Correcting The Guide to Objective Chance

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1. Introduction

In his "A Subjectivist's Guide to Objective Chance", David Lewis argues persuasively that a certain relationship holds between reasonable credence and objective chance (1980).[1] Both items involve varieties of probability: in the case of credence, we use subjective probability to describe gradations in degree of belief; in the case of chance, we use objective probability to describe irreducibly indeterministic processes (e.g., radioactive decay). While these two uses are not to be conflated, Lewis points out an important connection between them: roughly, rationality requires conforming one's subjective credence in a proposition to one's estimate of the objective chance that the proposition will come true. As a precise statement of this connection, Lewis offers his "Principal Principle"—only to find that it conflicts with his cherished thesis of reductionist metaphysics dubbed "Humean Supervenience". So we seem to face a dilemma: either we abandon an attractive and plausible principle relating credence and chance, or we give up the reductionist thesis.

Wrong. The Principal Principle (hereafter: the "Old Principle") does not adequately formulate the connection between credence and chance. Close inspection reveals that it embodies a crucial oversight for which there is, happily, a natural repair. Undertaking this repair, we arrive at a New Principle which all parties, regardless of their *metaphysical* commitments, should find compelling. This New Principle functions much better than the Old: it is more precise, holds without qualification, explains the initial plausibility of the Old Principle while yielding that Principle as an approximation, is perfectly immune to the problem raised in Lewis (1980) and provides an analysis of an important range of cases about which the Old Principle fell silent.

In what follows, I rehearse the Old Principle together with familiar problems confronting it, paying special attention to the notion of "admissibility" required for its formulation. This notion is, ultimately, dispensable: where "inadmissible" information was taken to *break* the connection between credence and chance, I show that on the contrary there is a perfectly straightforward way to *incorporate* such information into the connection. Following this line of thought leads directly to the New Principle.

All of which will require some background, which follows.

[1] Reprinted in Lewis (1986, pp. 83-113). All page references to "A Subjectivist's Guide to Objective Chance" are to the latter printing. See also Mellor (1971) for a similar view.

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2. *Terminology and assumptions*

I assume, as an idealization, that reasonable opinion is described by a probability distribution—called a "credence function"—over the space of possible worlds. I assume further that it can always be *represented* as coming from some reasonable *initial* credence function C by conditionalising on our total evidence E, so that for any proposition A, $\text{Credence}(A) = C(A/E) =_{\text{df}} \frac{C(AE)}{C(E)}$.

I assume that for each world w and each time t there is a unique probability function P_{tw} which describes the objective chances for world w at time t (the "t-chances"), and which is also a probability distribution over the space of possible worlds. I assume that what is past is no longer chancy: if A is a proposition true at w and entirely about matters restricted to times no later than t, then $P_{tw}(A) = 1$. Further, later chances come from earlier chances by conditionalising on intervening history[2]: if t is later than t', then $P_{tw}(\text{-}) = P_{t'w}(\text{-}/B)$, where B is the proposition which fully describes just the course of history from t' to t.

For each world w, let T_w be the proposition which completely characterizes the laws at w, probabilistic and otherwise: T_w holds at a world v iff v has exactly the same laws as w (in which case, of course, $T_w = T_v$). Let H_{tw} be the proposition which completely characterizes w's history up to time t. I assume that T_w specifies what chances would follow any initial segment of history (but unlike Lewis, I don't identify T_w with the set of implied history-to-chance conditionals). If so, the t-chance of any proposition A is x iff the conjunction $H_{tw}T_w$ implies that the t-chance of A is x. Hereafter, P will always denote a chance function, T a complete law-proposition, and H a complete history up to some time.

