier of IBM, Merck, Intel, AT&T, JP Morgan and GE



Observation: When more assets are included, the portfolio frontier improves, i.e., moves toward upper-left: higher mean returns and lower risk.

Intuition: Since one can choose to ignore the new assets, including them cannot make one worse off.

## 5 Portfolio Frontier with A Safe Asset

When there exists a safe (risk-free) asset, each portfolio consists of the risk-free asset and risky assets.

Observation: A portfolio of risk-free and risky assets can be viewed as a portfolio of two portfolios:

- (1) the risk-free asset, and
- (2) a portfolio of only risky assets.

**Example.** Consider a portfolio with \$40 invested in the risk-free asset and \$30 each in two risky assets, IBM and Merck:

- $w_0 = 40\%$  in the risk-free asset
- $w_1 = 30\%$  in IBM and
- $w_2 = 30\%$  in Merck.

However, we can also view the portfolio as follows:

- (1)  $1 - x = 40\%$  in the risk-free asset
- (2)  $x = 60\%$  in a portfolio of only risky assets which has
  - (a) 50% in IBM
  - (b) 50% in Merck.

Consider a portfolio  $p$  with  $x$  invested in a risky portfolio  $q$ , and  $1 - x$  invested in the risk-free asset. Then,

$$\tilde{r}_p = (1-x)r_F + x\tilde{r}_q$$

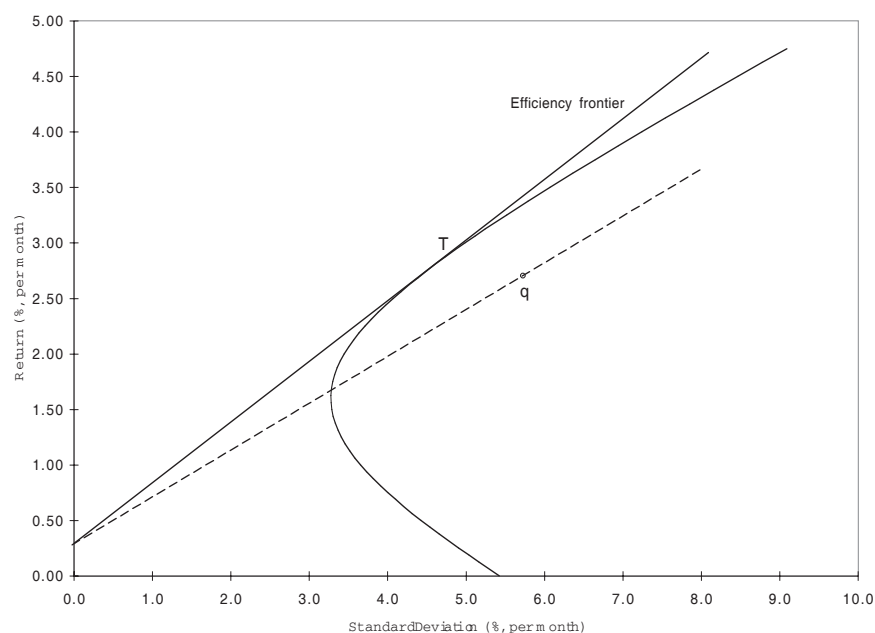
$$\bar{r}_p = (1-x)r_F + x\bar{r}_q$$

$$\sigma_p^2 = x^2\sigma_q^2.$$

When there is a risk-free asset, the frontier portfolios are combinations of:

- (1) the risk-free asset
- (2) the tangent portfolio (consisted of only risky assets).

#### Portfolio Frontier with A Risk-Free Asset (Capital Market Line or CML)



Frontier portfolios have the highest Sharpe ratio:  $\frac{\bar{r}_p - r_F}{\sigma_p}$ .



## 6 Summary

Main points of modern portfolio theory:

1. Risk comes in two types:
  - Diversifiable (non-systematic)
  - Non-diversifiable (systematic)
2. Diversification reduces (diversifiable) risk.
3. Investors hold frontier portfolios.
  - Large asset base improves the portfolio frontier.
4. When there is risk-free asset, frontier portfolios are linear combinations of
  - the risk-free asset, and
  - the tangent portfolio.

