

NAME: _____

BEM 103 QUIZ 3 Value of information.

The Shire has decided to set up a company to bring water from a distant lake to its inhabitants. The project will succeed or fail depending on how costly it is to build the pipe through the desert. At time 0 the project is equally like to succeed (and earn a return of 7 million dollars) or fail (return 2 two million dollars). To fund the project the Shire will issue 1000 shares.

1. Ignoring discounting, what is the expected value of the project and thus what should share prices be?

Expected value $0.5 \times 7 + 0.5 \times 2 = 4.5$ million

Share price \$4500

2. Each inhabitant receives an independent signal if the signal is good the project succeeds with probability 0.8 and fails with probability 0.2. If the signal is bad the project fails with probability 0.8 and succeeds with probability 0.2. Ms Smith, the richest woman in the shire is given the opportunity to buy the whole project after observing her signal. What is she willing to pay (again ignoring discounting) if her signal is good?

Her expected value conditional on the good signal is $0.8 \times 7 + 0.2 \times 2 = 6$ million

3. Alternatively, the village elders decide to have an auction and allow each of 1500 inhabitants to bid for one share and 1000 shares for sale. Should someone with a good signal bid the price from the expected value or \$7,000? Hint: (Will the auction reveal the state Succeed or Fail?)

There are 1000 shares for sale, if the project will succeeds 80% of the inhabitants (1200) will get a good signal (20% will get a bad one), the marginal bidder has a good signal. If the project will fail only 20% of the inhabitants (300) will get a good signal (80% will get a bad one), the marginal bidder has a bad signal. The auction reveals the state so someone who gets a good signal should bid \$7000 (if she bids less she will not get a share when the project will succeed). Because someone who gets a bad signal bids \$2000, when the project fails, the price is \$2000.

4. How Do you recommend the village elder raise the money for the project if they need to raise at least 4 million dollars?

Ms Smith will pay 6 million if she gets the good signal, Ms Smith will pay only $0.8 \times 2 + 0.2 \times 7 = 3$ million. So you only fund this way if the signal is good (probability $\frac{1}{2}$, 80% when project succeed and 20% when project fails).

The auction raises 7 million if the project will succeed and 2 million if it will fail. So you only fund when project succeeds.

In either case the project only gets funded with probability $\frac{1}{2}$ but when you do the auction you only fund when it will succeed which is better.