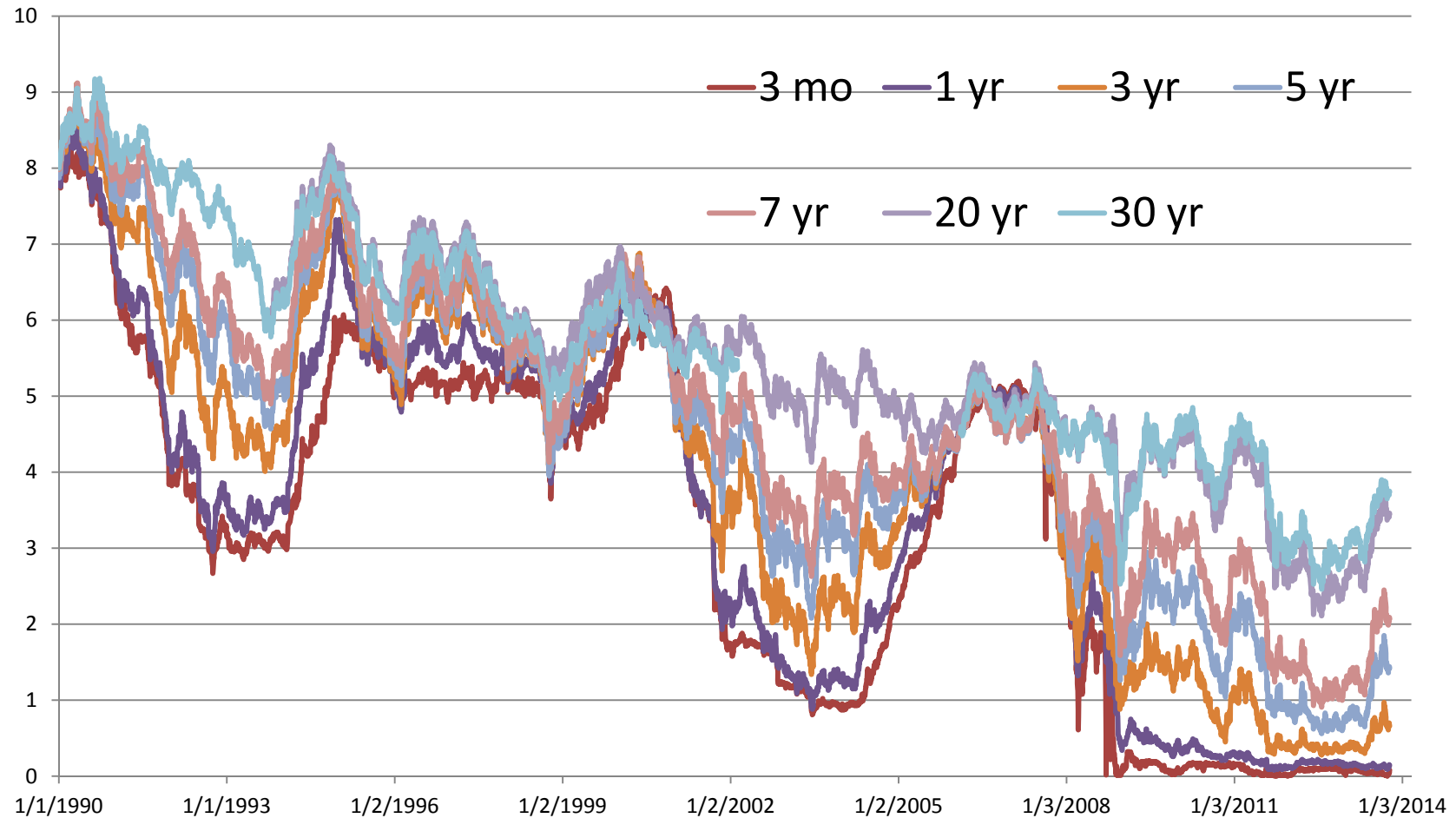


10-16 Class 6: Pricing riskless Bonds

- Net Present value, again;
- Fed policy and inflation;
- Expectations and the yield curve;
- Implication for assets with risk;
- Digression junk bonds and bankruptcy;
- Application: QE2 and the unwinding of the Fed Position

