BEM 103: Introduction to Finance. Midterm

Solutions

November 16, 2013

1. **Investment**

   A company faces a strategic choice:

   (A) . *It could build a new plant at the cost of $100 million which has no resale value. The plant start yielding profits in a year and will last 10 years. If it does so it can expand output by $30 million per year and all other costs (labor, energy, raw materials) will come to $10 million.*

   (B) . *It could produce the same output by running its current factory at double shift. Its annual costs would then come to $25 million.*

   1 pt *1.A. What is the more profitable option if interest rates are 5%?* A

$$
\begin{aligned}
PV_A &= -\$100m + (\$30m - \$10m)\sum_{t=1}^{10}\frac{1}{1.05^t} = \$54.43m \\
PV_B &= (\$30m - \$25m)\sum_{t=0}^{10}\frac{1}{1.05^t} = \$43.61m
\end{aligned}
$$

   1 pt *1.B. What is the more profitable option if interest rates are 10%?* B

$$
\begin{aligned}
PV_A &= -\$100m + (\$30m - \$10m)\sum_{t=1}^{10}\frac{1}{1.1^t} = \$22.89m \\
PV_B &= (\$30m - \$25m)\sum_{t=0}^{10}\frac{1}{1.1^t} = \$35.72m
\end{aligned}
$$

   1 pt *1.C. What maximizes profits if interest rates are 5%, building the plant, running the old factory intensely or both?* Since the present values of both projects are positive, it is better to do both.

1 pt *1.D. Suppose year 0 interest rates are 6% and the firm builds the plant then year 3 interest rates jump to 11% what should it do?* The firm has already built the plant, so \$100m is sunk cost. Thus, there is no point in closing the new plant as it has no resale value. Note that both projects still have positive present value, so if the firm can afford it, it should do both. If not, stick to option A for the rest of 7 years.

2. **Bond Yields**

An island economy faces booms and bust because tourists only come when the weather is good. The central bank uses a countercyclical interest policy to encourage investments in busts. The transition matrix and the interest rate conditional on the state are given in the matrix below.

		This period		
		Boom	Bust	Interest rate
Next period	Boom	0.4	0.6	10%
	Bust	0.6	0.4	5%
Interest rate		10%	5%	

1.5 pt *2.A. Suppose the central bank markets two zero coupon bonds (face value 100) one with a one year term and the other with a two year term. What are their prices if the current state is bust?* The price for the one-year bond:

$$p_1 = \frac{B}{1+r_L} = \frac{100}{1+0.05} = 95.24$$

The price for the two-year bond:

$$p_2 = Pr(Boom)\frac{B}{(1+r_H)(1+r_L)} + Pr(Bust)\frac{B}{(1+r_L)^2} = 0.6\frac{100}{1.1\times 1.05} + 0.4\frac{100}{1.05^2} = 88.23$$

1.5 pt *2.B. Compute the two bonds' yield, is there a yield curve?* The yield of the one-year bond:

$$R_1 = r_L = 0.05$$

The yield of the two-year bond:

$$\frac{B}{(1+R_2)^2} = P_2 \Rightarrow R_2 = 0.0646$$

Note that $R_1 \neq R_2$, so there is a yield curve.

1.5 pt *2.C. Suppose the central bank markets two 4% coupon bonds (face value 100). One with a one year term and the other with a two year term. What are their prices if the current state is bust?* The price of the one-year bond:

$$p_1' = \frac{B(1+c)}{1+r_L} = 99.04$$

The price of the two-year bond:

$$p_2' = \frac{c}{1+r_L} + 0.6\frac{B(1+c)}{(1+r_H)(1+r_L)} + 0.4\frac{B(1+c)}{(1+r_L)^2} = 95.57$$

1.5 pt *2.D. Is the yield curve different if there is a coupon vs. not?* The yield of the one-year bond:

$$R_1' = r_L = 0.05$$

The yield of the two-year bond:

$$\frac{c}{1+R_2'} + \frac{B(1+c)}{(1+R_2')^2} = 95.57 \Rightarrow R_2' = 0.0643$$

Note that $R_1' = R_1$ and $R_2' \neq R_2$, so the yield curve is different.

3. **Personal finance**

Consider the situation where you start working on January 1 of year 2018 aged 25, your initial salary is \$100,000, and your income grows at 2.5% a year until you decide to retire. You intend to retire as long as you have saved enough money to buy an annuity that pays as much as you earn your last year of paid work. You expect to be able to buy an annuity that will pay 8% of its capital value for the rest of your life provided you are at least 65 when you buy it.

1 pt *3.A. Your only investment option returns 7% a year with a variance of 8%. What constant savings rate produces a retirement income equal to your terminal salary at age 65?* $s = 12\%$

$$\left(\sum_{t=1}^{40} \$100,000 \cdot 1,025^{t-1} \cdot s \cdot 1.07^{40-t}\right) \cdot 0.08 = \$100,000 \cdot 1.025^{39}$$

1 pt *3.B. Suppose that age 25 you go to graduate school and spend 5 years getting a Phd and then have a 3 year post doc during which you do not save. Then you get a job, your initial salary is \$140,000 and grows at*

the same 2.5% a year henceforth. What constant savings rate produces a retirement income equal to your terminal salary at age 65? $s = 19\%$

$$\left(\sum_{t=1}^{32} \$140,000 \cdot 1,025^{t-1} \cdot s \cdot 1.07^{32-t}\right) \cdot 0.08 = \$140,000 \cdot 1.025^{31}$$

