BEM 103: Introduction to Finance.
Homework 5: Portfolios

Solutions

October 27, 2013

1. **Financial Literacy**

   Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify if need be your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

   (a) *Mean-Variance trade-off*: curve that shows the lowest variance for any given level of expected return.

   (b) *Risk Aversion*: expected utility is less then utility of expected value.

   (c) *Expected Utility*: sum over all possible values of a portfolio (states of the world), of the utility in a given state times the probability of this state:

   $$\sum_i U(x_i)p_i$$

   (d) *Utility of the expected value*:

   $$U\left(\sum_i x_i p_i\right)$$

   (e) *Portfolio with Short sales*: portfolio that has a short position on one or more securities; having short position means selling securities that are not currently owned, with the intention of subsequently repurchasing them ("covering") at a lower price.

(f) *Risk*: involves issues where we do not know the exact outcome (e.g. future price), but we can accurately measure the odds (e.g. we know the distribution).

(g) *Portfolio Separation*: the property that a (large) market can be reduced to a few indices ("funds") without the agents being worse off.

2.

3.

4. **Personal finance**

Your are a financial advisor, you are advising a client about what investment portfolios to choose. Using the linked CSV file that you can import into Excell or a statistical package propose an optimal portfolio in each of the cases below. In each case your only investment choices are 3-year T-Bills, and three stock funds (NASDAQ, DOW and FTSE100)

(a) *Compute the monthly returns for each investment opportunity, then the mean return and the variance-covariance matrix. For each five year period 1991-5 1996-2000, and 2011-present. . . .as well as for the whole period.*

Denote p_i the price in month i of one of the investments opportunities (3-year T-Bills, NASDAQ, DOW or FTSE100). Then the return for that month is

$$r_i = \frac{p_{i+1}}{p_i}$$

The mean return for 5 years is

$$\frac{1}{60}\sum_{i=1}^{60} r_i$$

The covariance of the returns for $\{p_i\}$ and $\{p'_i\}$ is

$$\frac{1}{60}\sum_{i=1}^{60} r_i r'_i$$

Period	Mean			
	3-year T-Bills	NASDAQ	DOW	FTSE100
1991 — 1995	1.004744528	1.01651883	1.012618231	1.00617018
1996 — 2000	1.004695942	1.02023991	1.012714509	1.008931072
2001 — 2005	1.002626324	0.999853328	1.000964229	1.002397547
2006 — 2010	1.002274013	1.004479086	1.001588771	1.000079894
2011 — 2013	1.000453115	1.012077652	1.01063849	1.005166594
1990 — 2013	1.003350935	1.010147334	1.00689557	1.00467214

Period	Variance-Covariance Matrix
1991 — 1995	$\begin{pmatrix} 7.28265 \cdot 10^{-7} & 4.19985 \cdot 10^{-6} & 3.47596 \cdot 10^{-6} & 3.00341 \cdot 10^{-6} \\ 4.19985 \cdot 10^{-6} & 0.001524413 & 0.000432739 & 0.000756144 \\ 3.47596 \cdot 10^{-6} & 0.000432739 & 0.000910511 & 0.0004357 \\ 3.00341 \cdot 10^{-6} & 0.000756144 & 0.0004357 & 0.001901752 \end{pmatrix}$
1996 — 2000	$\begin{pmatrix} 2.24411 \cdot 10^{-7} & -7.32029 \cdot 10^{-6} & -3.62887 \cdot 10^{-6} & -1.4078 \cdot 10^{-6} \\ -7.32029 \cdot 10^{-6} & 0.008184597 & -0.000375705 & 0.001949826 \\ -3.62887 \cdot 10^{-6} & -0.000375705 & 0.002207588 & -0.000221626 \\ -1.4078 \cdot 10^{-6} & 0.001949826 & -0.000221626 & 0.001749782 \end{pmatrix}$
2001 — 2005	$\begin{pmatrix} 5.23306 \cdot 10^{-7} & -1.80578 \cdot 10^{-5} & -5.91663 \cdot 10^{-6} & -7.64927 \cdot 10^{-6} \\ -1.80578 \cdot 10^{-5} & 0.00576979 & 1.98054 \cdot 10^{-5} & 0.002225922 \\ -5.91663 \cdot 10^{-6} & 1.98054 \cdot 10^{-5} & 0.001887287 & 2.30431 \cdot 10^{-5} \\ -7.64927 \cdot 10^{-6} & 0.002225922 & 2.30431 \cdot 10^{-5} & 0.001791478 \end{pmatrix}$
2006 — 2010	$\begin{pmatrix} 1.62443 \cdot 10^{-6} & -9.03358 \cdot 10^{-6} & 7.25068 \cdot 10^{-6} & -9.36944 \cdot 10^{-7} \\ -9.03358 \cdot 10^{-6} & 0.003665735 & 0.000268793 & 0.002831895 \\ 7.25068 \cdot 10^{-6} & 0.000268793 & 0.002206993 & 0.000381277 \\ -9.36944 \cdot 10^{-7} & 0.002831895 & 0.000381277 & 0.003300254 \end{pmatrix}$
2011 — 2013	$\begin{pmatrix} 5.57842 \cdot 10^{-8} & -1.78206 \cdot 10^{-6} & 1.16091 \cdot 10^{-6} & -8.70252 \cdot 10^{-7} \\ -1.78206 \cdot 10^{-6} & 0.001701492 & 9.59403 \cdot 10^{-5} & 0.001223007 \\ 1.16091 \cdot 10^{-6} & 9.59403 \cdot 10^{-5} & 0.001081424 & -4.73667 \cdot 10^{-5} \\ -8.70252 \cdot 10^{-7} & 0.001223007 & -4.73667 \cdot 10^{-5} & 0.001381979 \end{pmatrix}$
1990 — 2013	$\begin{pmatrix} 3.23378 \cdot 10^{-6} & -8.52268 \cdot 10^{-7} & 3.22362 \cdot 10^{-6} & 2.11646 \cdot 10^{-6} \\ -8.52268 \cdot 10^{-7} & 0.004452642 & 0.00016297 & 0.001891651 \\ 3.22362 \cdot 10^{-6} & 0.00016297 & 0.001756636 & 0.000153026 \\ 2.11646 \cdot 10^{-6} & 0.001891651 & 0.000153026 & 0.002141459 \end{pmatrix}$

