

BEM 103: Introduction to Finance.

Homework 9:

Solutions

November 23, 2013

1. Financial Literacy

Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify if need be your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

- (a) *Financial incentives*: some form of material reward — especially money — given to an agent to motivate him/her to act in a particular way.
- (b) *Transaction costs*: a cost incurred in making an economic exchange.
- (c) *Incomplete contracts*: contract where the parties did not specify their respective rights and duties for *every* possible future state of the world.
- (d) *Leveraged buy out (LBO)*: when a company is purchased with a combination of equity and significant amounts of borrowed money, structured in such a way that future cash flows are used as the collateral to secure and repay the money borrowed to purchase the company.
- (e) *Stock buyback*: reacquisition (repurchase) by a company of its own stock.

2. Firm Finance

The GooT travel agency is owned by Mr Brown, he has hired a manager, Dr Blue. He pays Dr Blue a wage $w = 600$ francs. The firm's revenues are $500 + 1600 \log(E)$ where E is Dr Blue's effort per period. Dr Blue like to surf so spending long hours at the travel agency is not on her agenda. In fact her disutility for effort is $0.5E^2$.

- (a) *Dr Blue knows that if she puts in below 20 units a year Mr Brown will fire her. Given the contract they have, what does she do?*

She will spend the minimum amount of effort which is 20. So, the firm's profit is

$$500 + 1600 \log(20) - 600 = 4693.17$$

- (b) *What is the discounted present value of the firm if it last 10 years and the interest rate is 5%?*

$$\sum_{t=0}^9 \frac{4693.17}{1.05^t} = \frac{4693.17 \left(1 - \frac{1}{1.05^{10}}\right)}{1 - \frac{1}{1.05}} = 38014.69$$

- (c) *Suppose Dr Blue is poor and comes to Mr Brown with an offer to buy him out in a leveraged buyout for 8,000 francs with a coupon rate of 6%. Should he accept the offer? Note to answer this question you must first find Dr Blue's effort level if she were the owner of the leverage firm, then you must find the firm's profit net of interest costs And then let Dr Blue deposit these profits in an interest bearing account so she can pay off the loan at the end.*

Dr Blue's effort level if she were the owner of the leverage firm is

$$\max_{E \geq 0} -100 + 1600 \log(E) - 0.5E^2$$

$$FOC : \frac{1600}{E} - E = 0 \quad \Rightarrow \quad E = 40$$

So, the firm's profit net of interest costs is:

$$500 + 1600 \log(40) - 600 - 0.06 \cdot 8000 = 5322.21$$

At the end she will get

$$\left(\sum_{t=0}^9 5322.21 \cdot 1.05^t \right) - 8000 = 5322.21 \frac{1 - 1.05^{10}}{1 - 1.05} - 8000 = 32974.36 > 0$$

So, Dr Blue will be able to pay off the loan at the end.

From the agreement, Mr Brown will get

$$8000 \sum_{t=0}^9 \frac{0.06}{1.05^t} + \frac{8000}{1.05^9} = 8000 \cdot 0.06 \cdot 8.1 + \frac{8000}{1.05^9} = 9045 < 38014.69$$

So, he should not sign the contract.

- (d) *Suppose however that Mr Brown does not want to sell the whole firm, at what price should he be willing to sell Dr Blue half the firm? Would doing so improve profits?*

If he sells half the firm, then the effort level would be

$$\max_{E \geq 0} \frac{1}{2} (-100 + 1600 \log(E)) - 0.5E^2$$

$$FOC : \frac{800}{E} - E = 0 \Rightarrow E = 28.28$$

The condition under which Dr Blue will be able to pay off the loan is

$$\begin{aligned} \left[\frac{1}{2} (-100 + 1600 \log(28.28)) - 0.06 \cdot D \right] \sum_{t=0}^9 1.05^t &\geq D \\ \frac{1}{2} (-100 + 1600 \log(28.28)) - 0.06 \cdot D &\geq \frac{D}{8.1} \\ 2623.72 - 0.06 \cdot D &\geq \frac{D}{8.1} \\ D &\leq 14301.56 \end{aligned}$$