In the discussion to come we will need to make use of a certain equation involving conditional credence. For any A and E, probability theory tells us that we can express $C(A/E)$ as the following sum:[3]

$$C(A/E) = \sum_w C(A/H_{tw}T_wE)C(w/E).$$

In other words, $C(A/E)$ is equal to a weighted sum of the quantities $C(A/H_{tw}T_wE)$, where the weights are the corresponding quantities $C(w/E)$ (the conditional credence, given E, that the actual world is w). Hereafter, we will refer to this sum

[2] What I have adopted as assumptions Lewis tries to derive, using the Old Principle (1986, pp. 92-102). Thau (1994) has given convincing reasons why this derivation fails.

[3] We get the sum as follows: begin with the coarse-grained partition of conjunctions $H_{tw}T_w$ (a partition in the sense that exactly one such conjunction is true at any world; coarse-grained in the sense that any such conjunction is true at many worlds). Summing not over worlds but over elements of this partition, we have, by the additivity of $C(\text{-}/E)$ and the definition of conditional probability, $C(A/E) = \Sigma\, C(AH_{tw}T_w/E) = \Sigma\, C(A/H_{tw}T_wE)C(H_{tw}T_w/E)$. We then expand each term into a sum over the worlds which belong to the corresponding history-law conjunction: $C(A/H_{tw}T_wE)C(H_{tw}T_w/E) = \sum_v C(A/H_{tw}T_wE)C(v/E)$ where the worlds v are those at which $H_{tw}T_w$ is true. But when $H_{tw}T_w$ is true at v, $H_{tw}T_w = H_{tv}T_v$. Thus, $C(A/E) = \Sigma\sum_v C(A/H_{tv}T_vE)C(v/E)$, where the first sum is over the elements of the partition of history-law conjunctions, and the second sum is over the worlds belonging to each such element. And that's just the same as the single sum over worlds given in the text.

as the *expectation* of $C(A/H_{tw}T_wE)$ relative to the given credence $C(-/E)$. Notice that if, for each w, $C(A/H_{tw}T_wE) = x$, then $C(A/E) = x$; likewise, if, for each w, $C(A/H_{tw}T_wE) \approx x$, then $C(A/E) \approx x$. These observations will be important later.

3. *Admissibility and the Old Principle*

We want a principle which gives precise form to the intuition that one's credence should conform to one's estimate of objective chances. Here is Lewis's candidate (1980), which I call the Old Principle:

(OP) Let X be the proposition that the t-chance of A is x. Then for any C and proposition E, the following holds, if E is admissible at time t and is compatible[4] with X: $C(A/EX) = x$.

OP only gains substance once we say which propositions are admissible. Unfortunately, the straightforward account of admissibility that Lewis (1980) spells out runs into serious problems. First the account, then the problems.

According to Lewis (1986, p. 92), "Admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes." Inadmissible propositions, by contrast, give information that bears too directly on the truth of A for the connection described by OP to hold: for example, a description of how a chance process turns out should be inadmissible at any time *before* the outcome of that process is settled.

Lewis suggests that, in general, two sorts of information are admissible at t: information about history up to t, and information about how chance depends on history. Both parts of this suggestion run into trouble; but for the moment let us follow it without reservation, and interpret it as saying that all complete historical propositions H_{tw} are admissible at t, and that all complete law-propositions T_w are admissible at t. Let us also agree with Lewis (1986, p. 96) that truth-functional combinations of admissible propositions are admissible. Then all complete history-law conjunctions $H_{tw}T_w$ are admissible at t, whence we have a special case of OP:

$$C(A/H_{tw}T_wX) = x.$$

The sum could be finite or countable, if we pretend that the credence $C(-/E)$ counts as subjectively possible only a finite or countable number of worlds. If we don't pretend, then it could be a non-standard sum of infinitesimal quantities; for details, see Skyrms (1980, Appendix 4). Or, we could make use of the standard method of integration over densities.

[4] "Compatible" here doesn't just mean logically consistent; it also means that $C(EX) \neq 0$ (else the conditional probability is undefined). Lewis equates these conditions *via* the assumption that reasonable initial credence functions give zero *only* to contradictions. I do not wish to follow him in this assumption and therefore reserve the term "compatible" for the stronger condition and "consistent" for the weaker.