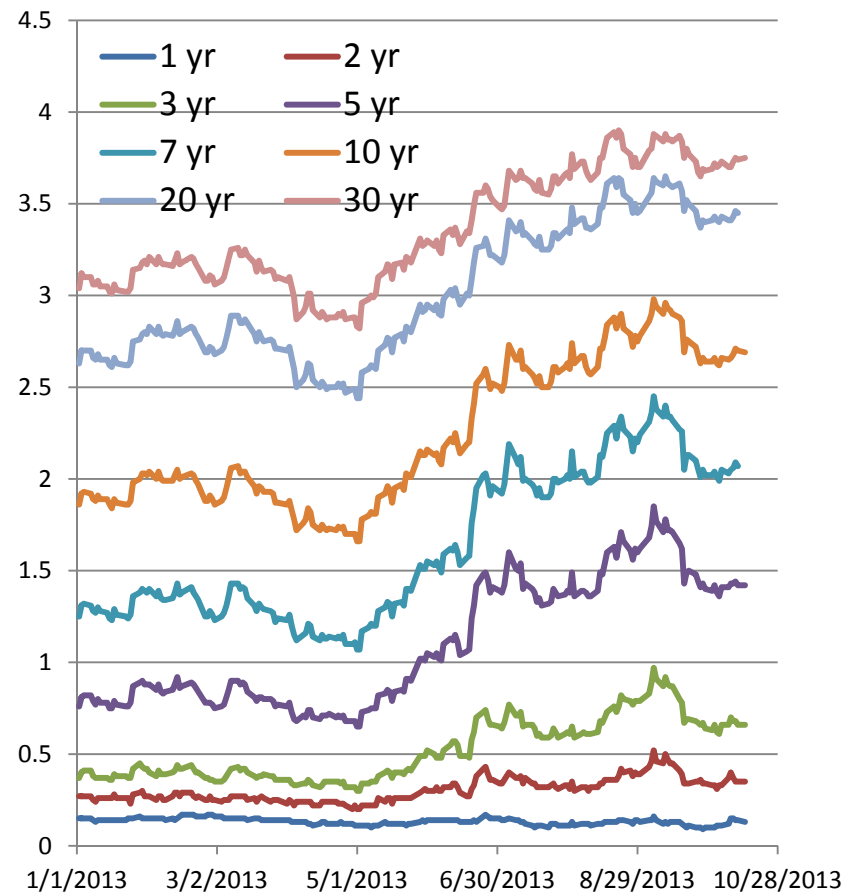
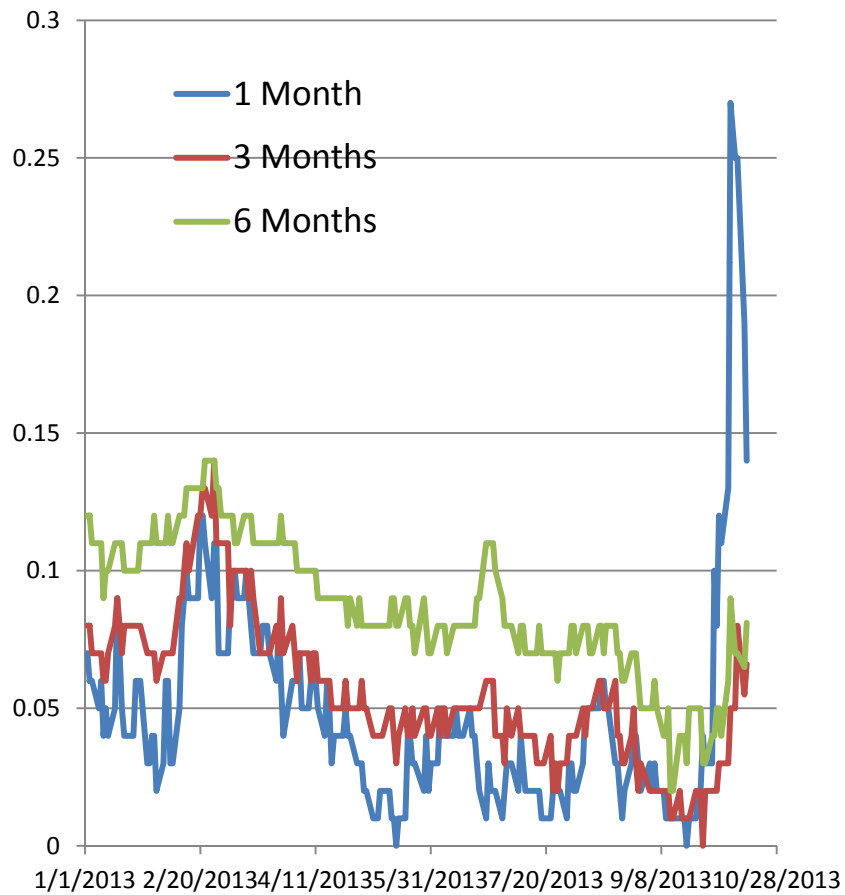
A Quarter Century of Bond Yields



Things to remember

- For a 'long' time yields have been falling
- There are periods where the long yield (3 years or more) is much higher than the short yield (3 months or less)
 - There is a positive yield curve
- There are period where the long yield is very close to the short yield.
 - This happens both when short yields are high (1990) or lower (2006)
 - But not when short yields are very low(2001-2005, post 2008)
- Why?