On the other hand, Mr Brown will get

$$\begin{aligned} &\left(\frac{1}{2} (-100 + 1600 \log(28.28)) + 0.06D \right) \sum_{t=0}^9 \frac{1}{1.05^t} + \frac{D}{1.05^9} \\ &= 8.1 (2623.72 + 0.06D) + \frac{D}{1.05^9} = 21252.132 + 1.13D \end{aligned}$$

So, the condition under which Mr Brown agrees to sign the contract is

$$21252.132 + 1.13D \geq 38014.69$$

$$D \geq 14834.12$$

So, such contract does not exist.

- (e) *Going back to the beginning, if contracts were complete what could Mr Brown do?*

Mr Brown could contract the level of effort equal to the optimal one, that is $E = 40$, without selling the firm.

3. Company C

Company C faces some difficulties completing its most important project and hires Ms Tech as project manager because she has some engineering expertise

that is particularly relevant. The firm stock price is S and to induce Ms Tech to get the project back on the right track they offer her 1000 options at strike price S . Consider Ms Tech's choices of strategy to fix the problem in different scenarios.

- (a) On her first day of work the stock price is S and she has to choose between a safe and a risky project completion path. If she picks the safe path, the fix will be expensive but work for sure and the stock price will be S with probability 0.5 and $1.2S$ with probability 0.5. If she picks the risky approach, the project does not get completed on time with probability 0.5 and the stock price will fall to $0.8S$. With equal probability she will find very cheap fix and the stock price will jump to $1.3S$. Which strategy does she pick? What would share holders want her to do?

She will pick the risky strategy since it gives her the expected payoff $0.5 \cdot 0.3S \cdot 1000$ whereas the safe strategy will give her only $0.5 \cdot 0.2S \cdot 1000$.

The shareholders want her to choose the safe path since

$$0.5 \cdot S + 0.5 \cdot 1.2S > 0.5 \cdot 0.8S + 0.5 \cdot 1.3S.$$

- (b) On her first day of work the stock price is $S_1 = 1.2S$ and she has to choose between a safe and a risky project completion. If she picks the safe path, the fix will be expensive but work for sure and the stock price will be S_1 with probability 0.5 and $1.2S_1$ with probability 0.5. If she picks the risky approach, the project does not get completed on time with probability 0.5 and the stock price will fall to $0.8S_1$. With equal probability she will find very cheap fix and the stock price will jump to $1.3S_1$. Which strategy does she pick? What would shareholders want her to do?

Ms Tech's expected payoff from the safe approach is

$$1000[0.5(S_1 - S) + 0.5(1.2S_1 - S)] = 1000[0.5 \cdot 0.2S + 0.5 \cdot 0.44S] = 320S$$

Ms Tech's expected payoff from the risky approach is

$$1000[0.5 \max\{0.8S_1 - S, 0\} + 0.5(1.3S_1 - S)] = 280S$$

So, she will pick the safe project.

The shareholders want her to choose the safe path since

$$0.5 \cdot S_1 + 0.5 \cdot 1.2S_1 > 0.5 \cdot 0.8S_1 + 0.5 \cdot 1.3S_1.$$

- (c) *Can you just change the success probabilities in case 1 to make the shareholders agree with her choice of strategy? How?*

Let p_s be the probability of success for the safe strategy, and p_r be the probability of success for the risky strategy. Then Ms Tech will choose the safe project if and only if

$$0.2Sp_s \geq 0.3Sp_r \quad \Leftrightarrow \quad 2p_s \geq 3p_r$$

The shareholders want the safe strategy iff

$$S(1 - p_s) + 1.2Sp_s \geq 0.8S(1 - p_r) + 1.3Sp_r \quad \Leftrightarrow \quad 2 + 2p_s \geq 5p_r$$

