

BEM 103: Introduction to Finance.

Homework 7: Future and Options

Solutions

November 11, 2013

1. Financial Literacy

Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify if need be your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

- (a) *Forward contract*: a promise to purchase a specific quantity of some item at a specified date.
- (b) *Futures contract*: a kind of forward contract which is standardized and traded on an exchange; each party contracts with an intermediary (not with a party on the other side) and has to establish a collateral account.
- (c) *Call option*: an agreement that gives an investor the right (but not the obligation) to buy a stock, bond, commodity, or other instrument at a specified price within a specific time period.
- (d) *Replicating portfolio*: a replicating portfolio for a given asset or series of cash flows is a portfolio of assets with the same properties (especially cash flows).
- (e) *Collateral account*: account that is used as a collateral against the counterparty risk in futures; parties should keep enough money on their accounts to guarantee the execution of the contract.
- (f) *Counterparty risk*: a likelihood that one party in a contract will be unable or unwilling to execute it.

- (g) *Option delta*: the sensitivity of an option price relative to changes in the price of the underlying asset.

2. Future

The NbN airline company sells tickets a year in advance and it flies from Southbend to Southpark. The fuel consumption for each flight are of the form $a + bq$ where q is the number of passengers. The Ceo, Alfred H. Sir, contemplates hedging.

- (a) *On the first day of the month it has to commit to a schedule (number of flights per month) for that month a year hence and at the end of each month it gets an update on how many tickets it has sold. What should it do if it wants to hedge fuel costs?*

Every month you buy futures a year forward for $a \cdot n$ to cover the flights you commit to (n is the number of flights), plus $b \cdot \Delta q$ futures for each month, where Δq is the number of additional tickets you have sold for that month. If you do that you fully hedge fuel costs

- (b) *Suppose NbN has raised all its capital as equity and in the absence of any hedging has a $\beta = 0.5$. Does hedging increase or decrease its β if fuel price volatility is perfectly correlated with market volatility?*

Decrease since

- $\beta_i = \sigma_{iM} / \sigma_M^2$,
- $\sigma_{iM} = \text{Cor}(r_i, r_M) \cdot \sigma_i \cdot \sigma_M$,
- fuel price volatility is perfectly correlated with market volatility, thus its market β is positive,
- hedging decreases σ_i as we remove fuel price volatility.

- (c) *In case (b), Alfred H. Sir owns 20% of NbN and is risk averse, does he want to hedge? Yes the CEO wants to hedge because doing so reduces the volatility of the firm and he is over invested relative to market (so his portfolio is not CAPM).*

- (d) *Does its β increase or decrease if fuel price volatility is uncorrelated with market volatility? Do you have enough data to give a formula for that change? If fuel prices are uncorrelated with market volatility then they are idiosyncratic risk and removing it will not change the firm's β .*

- (e) Suppose NbN has raised 90% of its capital as equity and in the absence of any hedging has a $\beta = 4.5$. Does hedging increase or decrease its β if fuel price volatility is uncorrelated with market volatility? Again this is idiosyncratic risk so it has no affect on the firm's β . The issue of leverage does not matter here.
- (f) In case (e) Alfred H. Sir owns 20% of NbN and is risk averse, does he want to hedge? The CEO still wants to hedge because he is over exposed.

3. Future again

You are a construction operator and are bidding to build a road to do so you will need 100,000 tons of granite rocks every six months to face earthworks.

- (a) Supposing the price of granite is \$10 today, the price of storage is zero, the real interest rate is 0.25% a month. What should future prices be for you to decide to hedge your granite rock needs? Give the sequence of price for 6 months, 1 year, and 1.5 years.

The generale formula is $F^T = (1 + r + c)^T S_0$, $c = 0$.

$$F^{6 \text{ months}} = 1.0025^6 \cdot \$10 = \$10.15$$

$$F^{1 \text{ year}} = 1.0025^{12} \cdot \$10 = \$10.3$$

$$F^{1.5 \text{ years}} = 1.0025^{18} \cdot \$10 = \$10.46$$

So, the price should be less (or equal) to F^T .