But $H_{tw}T_w$ completely specifies the t-chances. Therefore, $H_{tw}T_w$ is compatible with X only if it implies X, and only if the value x is the one given by P_{tw}. So the special case simplifies:

$$C(A/H_{tw}T_w) = P_{tw}(A).$$

Next, the special case is not so special. For recall that $C(A/E)$ is the expectation of $C(A/H_{tw}T_wE)$ (relative to credence $C(-/E)$). Putting EX in for E, $C(A/EX)$ is the expectation of $C(A/H_{tw}T_wEX)$ (relative to $C(-/EX)$). Suppose that E is "built up" out of HTs, in the sense that any HT consistent with E in fact implies E.[5] Then $C(A/HTEX) = C(A/HTX) = x$, so that $C(A/EX)$ is the expectation of a quantity whose value is always x. Then $C(A/EX) = x$. So we have a sufficient condition for admissibility: E is admissible at t if E is built up out of HTs. This sufficient condition—if correct—is general enough to guarantee the usefulness of OP, since we can typically take our *own* evidence to be of this sort.

We can also get a necessary condition. Suppose that E is admissible at t, and consider the case where A is E itself. Then, by OP,

$$C(E/EX) = x;$$

while, by ordinary probability theory,

$$C(E/EX) = 1.$$

In other words, E's admissibility implies the following: E is compatible with the proposition that the chance of E is x only if $x = 1$. Equivalently: if E is admissible at t, then for any reasonable credence C it must be that C(chance at t of E is $1/E$) $= 1$. On the supposition that E is true, it is unreasonable to give any credence to the claim that it has some objective chance of being false. (That's nontrivial: even on the supposition that the coin I'm about to toss will land heads, it's still reasonable to give credence to the claim that it has a non-zero *chance* of landing tails.)

Thus far the Old Principle seems to provide a useful and reasonably precise account of the relationship between credence and chance. But its utility and precision are hostage to the two-part assumption that information about either history or law is admissible. Each part faces a problem.

The first problem concerns the admissibility of historical information. Lewis himself notes (1986, p. 94) that there are possibilities (involving such things as time travellers, seers, and circular spacetimes) in which the past carries news from the future which, if known, breaks the connection between credence and chance. When the past does carry such news, I will say that it contains "crystal balls" (whether or not they take the form of magical quartz).

There needn't actually *be* crystal balls, there need only reasonably *seem* to be. That is, a proposition E will be inadmissible at time t if it provides reasonable warrant for a hypothesis that there are crystal balls (along with information about what the balls say)—even if that hypothesis is false. Don't suppose that for E to provide such warrant, it must include information directly about the future and so be inadmissible on familiar grounds. Example: we have, on thousands of occa-

[5] Here and elsewhere, I drop the subscripts "t" and "tw" unless clarity demands otherwise.

sions before this one, consulted the Oracle about what the chancy future would bring—and every time, her predictions have been vindicated, in minute detail. This time, she tells us that the coin will land heads. All of our information is purely historical, concerning only the record of her past successes, plus her most recent prediction. What should we believe about the outcome of the toss, on the supposition that it has a 50% chance of landing heads? Answer: we should be certain, or nearly so, that it will land heads, favouring the reliable words of the Oracle over the guidance of objective chance. We should treat the Oracle as a crystal ball—even though she might merely be lucky, and even though the evidence guiding our opinions contains no information directly about the future.

The second problem concerns the admissibility of information about the laws. We have supposed that for any time t and any complete law-proposition T, T is admissible at t. Then T must be incompatible with any proposition that says that T's t-chances of coming true are less than 1. And the choice of time is irrelevant: roughly, T must say of itself that it *always* was certain to come true (unlike historical propositions, which are certain *now*, but once weren't).

