BEM 103: Introduction to Finance.
Homework 2: time and money

Solutions

October 5, 2013

1. **Financial Literacy**

   Answer these questions in 3 steps. (1) give an answer to the question, (2) look over the material assigned for class and find a definition (3) modify if need be your first answer. The goal is not for you to memorize a given answer but to be sure you can explain the concept to someone. If you can't, then you do not control that concept.

   (a) *Equity*: ownership in any asset after all debts associated with that asset are paid off;

   (b) *Shares*: security that signifies ownership in a corporation and represents a claim on part of the corporation's assets and earnings;

   (c) *Discount rate*: the multiplier used in discounted cash flow analysis to determine the present value of future cash flows; the discount rate takes into account the time value of money (the idea that money available now is worth more than the same amount of money available in the future because it could be earning interest) and the risk or uncertainty of the anticipated future cash flows;

   (d) *Arbitrage opportunity*: a trade that profits by exploiting price differences without taking any risk; there are no arbitrage opportunities in perfect markets;

   (e) *Expected value*: anticipated value for a given investment; in statistics and probability analysis, expected value is calculated by multiplying each of the possible outcomes by the likelihood that each outcome will occur, and summing all of those values;

(f) *Yield curve*: a line that plots the interest rates (yields), at a set point in time, of similar bonds (having equal credit quality), but differing contract lengths (maturity dates);

(g) *Spread*: the difference between the bid (maximal price at which you can sell) and the ask (minimal price at which you can buy) price of a security or asset.

2. **Present value**

Early Tuesday October 1, Tesla Motors shares traded at \$190, thus it equity value was in excess of 20 billion dollars. This year it expects to sell 20,000 cars at 70,000 dollars each.

(a) *Supposing a generous 25% net profit per car, how many cars per year does Tesla have to sell 5 years hence (assume linear growth in car sales) and for the next 20 years to justify a market cap of 20 billion dollars. Assume that the interest rate is 5%.* 111,616.

Denote α the growth in car sales. Then

$$\$20,000,000,000 = 0.25 \cdot \$70,000 \cdot 20,000 \left(\sum_{i=0}^{5} \frac{\alpha^i}{1.05^i} + \sum_{i=6}^{20} \frac{\alpha^5}{1.05^i} \right)$$

$$20,000 = 350 \cdot \left(\sum_{i=0}^{5} \frac{\alpha^i}{1.05^i} + \alpha^5 \cdot 8.1327 \right)$$

Thus, $\alpha \approx 1.4$. So, the number of sales per year should be $20,000 \cdot \alpha^5 \approx 107,000$.

(b) *How likely is it that there is enough demand for \$70,000 electric cars that Tesla will reach that target number of sales?* Given the calculated growth of demand 1.4, it is highly unlikely.

(c) *What data could you use to give a more accurate answer to the question (you do not need to find the data)?* Growth of population, growth of percentage of people in population using luxury cars, growth of Tesla's share in the market of luxury cars.

(d) *Suppose five years hence Tesla has 30% of the luxury car sales market (200,000 units of its current model per year). Tesla has announced that it intends to introduce a \$30,000 sedan in the next three years. How many* *such cars at the same net profit margin would it need to produce to justify the market cap it has reached recently?* 102,000.

The demand for \$70,000 cars five years hence is $0.3 \cdot 200,000 = 60,000$ units per year. Then the growth rate for these five years is 1.316.

$$60,000 = 20,000 \cdot \alpha^5$$

$$\alpha \approx 1.246$$

Denote x the number of sedan cars Tesla has to produce each year.

$$\begin{aligned}\$20,000,000,000 = 0.25 \cdot \$70,000 \cdot 20,000 &\left(\sum_{i=0}^{5} \frac{1.246^i}{1.05^i} + \sum_{i=6}^{20} \frac{1.246^5}{1.05^i} \right) \\ &+ 0.25 \cdot \$30,000 \cdot x \sum_{i=3}^{20} \frac{1}{1.05^i}\end{aligned}$$

$$x \approx 102,000.$$

(e) *Suppose Tesla runs into development problems and instead of introducing the new model in 2016 it has to delay to 2018. How would that affect your calculations in D?* 121,000.

$$\begin{aligned}\$20,000,000,000 = 0.25 \cdot \$70,000 \cdot 20,000 &\left(\sum_{i=0}^{5} \frac{1.246^i}{1.05^i} + \sum_{i=6}^{20} \frac{1.246^5}{1.05^i} \right) \\ &+ 0.25 \cdot \$30,000 \cdot x \sum_{i=5}^{20} \frac{1}{1.05^i}\end{aligned}$$

$$x \approx 121,000.$$

(f) *What do you think Tesla Price should be? What calculation justifies your answer?*

