BEM 103: Introduction to Finance.
Homework 8:

Solutions

November 17, 2013

1. **Financial Literacy**

   Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify if need be your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

   (a) *Binomial option*: An option whose underlying asset has two possible values at the expiration date of the option.

   (b) *Strike price*: The price at which a specific derivative contract can be exercised.

2. **Pondering Options**

   Consider the case of the widget company that acquires a new technology from an inventor Ms Smith. It forms a new company to use the new technology which Widget Co owns 50% and Ms Smith owns 50%. The technology has a probability p of being successful in a year and then it will generate $1/p$ million dollars in profits, if it fails it returns 0. Interest rate is r.

   (a) *What is the expected value of the firm? What is its the net present value? And what is its price?*

   Expected value is $p \cdot (1/p) = 1$ million dollars. The net present value is $1/(1 + r)$ millon dollars. The price is equal to the net present value.

(b) *The widget company can invest effort to adapt its product to the technology. If it invests the likelihood of success is ph (1/p million dollars in profits). That effort can not be contracted up. What kind of contract would Ms Smith like to sign to induce the Widget co to make the effort? Can you think of a price for that contract?*

Ms Smith should sell an option to Widget Co with the profit of a new company as the underline asset (strike price is zero). In other words, the widget company pays x to Ms Smith now and owns 100% of the new company. x should be equal to $\frac{1}{2(1+r)}$, which is half of the net present value of this company conditional on the widget company not investing effort.

Or Ms Smith can sign a future contract which claims to sell her share one year later to the company at price 1, the expected value given that the company does not invest in effort. This will make Widget Co to be more motivated to invest effort while making Ms Smith indifferent to sell her share now or in the future.

(c) *Once the technology has been successful the Widget company can make further effort to adapt this technology but it does not want to share those additional profits. What kind of contract would it want to sign?*

Essentially it should be the same contract as in (b) which gives all rights to the profit to the Widget company.

(d) *In the case of success the Widget company will need to expand its operations. Should it sign a forward contract to lease space or an call option? why*

Call option since it wants to lease space only in the case of success.

3. **Pondering Futures**

You are a natural gas producer that has signed a large number of leases with foreign governments that requires you to produce a given level of output from their natural gas fields and pay them a given set of per BTU royalties.

(a) *What set of risks does this lease arrangement make you sensitive to?* Price risk on natural gas.

(b) *What might you want to do?* Sign a forward contract on natural gas.

(c) *Suppose (unrealistically) there are not organized markets for insurance, futures, or options for natural gas but these markets exist in petroleum, what might you do to hedge? Explain why you think this might work.*

You might want to partially hedge price risk on natural gas by sign a future contract on petroleum hoping that prices for natural gas and petroleum are highly correlated.

(d) *The Dutch have a large greenhouse industry that produces both vegetables and flowers. These greenhouses are entirely heated by natural gas. How might you use this fact to design a contract that allows you to hedge your price exposure in the gas market? Explain why you think this might work?*

You can sign a future contract to hedge price of Dutch vegetables and flowers. The logic is the same as in previous point: prices for Dutch vegetables and flowers and for natural gas are positively correlated.

4. **Computing Options**

(a) *The current price of Ampere Co is 136. The price of an option with strike price one month ahead (i.e.) December 2013 is 136 is 9.50, assuming the interest rate is 0.25% a month. What is the expected price if the option were to be exercise next month (assume that the expected price if the option is not exercised is symmetric about the current price).* 154.7

$S_0 = K = 136$, $p = 9.5$, $r = 0.0025$.

$$\begin{aligned} a(S_0 + \Delta) + b(1 + r) &= \Delta \\ a(S_0 - \Delta) + b(1 + r) &= 0 \end{aligned}$$

$a = \frac{1}{2}$, $b = -\frac{S_0 - \Delta}{2(1+r)}$.

Since $p = aS_0 + b$, we have the following equation for Δ:

$$9.5 = \frac{1}{2} \cdot 136 - \frac{136 - \Delta}{2(1 + 0.0025)},$$

which implies $\Delta = 18.7075$. Thus, the expected price if the option were to be exercise next month is $136 + 18.7075 = 154.7075$.

(b) *Currently Ampere makes about 6000 cars a quarter, recently there have been some fires in cars crashes. Suppose that if this problem persists the price of shares will fall to 100 dollars by February 13 (3 months hence).*

If the problem is quickly solved the stock price will bounce back to 172. Assuming the interest rate is 0.25% a month, what should be the value of binomial call option with strike price 136? 18.37

$S_0 = K = 136$, $r = 0.0025$.

$$
\begin{aligned}
172a + b(1+r)^3 &= 172 - 136 \\
100a + b(1+r)^3 &= 0
\end{aligned}
$$

$a = \frac{172-136}{172-100} = 0.5$, $b = -\frac{50}{(1+r)^3} = -\frac{50}{1.0025^3} = -49.63$. So, the price is $p = 136a + b = 68 - 49.63 = 18.37$.

(c) *Right now Ampere is facing problems securing enough batteries suppose each month supply can either go up by 5% or down by 5% and that will be fully reflected in the stock price because it is the only constraint on increasing output. What is a January call option worth today (assuming the interest rate is 0.25% a month).*

$S = 136$, $\Delta = 0.05$, $r = 0.0025$. Denote $C_{1,u}$ the price of the January call option in December if the price does up.

$$
\begin{aligned}
S(1+\Delta)^2 a + b(1+r) &= S(1+\Delta)^2 - S \\
S(1+\Delta)(1-\Delta)a + b(1+r) &= 0
\end{aligned}
$$

$a = \frac{(1+\Delta)^2-1}{(1+\Delta)^2-(1-\Delta^2)} = \frac{2\Delta+\Delta^2}{2\Delta+2\Delta^2} = \frac{2+\Delta}{2+2\Delta}$, $b = -\frac{S(1+\Delta)(1-\Delta)a}{1+r}$. So, $C_{1,u} = a(1+\Delta)S + b = aS(1+\Delta)\left(1 - \frac{1-\Delta}{1+r}\right) = \frac{S(2+\Delta)(r+\Delta)}{2(1+r)}$

Next:

$$
\begin{aligned}
S(1+\Delta)a + b(1+r) &= C_{1,u} \\
S(1-\Delta)a + b(1+r) &= 0
\end{aligned}
$$

$a = \frac{C_{1,u}}{2S\Delta}$, $b = -\frac{S(1-\Delta)a}{1+r}$. So, the price today is equal to

$$
\begin{aligned}
p &= aS + b = aS\left(1 - \frac{1-\Delta}{1+r}\right) = \frac{aS(r+\Delta)}{1+r} = \frac{C_{1,u}(r+\Delta)}{2\Delta(1+r)} \\
&= \frac{S(2+\Delta)(r+\Delta)^2}{4\Delta(1+r)^2} = 3.823
\end{aligned}
$$