The recent past



Thinks Remember

- The very short rates are spiking last week but coming back down
- The long rates are flat recently (the peak was in late August)
- Short rates have been falling since March
- Long rate have been increasing since May

Explaining these patterns

- Two steps
- Suppose these bonds are riskless, how should they be priced?
- Then bring uncertainty and default risk back in.

Definitions

- Riskless bond: A bond that has a zero probability of not making payments on time.
- Collateralized bond: if payment(s) are missed creditors can foreclose on assets pledged as collateral
- General obligation bonds: if payment(s) are missed creditors can seek to seize assets of the debtor.
- Unsecured debt: if payment(s) are missed creditors cannot seize assets
- Sovereign bond: Sovereign states cannot be sued for debt recovery. So they choose to pay or not, all such debt is unsecured.

Default and other bad news

- Default: a missed payment
- Repudiation: borrower announces that she/he/it will never repay
- Suspension: borrower announces that she/he/it is not paying now (but will pay later)
- Resumption: borrower starts making payments again
- Hair cut: a reduction of the credit terms (capital, interest or both) that brings the debtor to resume
- Insolvency: debtor does not have the cash on hand to meet the scheduled payments on his or her obligations
- Bankruptcy: an organized process to deal with insolvency.

- BUT: for now no bad news

Lets price a simple bond

- Government issues a 1 month treasury bill (no coupon)
- Pay B at time 1 (November 16). What is its price (x) at time 0 (October 16)?
- Investor choice
 - Hold bond or hold cash?
 - Price of bond is just B because the return on the alternative is zero.
 - Hold bond and consume B at time 1 or consume X at time 1.
 - Investor is impatient so discounts the future by d ($d < 1$) So investor has to be paid to hold the bond ($X < B$)
 - Hold bond if $U(X) < d * U(B)$.
 - At margin indifferent. So $U(X) = d * U(B)$
 - Simplify: U is linear. Then $u(X) = X = dB$ or $X/B = d$.
 - But we do not consume money we consume goods. Bond pays in money so need to adjust for prices.
 - Let price level at time 0 be 100, and the price level at time 1 be p.
 - So $U(X) = d * U(B)$ becomes $U(X/100) = d * U(B/p)$
 - $X/B = d$ now becomes $X/B = d(100/p)$.
- If there is inflation $100/p < 1$ so $d(100/p) < d$ investor must be paid to hold the bond for his value of time *and* inflation.

Computing yield

- interest rate is r per year. The price of a one year bond with no coupon is $X = B/(1+r)$.
- Now suppose you observe price (X)
- Then yield price of a one year bond with no coupon is the interest rate such that $r = (B/X) - 1$
- That is the nominal yield
- More general bond (coupon c , matures at T) the yield is the interest rate that solves

$$\sum_{t=1}^T (cB)(1+r)^t + B(1+r)^T$$

The yield curve without uncertainty

- Suppose the long term interest rate is r per year and the change in the price level per year is ΔP . Then the price of a one year bond with no coupon is
- $X = B/(1+r)$.
- Now suppose I offer a one year riskless bond that pays r
- Its price is $X = 100 \cdot (1+r)/(1+r) = 100$
- Now suppose I offer a two year riskless bond that pays r per year
- Its price is $X = 100r/(1+r) + 100 \cdot (1+r)/(1+r)^2$

$$= 100R/(1+r) + 100/(1+r) = 100(1+r)/(1+r) = 100$$
- Repeat as many periods as needed. You will see that the price of a bond riskless that pays r is constant. Its yield is r no matter what length the bond runs.
- If it pays less than r coupon its price will fall as maturity length but its yield will be constant.
- If there is inflation that $X = B/((1+r)(1+\Delta P))$. Just relabel $(1+r)(1+\Delta P) = (1+R)$ and now the gross yield will be R the inflation adjusted yield is r
- If there is uncertainty about r or inflation, but the mean of the distribution is the current level (of r or inflation), then no yield curve