So what? In fact, why not just build this result into our metaphysics by endorsing the plausible thesis that the law-proposition for a given world never has any chance of being false at that world? The answer is that such a constraint places an onerous burden on various philosophical theories about chance, as Lewis persuasively argues (1980 and 1994). Further, while Lewis focuses on the trouble the constraint raises for the thesis of Humean Supervenience, Thau (1994) shows that it conflicts with a much wider range of philosophical positions about chance. I will not attempt to recapitulate their discussions, but will briefly review a standard way—as well as a non-standard way—of spelling out the problem for Humeanism.

Humeanism is, in part, the view that any proposition which holds at a world is in fact a consequence of the proposition which completely characterizes the total history of that world. (The other part concerns the sorts of things history can include.) Assume this supervenience claim, and consider, for some time t, some conjunction HT, where H specifies history only up to time t. HT assigns chances to various possible courses of future history. Suppose there is some such future F (not necessarily a *complete* future) with the following features: (i) HT assigns F a non-zero chance of coming about; (ii) no *total* history consistent with the conjunction HF has T as a consequence—in which case, by the supervenience claim, HF must *contradict* T. Such an F I will call, following Lewis, an *undermining future*: a course of events which has some chance of coming about, according to HT, but whose occurrence would, in conjunction with H, contradict T.

It deserves its name: for by (ii), $C(F/HT) = 0$; whereas by (i), OP, and the admissibility of HT, $C(F/HT) \neq 0$—a contradiction. Lewis argues that any plausible Humeanism will be committed to undermining futures; Thau shows how this commitment extends to other views on which complete enough information about history could justify certainty about the laws that govern chances.

You might think that the problem turns on allowing that a *complete* theory T is admissible, since we applied OP to $C(F/HT)$. Not at all: let A describe some total

world history. By the supervenience claim, there is some number y such that A implies that the t-chance of A is y. Let X be the proposition that the t-chance of A is x, with x positive but not equal to y. Then—taking E to be a tautology, and so admissible—we have $C(A/XE) = C(A/X) = x$. But this is impossible, since A contradicts X. If OP is correct, then its use here cannot be—whence X itself must contain inadmissible information!

The Old Principle looks to be in trouble, since the account of admissibility which gives it substance appears flawed. If we cannot in general assume that historical information is admissible, or even that information about the chances is admissible, then it becomes unclear how we are to apply the Old Principle. One can try to dismiss these problems. Crystal ball cases seem, perhaps, too esoteric to take seriously. And undermining futures may seem the sole province of the Humean—so too bad for his metaphysics if it conflicts with a sound, intuitive principle relating credence and chance.[6] Therefore—so goes the response—we should hold onto OP, stick to our initial assumptions about admissibility, abandon Humeanism, and not worry too much about bizarre cases involving crystal balls.

Don't be tempted. A more careful look at the notion of admissibility shows that the Old Principle needs amending, anyway. Once the amendment is made, the two problems dissolve.

4. Admissibility reconsidered: the New Principle

You might have this impression about OP: as long as E is admissible, credence should go by estimate of chances; if it is *not* admissible, no such constraint applies, and the Old Principle simply falls silent about what credence should be. But this is wrong. There is a straightforward way to incorporate inadmissible information, as follows.

Let the proposition E be inadmissible, but assume that *HT is* admissible. Then the Old Principle says that $C(A/HT) = P(A)$ (where P is the chance function picked out by HT). What about $C(A/HTE)$? OP fixes this quantity as well, even though E is inadmissible:

$$\begin{aligned} C(A/HTE) &= \frac{C(AE/HT)}{C(E/HT)} && \text{by probability theory} \\ &= \frac{P(AE)}{P(E)} && \text{by OP} \\ &= P(A/E) && \text{by the definition of conditional probability.} \end{aligned}$$

The inadmissible proposition E does not break the connection between credence and chance, but merely alters it: instead of making credence conform to the *unconditional* chances, one makes credence conform to *conditional* chances—where the condition in question is just the inadmissible information.

