Chapter 3

On Value Additivity

Contents

3.1 Value Additivity

There is a principle that no grocery store will ever want to violate:

> The price of a basket of goods (apples, pears,...) equals the sum of the prices of the individual goods.

In finance we merely replace the goods with financial assets, such as

- cash flows to be delivered tomorrow, in one week, one month, ...
- portfolios of equities with random values one year from now.
- options.
- ...

3.2 Application To Corporate Finance: The Value of the Firm

A corporation holds a certain number of *assets*. Applying value additivity, the value of those assets is simply the sum of the values of the components. That is

called the *value of the firm*. The firm is held by a number of different creditors, such as equityholders, bondholders, banks and so forth. The value of the creditor's holdings, called liabilities, must by value additivity and no free lunches add up to the value of the firm.

We define *the value of a firm* as the price for which one could sell the stream of cash flows that the assets of the firm generates for the traditional creditors. Traditional creditors are: bondholders, warrantholders, shareholders, banks.

By the value additivity axiom and no free lunches,

> The value of the firm is always equal to the sum of the values of the company's liabilities to the traditional creditors.

For example, if the company's liabilities consist of equity with value E and debt with value B, and the firm value equals V, then:

$$V = E + B.$$

The summation on the RHS is justified by value additivity, the equality by "no free lunches."

But some care is advised here. The value of the firm is not necessarily equal to the price of the assets of the company. This because some of the cash flows go to third parties, in the form of taxes, lawyers and accountants fees and also because it may be advantageous to keep the company alive rather than selling it. The "live firm" may be more valuable e.g. because of its possibility for reducing future taxes. "Selling the stream of cash flows of the firm" does thus not necessarily mean "liquidation." Another thing to note is that the value of the firm almost never equals the book value of the firm's assets.

Example

The assets of a firm generate a perpetual, riskfree, after-tax cash flow of \$10 per year. The riskfree rate is 5% p.a. The value of the firm equals

$$\sum_{t=1}^{\infty} \frac{10}{(1+0.05)^t} = \frac{10}{0.05} = \$200.$$

3.3 A Couple of Brain Teasers

Let us list a couple of issues that may look like violations of the axioms of value additivity and no free lunches on first glance. The reader should return to these after having gone through the book and see if she can understand why these may not be violations after all.

Closed-end mutual funds are regular companies whose sole purpose it is to invest in other shares. Usually, the shares (of equity) in closed-end mutual funds trade at a discount to the net asset value (the value of the shares the fund holds, less any liabilities to bondholders and/or banks). The discount can be as high as 25%. Is this a violation of value additivity and no free lunch?

Options give the right to purchase (call) or sell (put) an underlying asset at a pre-determined price (the strike price) during a particular period. Merton proved in 1973 that the value of an option on a portfolio of assets (think of it as equity in a firm with multiple subsidiaries) can never exceed, and will usually be lower than, the sum of the values of options on the component assets of the portfolio (think of it as the combined equity in each of the subsidiaries of the firm). How can it ever be lower? That seems to violate value additivity and no free lunch, but it does not really. Merton's result has been used to explain why the stock price of companies that disintegrate (or spin off subsidiaries) increases.

Warren Buffett claims to have made a living out of violations of value additivity and no free lunch.

Problems

3.1 *Ketchup* [2]

As an empirical investigation, check your local supermarket. Does 2 ketchup bottles of 0.5 litres cost the same as one ketchup bottle of 1 liter? What does this tell you about value additivity in financial markets?

3.2 *Milk* [2]

Why is skimmed milk always cheaper than regular milk even if it is healthier?