Instructions: This exam is open book and notes, but it should involve your work only, you will need a calculator and can use a computer. For each question where a computation is needed your answer must consist of the derivation of the formulas you need to arrive at your answer and the actual numerical solution, two digit precision is good (on interest rates, that is in $100^{\text {th }}$ of percents). This exam should take you about an hour and half but you may use as much as three hours. To get full credit you must complete this exam by Nov 4 Start of class. You can turn it in to my box or to the box outside my office.
The exam will be posted after class on Wednesday October 30.

## 1. Investment

A company faces a strategic choice:
(A). It could build a new plant at the cost of $\$ 100$ million which has no resale value. The plant start yielding profits in a year and will last 10 years. If it does so it can expand output by $\$ 30$ million per year and all other costs (labor, energy, raw materials) will come to $\$ 10$ million.
(B). It could produce the same output by running its current factory at double shift. Its annual costs would then come to $\$ 25$ million.
$\mathbf{1} \mathbf{p t} 1$. A. What is the more profitable option if interest rates are $5 \%$ ?
$\mathbf{1}$ pt 1 . B. What is the more profitable option if interest rates are $10 \%$ ?
1 pt 1.C. What maximizes profits if interest rates are $5 \%$, building the plant, running the old factory intensely or both?

1 pt 1.D. Suppose year 0 interest rates are $6 \%$ and the firm builds the plant then year 3 interest rates jump to $11 \%$ what should it do?

## 2 Bond Yields

An island economy faces booms and bust because tourists only come when the weather is good. The central bank uses a countercyclical interest policy to encourage investments in busts. The transition matrix and the interest rate conditional on the state are given in the matrix below.

|  |  | This period |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | Boom | Bust | Interest rate |
| Next period | Boom | 0.4 | 0.6 | $10 \%$ |
|  | Bust | 0.6 | 0.4 | $5 \%$ |
| Interest rate |  | $10 \%$ | $5 \%$ |  |

1.5pts 2.A. suppose the central bank markets two zero coupon bonds (face value 100) one with a one year term and the other with a two year term. What are their prices if the current state is bust?
1.5pts 2.B. Compute the two bonds' yield, is there a yield curve?
1.5pts 2.C. suppose the central bank markets two $4 \%$ coupon bonds (face value 100) one with a one year term and the other with a two year term. What are their prices if the current state is bust?
1.5pts 2.D. Is the yield curve different if there is a coupon vs not?

## 3 Personal finance

Consider the situation where you start working on January 1 of year 2018 aged 25, your initial salary is $\$ 100,000$, and your income grows at $2.5 \%$ a year until you decide to retire. You intend to retire as long as you have saved enough money to buy an annuity that pays as much as you earn your last year of paid work. You expect to be able to buy an annuity that will pay $8 \%$ of its capital value for the rest of your life provided you are at least 65 when you buy it.

1 pt 3.A. Your only investment option returns $7 \%$ a year with a variance of $8 \%$. What constant savings rate produces a retirement income equal to your terminal salary at age 65 ?

1 pt 3.B. Suppose that age 25 you go to graduate school and spend 5 years getting a Phd and then have a 3 year post doc during which you do not save. Then you get a job, your initial salary is $\$ 140,000$ and grows at the same $2.5 \%$ a year henceforth. What constant savings rate produces a retirement income equal to your terminal salary at age 65 ?
1.5 pt 3.C. Assuming the distribution is normal and returns are independent from year to year with what probability will you have two consecutive negative returns?
1.5 pt 3.D. How many additional years would you have to work to get to a wealth sufficient to purchase an annuity that gives your age 65 salary, if you have two years of negative returns?

## 4. Efficient Frontier

Two stock one riskless assets.

|  | Stock A | Stock B | Riskless bond |
| :--- | :--- | :--- | :--- |
| R | 0.08 | 0.02 | 0.01 |
|  | Variance covariance matrix |  |  |
| Stock A | 0.03 |  |  |
| Stock B | 0.01 |  |  |

For each of the questions below a portfolio must report, its proportion of each asset, its expected return, and it expected variance.
2.5 pts 4.A. What are efficient portfolios by 0.01 increments from 0.02 to 0.08 target returns (no short sales)?

1 pt 4.B. Which of these portfolios are dominated?
2.5 pts 4.C. If you are allowed short sales what are efficient portfolios by 0.01 increments from 0.02 target returns to 0.12 returns?
$\mathbf{1} \mathbf{p t}$ 4.D. Which portfolios in the no short sale condition are dominated?
2 pts 4.E. Now consider the riskless bond that returns $0.01 \%$, in a $\mathrm{r}-\sigma$ environment. What is the tangent portfolio (an approximation to the nearest $0.1 \%$ will do )?
$\mathbf{1} \mathbf{~ p t} 4 . \mathrm{F}$. What is now the sequence of un-dominated portfolios from 0.01 to 0.12 target returns?