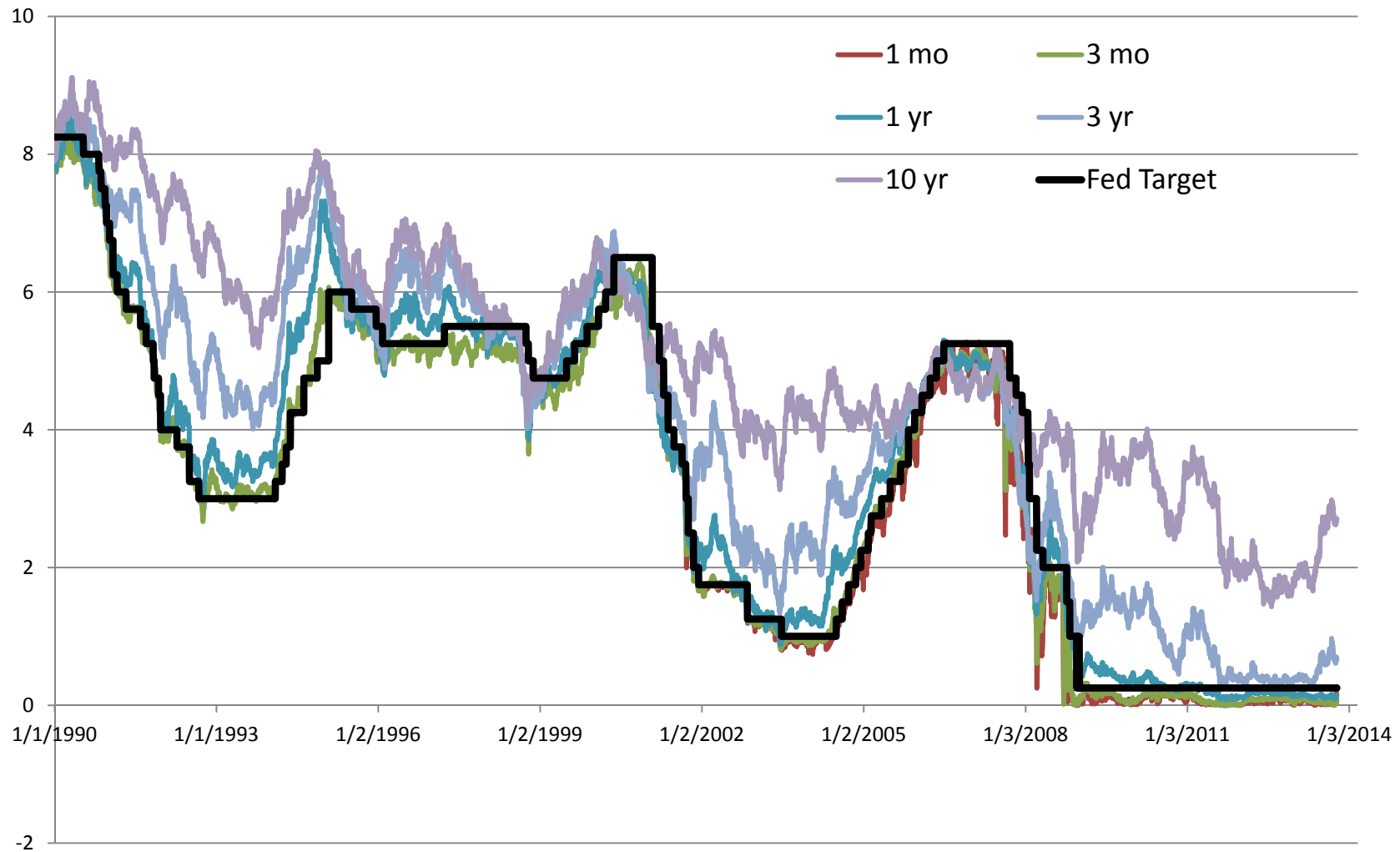
So much for Theory

- Prices are set to equalize yields both across types of bonds and over time.
- So absent something that is not evenly distributed there should be no yield curve
- So why do we see a yield curve about half the time in the data?
 - Federal reserve policy
 - Mean reversion in interest rates
 - The supply of funds is not infinitely elastic

What do prices tell us?

- January 1 yield on 1 month T-bill was 0.07% ($d=.9993$).
- January Inflation was zero. But inflation over the previous 12 months was 1.6 % which suggests a rate of interest of -0.029% per month.
- How could this be? And its been going on for some time
 - 2009 and 2010 $\approx 0.1\%$; 2010 $\approx 0.05\%$; 2009 $\approx 0.07\%$
- Who buys 1 month T-bills
 - What are their alternatives
 - Short term credit to banks, but they can money from the fed at essentially the same rates. So it's the discount policy of the Fed that drive very short term interest rates.
- Notice that this is not a standard situation
 - Even short term interest rates should be positive
 - And if you take a longer look they usually are
 - 2006 $\approx 3\%$; 2006 $\approx 4\%$; 2006 $\approx 4.5\%$; 2008 $\approx 2\%$

Fed Target rate and Bond Yields



Implication for the Yield curve

- Suppose the federal reserve is happiest when the economy grows between 2.5 and 3.5% and unemployment is between 3 and 5%.
- And the long term interest rate is 4%
- This keeps inflation in control (and thus allows it to set interest rates)
- When there is a negative shock (growth goes down and unemployment goes up). The fed lowers interest rates, and the short term rate falls
- But the market knows that when the shock has passed the federal reserve will raise interest rates.
- Notice now future interest rates are no longer symmetrically distributed around the current level, they have mean reversion around some long term trend.

