BEM 103: Introduction to Finance.
Homework 4: Bonds and risk

Solutions

October 20, 2013

1. **Financial Literacy**

   Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify, if need be, your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

   (a) *Riskless bond*: a bond that has a zero probability of not making payments on time.

   (b) *Default*: a missed payment.

   (c) *Winner's curse*: a tendency for the winning bid in an auction to exceed the intrinsic value of the item purchased; in short, the winner tends to overpay; it may occur in common value auctions with incomplete information.

   (d) *Market failure*: the situation when the allocation of goods and services by a free market is not efficient.

   (e) *Coupon*: a periodic interest payment that the bondholder receives during the time between when the bond is issued and when it matures.

   (f) *State dependence*: when the expected value of a future variable depends on the current state and is thus not necessarily the same as the current value

2. **Riskless bonds**

(a) *A. The yield on the 1 year T-Bill was 0.1% on May 7 2013, the yield on the 5 year T-Bill was 0.75%. Both have a coupon of 0.25% paid once a year. What is the expected interest rate from year 2 to 5?* 0.9%

The interest rate for year 1 coincides with the yield on the 1 year T-Bill, which is 0.1%. Denote the expected interest rate from year 2 to 5 as r. Then

$$\sum_{n=1}^{5} \frac{0.0025B}{(1.0075)^n} + \frac{B}{1.0075^5} = \frac{0.0025B}{1.001} + \sum_{n=2}^{5} \frac{0.0025B}{1.001(1+r)^{n-1}} + \frac{B}{1.001(1+r)^4}$$

$$r \approx 0.009$$

(b) *On July 1 2013 the yield on the 10 year T-Bill hit 2% up from 1.1% May 1 before. If those T-bills had a 1% coupon (paid annually), what were their prices on July 1 and May 1?*

Assuming face value=100, we have

$$\begin{aligned} p(1 \text{ July}) &= \sum_{n=1}^{10} \frac{Bc}{1.02^n} + \frac{B}{1.02^{10}} \\ &= \sum_{n=1}^{10} \frac{1}{1.02^n} + \frac{100}{1.02^{10}} \approx 73 \\ p(1 \text{ May}) &= \sum_{n=1}^{10} \frac{Bc}{1.011^n} + \frac{B}{1.011^{10}} \\ &= \sum_{n=1}^{10} \frac{1}{1.011^n} + \frac{100}{1.011^{10}} \approx 99 \end{aligned}$$

3. **Government default**

(a) *On 10/9/2013 the yield of the zero coupon one month T-Bill was 0.26%. Supposing that market actors thought that repudiation was out of the question and given that the 6 Month T-Bill was 0.08% was the expected delay in payment (beyond the month of the contract), implied by the premium on the one month bonds)? In other words when did investors expect to get paid?*

$$\frac{B}{1.0026^{1/12}} = \frac{B}{1.0008^t} \quad \Rightarrow \quad 1.0026^{1/12} = 1.0008^t \quad \Rightarrow \quad t \approx \frac{1}{4}$$

So, investors expect to get paid in three month implying two-month delay.

(b) *Conversely suppose that repudiation was the only alternative to extending the debt ceiling. Given the jump from 0.01% on 10/1/2013 to 0.26% how likely was repudiation to occur.*

$$\frac{B}{1.0026^{1/12}} = \frac{B(1-D)}{1.0001^{1/12}} \quad \Rightarrow \quad D \approx 0.0002$$

So, the probability of repudiation is 0.02%.

4. **Personal finance**

Using the linked CSV file that you can import into Excell or a statistical package consider the situation where you start working on January 1 of year 2018 aged 25, your initial salary is \$100,000, and your income grows at 2.5% a year until you decide to retire. You intend to retire as long as you have saved enough money to buy an annuity that pays as much as you earn your last year of paid work. You expect to be able to buy an annuity that will pay 8% of its capital value for the rest of your life provided you are at least 65 when you buy it.

For all calculations — see the file.

(a) *Your only investment choice is to invest in the* S&P *500. Based on the time series of the* S&P *in the CSV file what saving's rate allows you to reach that goal by 2058 (after 40 years of work)?*

The salary in 2057 would be $\$100,000 \cdot (1.025)^{39} \approx \$261,957$. So, the capital value of the annuity is $\$261,957/0.08 = \$3,274,468$ and therefore the saving's rate should be approximately 13.49%.

(b) *Suppose, however, that this exact sequence of returns is not guaranteed to occur but that the returns have the same mean and variance. How likely is it that you will reach that goal by 2058?*

The mean of returns is equal to 0.07, the variance is 0.035. The constant return that gives \$3,274,468 of total savings by 2058 (given the saving rate 13.49%) is 0.065. Assuming normal distribution of returns, the probability of reaching the goal is[1]

$$\mathbb{P}\left(R > 0.065\right) = 1 - \Phi\left(\frac{\sqrt{53}(0.065 - 0.07)}{\sqrt{0.035}}\right) = 0.586$$

[1] Number of observations is $n = 53$.

So, the probability is 59%.

(c) *Now fix your target income at what you earn at 65. How likely is it that you will have to continue working until age 70 (2063)?*

Now the average return should be 0.051. Again, assuming normal distribution of returns, the probability of hitting the target at age 70 is 78%.

(d) *How likely is it that you will face two consecutive years of negative returns (assuming that returns are independent over time)? If these should occur when you are 64 and 65, how long would you delay retirement (again to reach target retirement income = income at 65)?*

According to the data, the probability of negative return in one year is $\frac{19}{53} = 0.358$. So if independence holds, the probability of two consecutive negative returns is $0.358^2 = 0.129$

Note that the average negative return is $r = -0.124$. Suppose that the total value of assets at year 2056 (age 63) is x. If in year 2057 and 2058 the returns are both negative, then in the year 2058 the value becomes $x(1+r)^2 = 0.767x$. To reach the goal that the payment of the annuity equals 2058's salary, we need

$$x(1+R)^2 = 0.767x(1+R)^t$$

where $R = 0.071$ is the average return of S&P 500.

We can get that $t = 5.87$

So the retirement is delayed by nearly 6 years.

(e) *From the data how likely are two consecutive negative returns? Do you think this violates independence, how would you revise your answer to 4.D.*

By simply counting we can get the probability of two consecutive negative return is $\frac{8}{52} = 0.154$ which is greater than the case when we assume independence. This means that it is very likely to see more negative returns after two consecutive negative returns. So we should expect even longer delay in retirement than in 4.D.

(f) *Would the option to revise your saving's rate at age 45 be a valuable option? If so when and why?*

Notice that from year 2018 to 2038, the average rate of return is 8%, and the variance is 0.02. While from 2039 to 2058, the average rate of return is 7.8% and the variance is 0.038. So to reach the goal the investor should save more before age 45 (age 45 included) and less after 45.