(b) *Compute the efficient frontier for the data over the whole period*[1].

Portfolio: $w = (w_{3TB}, w_{NAS}, w_{DOW}, w_{100})'$.

Mean returns:

$$r = \begin{pmatrix} 1.003350935 & 1.010147334 & 1.00689557 & 1.00467214 \end{pmatrix}'$$

Variance-Covariance Matrix:

$$C = \begin{pmatrix} 3.23378 \cdot 10^{-6} & -8.52268 \cdot 10^{-7} & 3.22362 \cdot 10^{-6} & 2.11646 \cdot 10^{-6} \\ -8.52268 \cdot 10^{-7} & 0.004452642 & 0.00016297 & 0.001891651 \\ 3.22362 \cdot 10^{-6} & 0.00016297 & 0.001756636 & 0.000153026 \\ 2.11646 \cdot 10^{-6} & 0.001891651 & 0.000153026 & 0.002141459 \end{pmatrix}$$

The optimal portfolio problem[2]:

$$\begin{cases} \min\limits_{w} & w'Cw \\ \text{s.t.} & r'w \geq r_t \\ & i'w = 1 \\ & w \geq 0 \end{cases}$$

	Weights				
Return	3 yr-Tbill	NASDAQ	Dow	FTSE100	Variance
0.00335	1.000000001	0	0	0	$3.23378{\cdot}10^{-6}$
0.0035	0.970316711	0.013484625	0.016198664	0	$4.46546{\cdot}10^{-6}$
0.004	0.869534233	0.057387782	0.073077985	0	$2.81818{\cdot}10^{-5}$
0.005	0.667969164	0.145194147	0.186836689	0	0.000166112
0.006	0.466404127	0.233000497	0.300595376	0	0.000424707
0.007	0.26483909	0.320806847	0.414354063	0	0.000803964
0.008	0.063274053	0.408613198	0.52811275	0	0.001303886
0.009	0	0.647165988	0.352834013	0	0.002157985
0.01	0	0.95469136	0.04530864	0	0.004076001
0.011	0	1	0	0	0.004452642

[1] For the whole problem, we assume no short sales.

[2] $i = (1, 1, 1, 1, 1)'$

Target return 0.005.

(c) *What is the efficient portfolio given this return target?* $w = (0.667969164, 0.145194147, 0.186836689, 0)$.

(d) *Your client arrives January 1 1996 and has a five year horizon. What is the efficient portfolio given the data 1991-1995 given his target return? What is its performance given the realized returns 1996-2001?*

The efficient portfolio is $w = (0.974267033, 0.013550286, 0.01218268, 0)$.

Dividing prices at the end of 2001 to prices at the beginning of 1996, we got the following vector of realized returns $r = (1.318624669, 2.331141075, 1.9303125, 1.583878584)'$.

The realized return is $r \cdot w = 1.339796554$.

(e) *Your client arrives January 1 2001 and has a five year horizon. What is the portfolio you propose given the data 1996-2001? What is its performance given the realized returns 2002-2006?*

The efficient portfolio is $w = (0.968810978, 0.008916187, 0.018801964, 0.003470871)$.

The realized return is 1.158758724.

(f) *Would your client have been better off staying with his original weights (from question d)?*

With the original weights the return would be 1.14160398. So, he would be worse off.

(g) *Your client arrives January 1 2006 and has a five year horizon. What is the portfolio you propose given the data 2001-2005? What is its performance given the realized returns 2006-2011?*

The efficient portfolio is $w = (1, 0, 0, 0)$.

The realized return is 1.147867382.

Target return 0.01.

(h) *Your client arrives January 1 1996 and has a five year horizon. What is the efficient portfolio given the data 1991-1995 if his target return is 0.01 per month? What is its performance given the realized returns 1996-2001?*

The efficient portfolio is $w = (0.467895254, 0.273249658, 0.258855088, 0)$.

The realized return is 1.753632938.

(i) *Your client arrives January 1 2001 and has a five year horizon. What is the portfolio you propose given the data 1995-2001? What is its performance given the realized returns 2002-2006?*

The efficient portfolio is $w = (0.45196639, 0.154016552, 0.328091879, 0.065925178)$.

The realized return is 1.036588157.

(j) *Would your client have been better off staying with his original weights (from question b)?*

If he stays with the original weights, the return is 1.143616796. So, he would be better off.

(k) *Your client arrives January 1 2006 and has a five year horizon. What is the portfolio you propose given the data 2001-2005? What is its performance given the realized returns 2006-2011?*

The efficient portfolio is $w = (1, 0, 0, 0)$.

The realized return is 1.153578165.