## 7 Appendix: Solve Frontier Portfolios

	A	B	C	D	E	F	G	H	I
14	<b>Moments:</b>								
15		Stock	Std (%)	Ret (%)					
16		IBM	8.8147	1.4877					
17		Merck	5.9892	1.0012					
18		Intel	9.8947	4.5130					
19		Cov(IBM,Merck)	20.9513						
20		Cov(IBM,Intel)	11.8918						
21		Cov(Merck,Intel)	2.2930						
22									
23									
24	<b>Solver:</b>	<b>Reqd return</b>	<b>Weights</b>		<b>Check</b>	<b>Port. Variance</b>	<b>Port. Std</b>	<b>Port. Ret</b>	
25	-----> explains what is in row 26	r%---you specify this	w1--- solver finds this	w2--- solver finds this	Formula: =1-w1-w2	check: weights add to 1	=C26^2*\$C\$16^2 +D26^2*\$C\$17^2 +E26^2*\$C\$18^2 +2*C26*D26*\$C\$ 19+2*C26*E26*\$ C\$20+2*D26*E26 *\$C\$21	=G26^0.5	=C26*\$D\$16 +D26*\$D\$17 +E26*\$D\$18
26		0.00	0.1805	1.1296	-0.3101	1	63.3210	7.9574	0.00
27		0.25	0.1735	1.0646	-0.2379	1	54.1182	7.3565	0.25
28		0.50	0.1665	0.9993	-0.1658	1	46.2183	6.7984	0.50
29		0.75	0.1595	0.9341	-0.0936	1	39.6211	6.2945	0.75
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Set Target Cell:	\$H\$26	<b>Solve</b>
Equal to:	<input type="radio"/> Max <input checked="" type="radio"/> Min <input type="radio"/> Value of:	0
<b>By Changing Cells:</b>		
\$C\$26:\$D\$26	<b>Guess</b>	
<b>Subject to the Constraints:</b>		
\$I\$26 = \$B\$26	<b>Add...</b>	
	<b>Change...</b>	
	<b>Delete</b>	
	<b>Options...</b>	
	<b>Reset All</b>	
	<b>Help</b>	

Enter

We now outline the steps involved in obtaining the optimal portfolio using the Solver.

**Step 1** Enter the data on

- Expected return on each stock in the portfolio
- Standard deviation (volatility) of each stock
- Covariance among the stocks

This is done in the section of the spreadsheet labeled “Moments.”

**Step 2** Create the cells for the optimization using Solver.

- Specify a required rate of return for the portfolio. (In cell B26, we specified this to be 0.00%.)
- Enter some initial values for  $w_1$  and  $w_2$  in cells C26 and D26. We used the initial values of 0.25 and 0.35. These are the cells that Solver will change to find the optimal portfolio weights.
- Enter, in E26, the portfolio constraint:  $w_3 = 1 - w_1 - w_2$ .
- Cell F26 simply checks that the weights satisfy the constraint that their sum is equal to one.
- Enter the portfolio variance in G26 using the weights given in cells C26 and D26 and the information on variances and covariances in Step 1. Then, use this definition of portfolio variance to specify the portfolio StD, in cell H26.
- Finally, enter the definition of portfolio return in cell I26, again using the weights given in cells C26 and D26 and the information on expected returns that we entered in Step 1.

**Step 3** Open Solver (from the “Tools” menu). If Solver is not installed, you can try and install it from the “Add-ins” choice under “Tools”. If this does not work, then you will need to get professional help.

- In the target cell, choose \$H\$26 and select “Min.”

- The minimization will be done by changing the cells \$C\$26:\$D\$26 — these are the cells that contain the portfolio weights  $w_1$  and  $w_2$ .
- Now add the constraint that the portfolio expected return given in cell I26 must be equal to the return specified in cell B26.

**Step 4** Click on “Solve” and the optimal portfolio weights should appear in cells C26 and D26.

**Step 5** You can repeat this process for other values of portfolio expected return (as in B27, B28, ... and redoing Steps 2-4 with the appropriate changes).

**Step 6** Plotting the points in columns H and I (using x-y plots) will give the portfolio frontier.

## 8 Homework

### Readings:

- BKM Chapters 6.2, 7, 8.
- BMA Chapters 7, 8.1.

### Assignment:

- Problem Set 4.