State dependence

Price a two period bond Interest rates take on two values but the transition matrix is not symmetric . The low state is when the economy does badly and the fed lowers interest rates.

	L	H
L	0.25	0.25
H	0.75	0.75

First Look at bond with coupon c that runs one period. All that matters is the current interest rate

$$X_i = B(1+c)/(1+r_i) \quad \text{So } X_l > X_h$$

Now back up one period and suppose the interest rate is low

$$X_l^1 = B(1+c)/(1+r_l) \quad \text{Yield is } r_1 = r_l$$

What is the price of the two period bond?

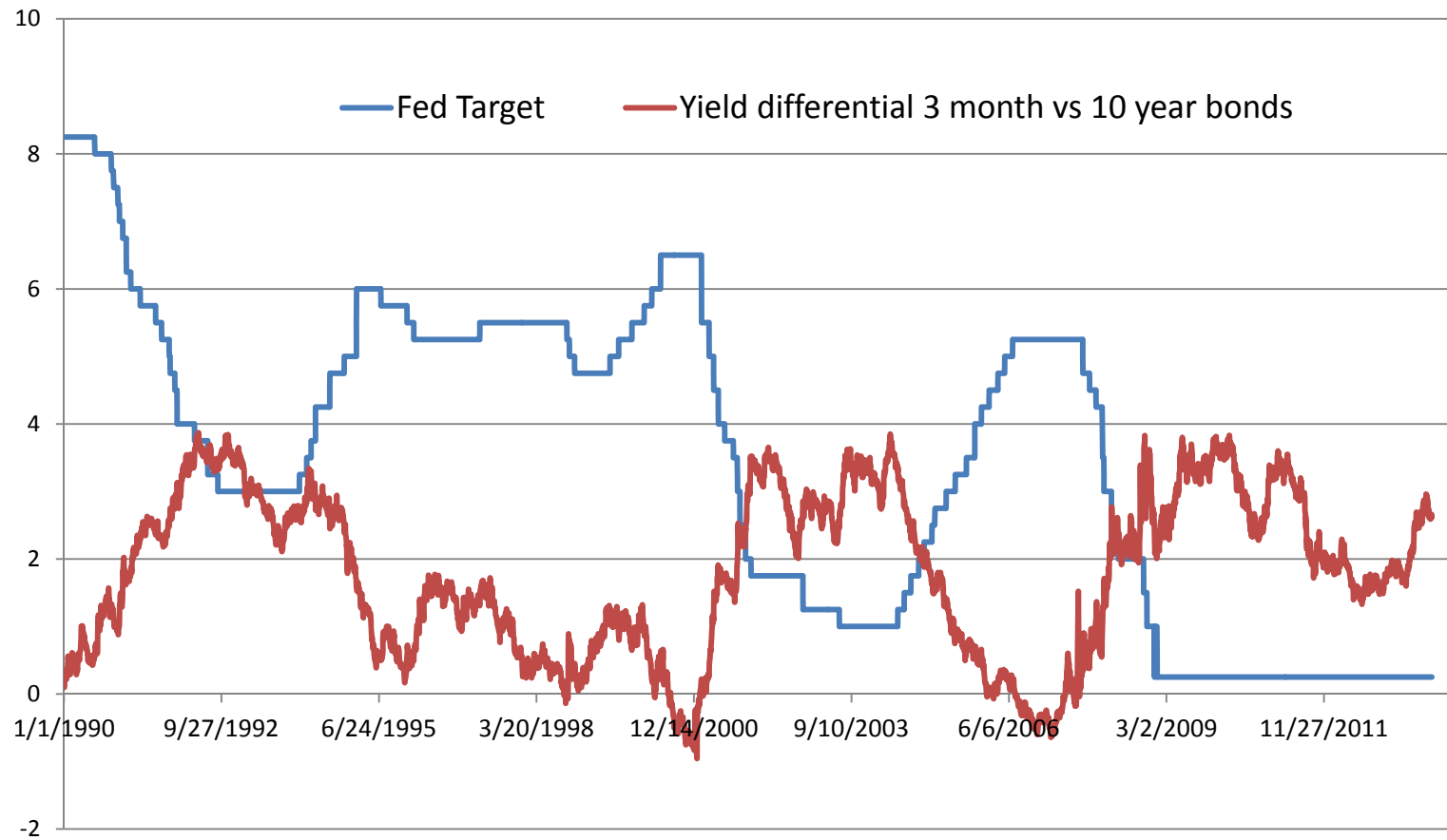
$$X_l^2 = Bc/(1+r_l) + 0.75(B(1+c)/(1+r_h)) + 0.25(B(1+c)/(1+r_l))$$