- (b) Suppose you are in the same situation but inflation runs at 0.1% a month. What should future prices be for you to decide to hedge your granite rock needs? Give the sequence of price for 6 months, 1 year, and 1.5 years.

The generale formula is $F^T = (1 + r - i)^T S_0$.

$$F^{6 \text{ months}} = 1.0015^6 \cdot \$10 = \$10.1$$

$$F^{1 \text{ year}} = 1.0015^{12} \cdot \$10 = \$10.18$$

$$F^{1.5 \text{ years}} = 1.0015^{18} \cdot \$10 = \$10.27$$

So, the price should be less (or equal) to F^T .

- (c) *Suppose you do hedge today for the next year and a half using futures. Six month later you take delivery of the first contracts. Then a week later a Canadian firm offers to deliver granite at \$5 a ton. What should you do?*

You cannot do anything for the next year contracts as you have already signed them. Once the Canadian firm offer appears, you have to put enough money on your collateral account to guarantee you have no incentives to break the deal.

- (d) *Suppose you do hedge today for the next year and a half using forwards. Six month later you take delivery of the first contracts. Then a week later a Canadian firm offers to deliver granite at \$5 a ton. What should you do?*

Now you can break the deal and buy Canadian granite at \$5 a ton (unless you do not want to damage your reputation). The difference is that there is no collateral account for forwards. This is the example when a counterparty risk is realized.

4. Options

Assume the current value of a stock is S_0 it can either jump up by Δ or go down by Δ . The interest rate is r .

- (a) *Using the replicating portfolio approach show that a put option with exercise price of S_0 should have a price of $(-rS_0 + \Delta)/2(1 + r)$.*

Consider a portfolio of a stocks and b bonds (the price of one bond is 1). The current value of this portfolio must be equal to the price of the put option:

$$p = aS_0 + b$$

Pricing rule:

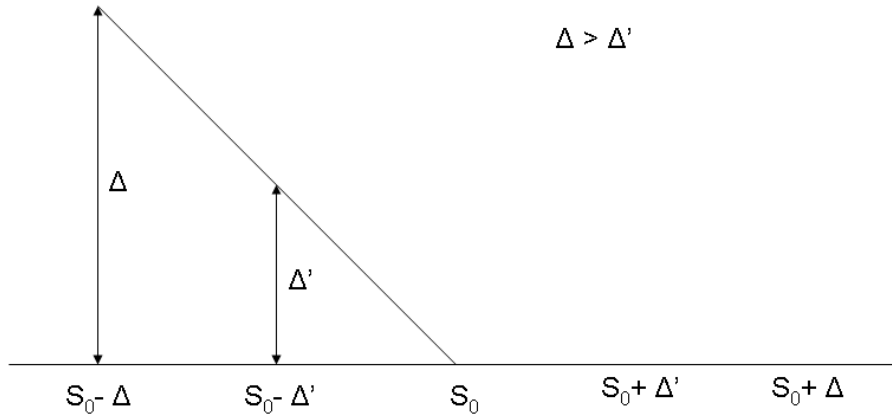
- if $S_T = S_0 + \Delta$, then $a(S_0 + \Delta) + b(1 + r) = 0$
- if $S_T = S_0 - \Delta$, then $a(S_0 - \Delta) + b(1 + r) = \Delta$

Thus, $a = -\frac{1}{2}$ and $b = \frac{S_0 + \Delta}{2(1+r)}$. So,

$$p = -\frac{S_0}{2} + \frac{S_0 + \Delta}{2(1+r)} = \frac{-S_0(1+r) + S_0 + \Delta}{2(1+r)} = \frac{-rS_0 + \Delta}{2(1+r)}$$

- (b) *If the formula above is correct increasing the volatility of the stock makes options more valuable even if the expected price stays the same. Explain with words and a graph why that is the case.*

The expected payoff $(S_0 - S_T)^+$ from the put option increases in volatility: the payoff is either Δ or zero.



- (c) *Does increasing volatility but keeping the expected price the same make a futures contract more valuable (as in the price of the future). No because futures are zero-sum games.*