Assuming the demand 60,000 in five years, the market cap should be

$$0.25 \cdot \$70,000 \cdot 20,000 \left(\sum_{i=0}^{5} \frac{1.246^i}{1.05^i} + \sum_{i=6}^{20} \frac{1.246^5}{1.05^i} \right) \approx \$12,000,000,000$$

The number of shares is

$$\frac{\$20,000,000,000}{\$190} \approx 105,000,000$$

So, Tesla Price should be

$$\frac{\$12,000,000,000}{105,000,000} \approx \$114.$$

3. **Perpetuities**

   (a) *What is the capital value of an annuity that pays x forever if the interest is r? x/r.*

   $$PV = \sum_{t=1}^{\infty} \frac{x}{(1+r)^t} = \frac{x}{r}.$$

   (b) *What should happen to its price if the interest jumps from r to $2r$?* It will be $x/(2r)$.

   (c) *Now consider a perpetuity that pay X the first year and grows at a rate g each year:*

   $$PV = \sum_{t=1} \infty \frac{X(1+g)^{t-1}}{(1+r)^t}.$$

   Show this simplifies to

   $$PV = \frac{X}{r-g}.$$

   $$\sum_{t=1} \infty \frac{X(1+g)^{t-1}}{(1+r)^t} = \frac{X/(1+r)}{1-(1+g)/(1+r)} = \frac{X}{r-g}.$$

4. **Modigliani Miller**

   Two Caltech undergraduates (J&J) *create a new windmill that is adaptable to extremely cold and wet climates. To develop this invention they need 50 million dollars in capital for plant, equipment, and advances to power companies in Greenland and Nova Scotia. Net of variable costs (labor, supplies, shipping...) the firm would clear 5 million dollars a year two years after starting.*

   (a) *What interest rate makes investors indifferent between starting the firm or not?* 9%

   $$50 = \sum_{t=2}^{\infty} \frac{5}{(1+r)^t}$$

   $$10 = \frac{1/(1+r)^2}{1-1/(1+r)}$$

   $$10 = \frac{1}{r(1+r)}$$

   $$r \approx 0.0916$$

(b) *If the interest rate is 5% and there is no equity premium show that* J&J *are indifferent between financing their capital needs by debt or by equity. What fraction of the firm's income do the two inventors retain?* 0.475.

The first part follows from Modigliani Miller theorem.

Let α be the part of annual income that the investors retain. Then

$$50 = \sum_{t=2}^{\infty} \frac{(1-\alpha)5}{(1.05)^t}$$

$$50 = \frac{(1-\alpha)5}{1.05 \cdot 0.05}$$

$$\alpha = 0.475.$$

So, the investors retain 0.475 of the firm's income.

(c) *Now suppose there is a 20% possibility that the government of Nova Scotia decides at the end of year 1 to ban the new windmill reducing income by 50%. Show again that if markets are efficient, combinations of debt and equity do not affect the value of the firm.*

Note that the firm cannot become a bankrupt here: is case of banning the maximal interest the firm must pay is $\$50,000,000 \cdot 0.05 = \$2,500,000$. On the other hand, the expected annual profit is $0.8 \cdot \$5m + 0.2 \cdot \$2.5m = \$4.5m$ which is greater than the annual profit $(1-\alpha)\$5m = \$2.625m$ needed to justify the investment.

Modigliani Miller theorem works here as the stream of profits the firm will make does not depend on the capital structure (the decision of banning does not depend on debt/equity ratio).

(d) *What financial structure implies that after an adverse regulatory decision by the government of Nova Scotia, the company would belong to bond holders?* All capital is in bonds.

Denote D is the amount of capital in bonds. Then $S = \$50,000,000 - D$ is the amount of capital in shares. After an adverse regulatory decision by the government of Nova Scotia, the company would belong to bond holders if

$$0.05 \cdot D \geq \$2,500,000$$

$$D \geq \$50,000,000$$

(e) *Suppose now that interest rates are 6% What financial structure implies that after an adverse regulatory decision by the government of Nova Scotia, the company would belong to bond holders?* If 42 million dollars are in bonds and 8 million dollars are in stocks.

Now:

$$0.06 \cdot D \geq \$2,500,000$$

$$D \gtrsim \$42,000,000$$

(f) *Suppose that* J&J *are risk averse, what would financial structure are they likely to choose? (1) if they are sure there will be no bad news, or (2) if they have to worry about regulatory decisions.* (1) indifferent (2) prefer stocks

If (1) then by Modigliani Miller theorem they are indifferent.

If (2), then J&J want to share the risk with shareholders so that the difference between the retaining earning in bad and good states would be the same.

Let D be the amount of capital in bonds. Then the retaining earnings would be

- in bad state: $\alpha(\$2.5m - rD)$
- in good state: $\alpha(\$5m - rD)$

To smooth them, we need $D = 0$.