Because $r_h > r_l$ $X_l^2 < Bc/(1+r_l) + 0.75(B(1+c)/(1+r_h)) + 0.25(B(1+c)/(1+r_l))$ so $r_2 > r_l$

So you get a yield curve

Note: you also get an inverted yield curve if the state is high (because interest rates could fall next period but the inversion is less because the likelihood of a fall is small).

Yield curve

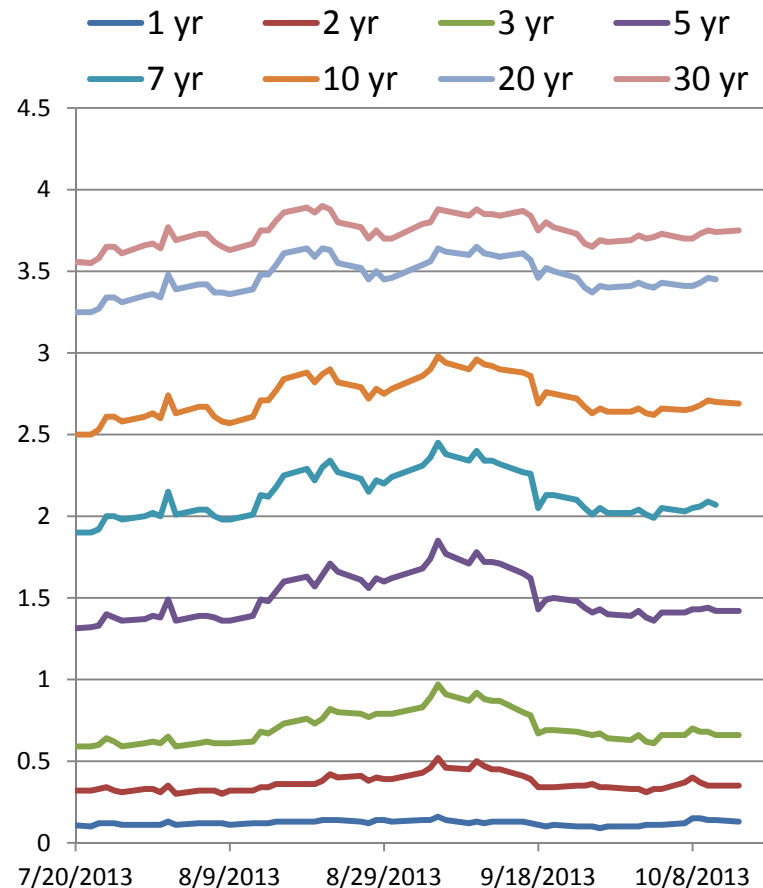


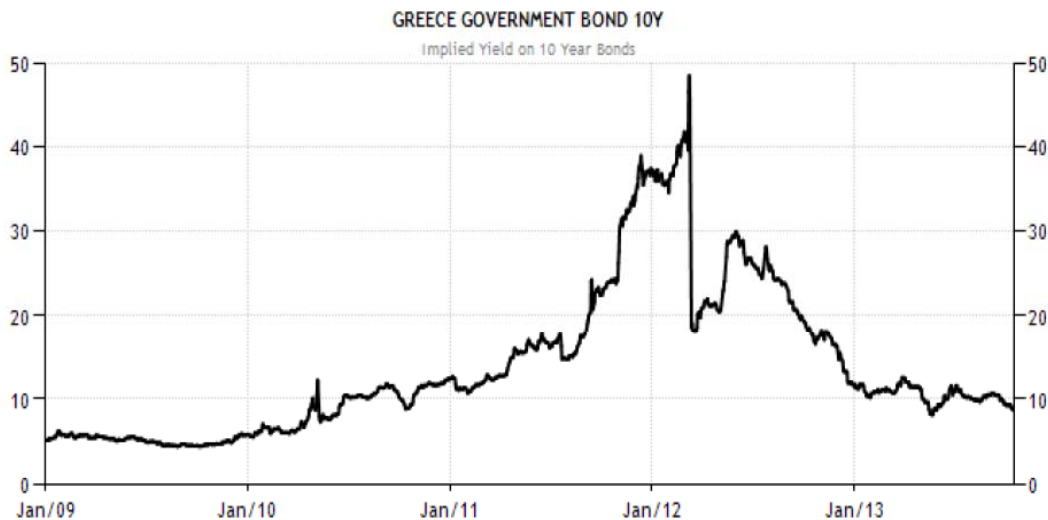
Beyond the Federal reserve

- Other elements that produce mean reversion
- Aggregate demand for funds is highest in boom times and easy to get (so expensive)
- So in bad times credit is rationed but cheap
- Now when the short term rate is low, people anticipate returning to more conventional interest rates