1.5 pt *3.C. Assuming the distribution is normal and returns are independent from year to year with what probability will you have two consecutive negative returns?* 16%

$$r \sim \mathcal{N}\left(0.07, 0.08\right), \quad \left[\mathbb{P}\left(r < 0\right)\right]^2 = \Phi^2\left(-\frac{0.07}{\sqrt{0.08}}\right) = 0.16$$

1.5 pt *3.D. How many additional years would you have to work to get to a wealth sufficient to purchase an annuity that gives your age 65 salary, if you have two years of negative returns?* 8.6 years

$$R_{-} \equiv \mathbb{E}\left(r \mid r < 0\right) = \frac{\int\limits_{-\infty}^{0} r \frac{1}{\sqrt{2 \cdot 0.08\pi}} e^{-\frac{(r-0.07)^2}{2 \cdot 0.08}} dr}{\mathbb{P}\left(r < 0\right)} = -\frac{0.08}{0.4} = -0.2$$

$$x(1 + R_{-})^2(1 + 0.07)^t = x(1 + 0.07)^2 \quad \Rightarrow \quad t = 8.6$$

4. Efficient Frontier

Two stock one riskless assets.

	Stock A	Stock B	Riskless bond
R	0.08	0.02	0.01
	Variance covariance matrix		
Stock A	0.03	0.01	
Stock B	0.01	0.02	

For each of the questions below a portfolio must report, its proportion of each asset, its expected return, and it expected variance.

2.5 pts 4.A. What are efficient portfolios by 0.01 increments from 0.02 to 0.08 target returns (no short sales)?

The portfolio in a stock portfolio is unique because $W_B=(1-W_A)$ if so the return from a portfolio with weight W_A on stock A is simply $W_A r_A+(1-W_A)\ r_B$. For a target return is r, the only solution is $W_A=(r-r_B)/(\ r_A-r_B)=$ (r-0.02)/(0.06). The variance is the $W_A^2 0.3+2\ W_A W_B(0.01)+\ W_B^2 0.2$.

The set of portfolios are thus

Target r	W_A	W_B	Var
0.200	0.000	1.000	0.678
0.030	0.165	0.835	0.132
0.040	0.330	0.670	0.129
0.045	0.430	0.570	0.130
0.500	0.500	0.500	0.132
0.060	0.660	0.340	0.141
0.070	0.840	0.160	0.156
0.080	1.000	0.000	0.173

1 pt 4.B. Which of these portfolios are dominated?

A portfolio is dominated if there is another portfolio with a lower variance and higher return.

So the first two are in this category (r=0.02 and 0.03)

2.5 pts 4.C. If you are allowed short sales what are efficient portfolios by 0.01 increments from 0.02 target returns to 0.12 returns?

Solution process is the same except we extend the range above to 0.12

Target r	W_A	W_B	Var	Sharpe ratio
0.020	0.000	1.000	0.678	0.015
0.030	0.165	0.835	0.132	0.150
0.040	0.330	0.670	0.129	0.231
0.045	0.430	0.570	0.130	0.275
0.500	0.500	0.500	0.132	0.302
0.060	0.660	0.340	0.141	0.352

0.070	0.840	0.160	0.156	0.387
0.080	1.000	0.000	0.173	0.404
0.090	1.160	-0.160	0.193	0.413
0.100	1.340	-0.340	0.217	0.416683
0.110	1.500	-0.500	0.240	0.417029
0.120	1.667	-0.667	0.265	0.416

1 pt 4.D. Which portfolios in the no short sale condition are dominated?

Again the first two are in this category (r=0.02 and 0.03)

2 pts 4.E. Now consider the riskless bond that returns 1%, in a r-σ environment. What is the tangent portfolio (an approximation to the nearest 0.1% will do)?

Any candidate tangent portfolio must be on the efficiency frontier. At the tangent portfolio the tangent of the efficiency frontier must have the same slope as the line that runs from the riskless return (0,0.1) to the tangent portfolio. That slope is $(r_p\text{-}r)/\sigma_p$

Moreover, the tangent portfolio is the portfolio with the highest adjusted Sharpe Ratio $(r_p\text{-}r)/\sigma_p$

That is computed in the table above as the portfolio that offers a return of 0.11. (in fact the tangent portfolio has a return just above 0.106 (var 0.232)).

1 pt 4.F.What is now the sequence of un-dominated portfolios from 0.01 to 0.12 target returns?

The un-dominated portfolios are all on the line defined by the tangency condition.

Target r	Wa	Wb	Wriskless	Var
0.010	0	0	1	0.000
0.020	0.149	-0.046	0.897	0.024
0.030	0.299	-0.092	0.794	0.048
0.040	0.448	-0.139	0.691	0.072
0.045	0.522	-0.162	0.639	0.084
0.050	0.597	-0.185	0.588	0.096
0.060	0.746	-0.231	0.485	0.120
0.070	0.896	-0.277	0.381	0.144
0.080	1.045	-0.323	0.278	0.168
0.090	1.194	-0.369	0.175	0.192
0.100	1.344	-0.416	0.072	0.215
0.107	1.448	-0.448	0.000	0.232
0.110	1.493	-0.462	-0.031	0.239
0.120	1.642	-0.508	-0.134	0.263

If you chose to answer question 1 with the riskless asset than your portfolios looks 4F for both question A and C and there are no un-dominated portfolios