Note that

$$2p_s \geq 3p_r \quad \Rightarrow \quad 2 + 2p_s \geq 5p_r$$

$$2 + 2p_s \leq 5p_r \quad \Rightarrow \quad 2p_s \leq 3p_r$$

Therefore, any pair $(p_s, p_r) \in [0, 1]^2$ such that $2p_s \geq 3p_r$ or $2 + 2p_s \leq 5p_r$ will make the shareholders agree with Ms Tech's choice of strategy. For example, $p_s = 0.5$ and $p_r = 0.3$, or $p_s = 0.2$ and $p_r = 0.5$, or $p_s = p_r = 0.7$.

4. Gambling for Resurrection

Sunset co has been investing heavily in real estate. Its financial structure is as follows it has 100 million dollars of debt, its equity is valued at 5 million dollars (each share is 5 dollars) and its management owns 1 million stock options with a strike price of \$15.

- (a) *It could just sell all its real estate holdings. Pay off its debt and distribute the residual to shareholders. If value additivity holds (or markets are efficient) what do shareholders expect to get?*

They expect to get 5 million dollars (or 5 dollars per share).

- (b) *Second it could undertake to finish a mall it has under construction and if all goes well it add 5 million dollars to shareholder value if they run into further problems with terrain stability the loss will be 15 million dollars. The probability of success is 75%.*

Then the shareholders will get (in expectation):

$$0.75 \cdot \$10m - 0.25 \cdot \$0m = \$7.5m$$

Note that if the project fails the shareholders will get nothing and bondholders will get \$90m only.

- (c) *Third it could undertake to complete the mall and build condos on the adjacent land it also owns. That would be a profitable operation only if the real estate market bounces back strongly, something experts put at 20% but in this case the next gain to equity would be 25 million dollars. In case of failure the firm loses 20 million dollars. The probability of success of the condos is independent of that of the mall. Compute the different net values of the firm and of the firm's stock.*

The net value of the firm is

$$\$100m + \$5m + 0.75 \cdot \$5m - 0.25 \cdot \$15m + 0.2 \cdot \$25m - 0.8 \cdot \$20m = \$94m$$

The net value of the firm's stock is

$$\begin{aligned} & 0.2 \cdot 0.75(\$5m + \$5m + \$25m) + 0.2 \cdot 0.25 \max\{\$5m - \$15m + \$25m, 0\} \\ & + 0.8 \cdot 0.75 \max\{\$5m + \$5m - \$20m, 0\} + 0.8 \cdot 0.25 \max\{\$5m - \$15m - \$20m, 0\} \\ & = \$6m \end{aligned}$$

and the price of each share is \$6.

- (d) *In what cases would management exercise their options remember that if management exercises the options the firm has to buy back at current prices enough shares to complete the transactions. So the share value has to be established after net of the cost of paying out the options.*

Management would exercise their options only if both projects succeed. In this case they will get $p - 15$ million dollars where p is defined from

$$35 - (p - 15) = p \quad \Rightarrow \quad p = 25$$

- (e) *If bondholders are in control what strategy do they pursue for the firm?*

They will prefer the first strategy when no project is undertaken as it is the only case when the firm will pay all its debt for sure.

- (f) *If shareholders are in control what strategy do they pursue for the firm?*

The second strategy as it leads to the highest stock price (\$7.5 per share).

- (g) *If management is in control what strategy do they pursue for the firm?*

The third strategy as it is the only strategy that gives positive payoff with positive probability.

(h) *What is socially efficient?*

The net value of the firm is

Strategy 1 : $\$105m$

Strategy 2 : $\$105m + 0.75 \cdot \$5m - 0.25 \cdot \$15m = \$105m$

Strategy 3 : $\$94m$

So, the social efficient strategies are first and second.