Interpreting recent events

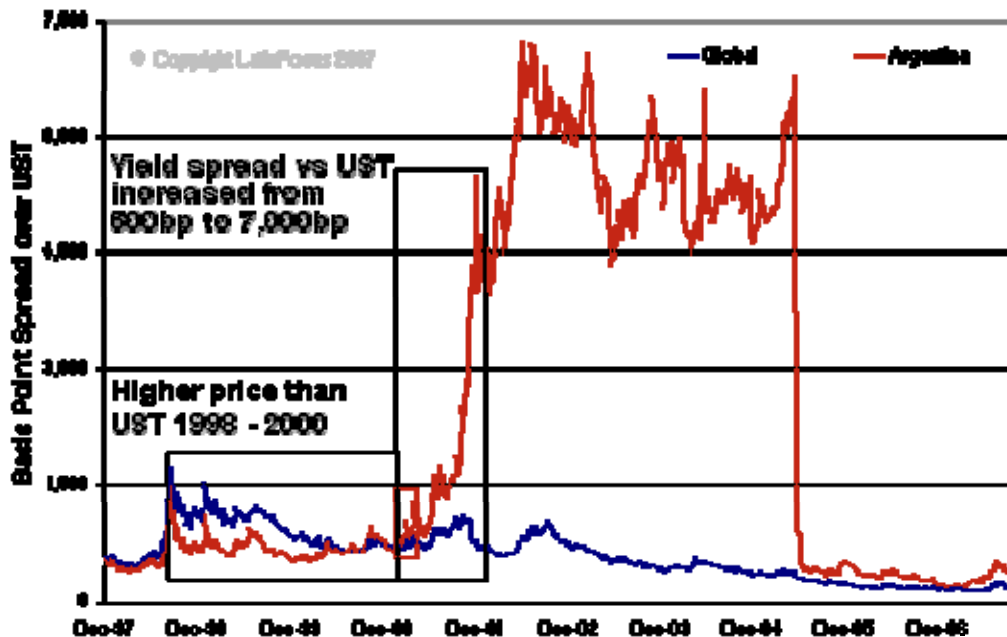
- Long term yields are flat
- That suggests the prospects for repudiation or a long term suspension of payments are **very very low**.
- If not yields should have gone up.
- Financial markets interpret the political fight as just that.





SOURCE: WWW.TRADINGECONOMICS.COM | PUBLIC DEBT MANAGEMENT AGENCY (PDMA)

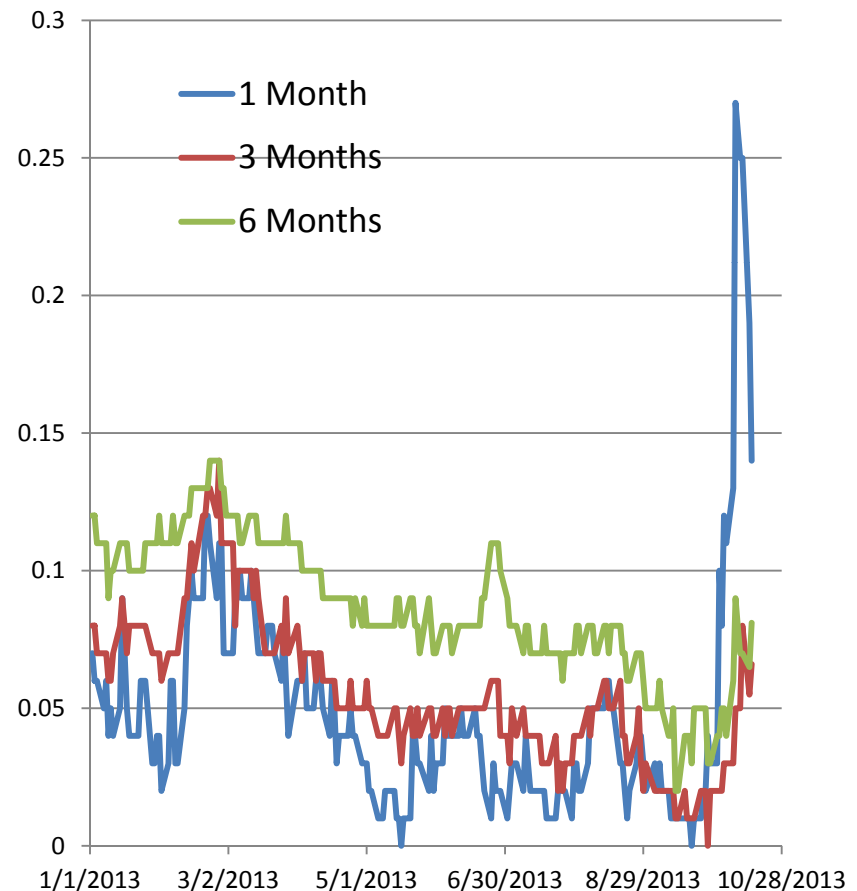
- When there are real risks of default yields jump (theses are % per year)



- Argentina actually defaults and the implied yields are now 70% per year

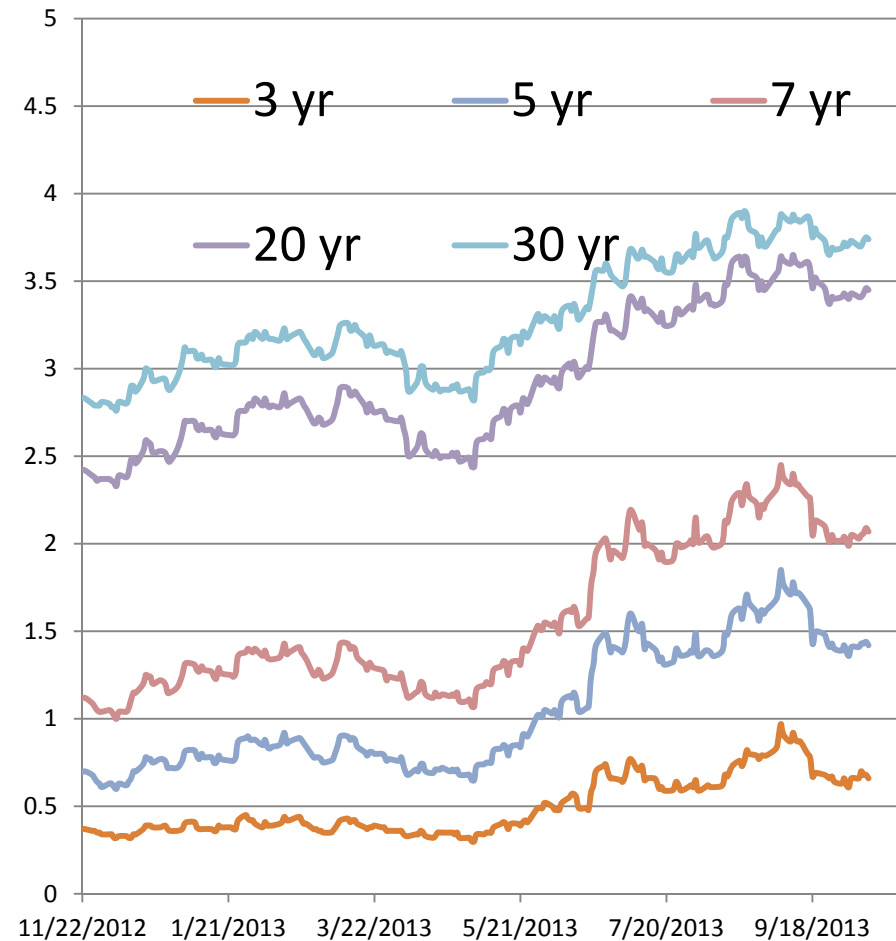
Interpreting recent events (2)

- In the US (today is yesterday)
- Rates were falling through the end of august but the have no exceeded their early year values until the shut down begins.
- The current jump in the premium, is consistent with a possible delay of 1 month. It matters for these short term zero coupon but not so much for anything else.



Interpreting recent events (2)

- From Dec 2012 the fed has bought 85 billion dollars of high quality bonds
- Then 19 June 2013, it announces that it will taper off these purchases by the end of the year
- Market reacts and the reaction are sharpest in the medium maturities.



Next time

- 0-21 Class 7 : Risk
- **Each Security has uncertain payoffs;**
- **Down side VS Upside;**
- **Risk aversion: Expected utility and utility of expected value;**
- **Basic mean variance trade off in utility**
- **Individual heterogeneity and sorting in markets.**