Now, if the problems raised in the last section are cogent, then it is bogus to assume *both* the Old Principle *and* the general admissibility of history-law con-

[6] But such a response had better be able to come to grips with Thau's argument.

junctions—so the above derivation is also bogus. Still, we can take from it a clue about how to reconceive the epistemic role of chance: rather than letting *unconditional* chances guide credence (our overall evidence permitting), we should let the chances, *conditional* on our evidence, guide credence—no matter what the content of that evidence. In that case, we should not have $C(A/HT) = P(A)$, but rather $C(A/HT) = P(A/HT)$. Assuming that what is past is no longer chancy, so that $P(H) = 1$, we arrive at the New Principle:

(NP) For any A and any complete history-law conjunction $H_{tw}T_w$, $C(A/H_{tw}T_w) = P_{tw}(A/T_w)$.

It would be nice if we could reach NP without a suspicious detour through OP. We can, by taking a more careful look at the motivation behind the search for a principle relating credence and chance.

Why should chance guide credence? Because—as far as its *epistemic* role is concerned—chance is like an expert in whose opinions about the world we have complete confidence.[7] Let us imagine that chance just *is* the credence of such an expert, called "the Oracle". Since the Oracle's credence is ideal, we should like our own to match hers. Of course, we do not know who the Oracle is; that would be like knowing, in complete detail, what the chances are. It is rather as if we have at our disposal a whole panel of *purported* experts (one for each chance-fixing conjunction *HT*), exactly one of whom is the Oracle; further, our evidence only goes so far toward determining who that true expert is. We don't know who the Oracle is; nevertheless, we still can—and should—rely on the opinions of the panellists in forming our own opinions, as follows.

Let A be any proposition, and suppose that E is a hypothesis rich enough to include not only our total evidence but also some conjunction *HT* (and perhaps more besides); thus E picks out one of the panellists—Cassandra, say—as the Oracle. We want to fix our conditional credence in A, given E; that is, we want to know how likely we should take A to be, on the supposition that E is true. But if E is true, then Cassandra is the Oracle. So we should fix our conditional credence by consulting Cassandra, and no one else. But in consulting her, we must take care to ask the right question: we don't want to know what her unconditional credence $P(A)$ is, but rather what her *conditional* credence $P(A/E)$ is—since it is our own such conditional credence about which we are seeking advice. In the special case where $E = HT$, we conclude that $C(A/E)$ should equal $P(A/HT)$, which is just what the New Principle says.

If the hypothesis *HT* which implies that Cassandra is the Oracle contains no information that is news to Cassandra—that is, if $P(HT) = 1$—then $C(A/HT) = P(A)$, which is just what the Old Principle says (when augmented by the assumption that *HT* is admissible). More generally, we get the Old Principle, augmented by the sufficient condition on admissibility (E is admissible if it is built up out of *HT*s), if we require that no conjunction *HT* be news to the panellist it singles out as the Oracle. But this requirement can't derive from the *epistemic* role of

[7] The idea that chance plays the role of an expert is not new. See, for example, Gaifman (1988) and van Fraassen (1989, pp. 197-201).

chance—it needn't be that each panellist must be *certain* that he or she is the Oracle. If there is reason to impose it, that reason must derive from some sort of (presumably anti-Humean) *metaphysics* of chance, not from its epistemology. Hence, to the extent that the (augmented) Old Principle was meant only to describe the distinctive epistemic role of chance, it overstepped its bounds by illicitly incorporating a bit of metaphysics. For the distinctive epistemic role of chance is just like the epistemic role of an expert, and *that* role is correctly captured by the New Principle, and not by the Old.

NP has some obvious virtues: since it dispenses entirely with the notion of admissibility in its formulation, no such notion need be analysed in order to clarify its content, whence it is more precise than OP. As we saw above, it also yields OP under the simplifying (but anti-Humean) assumption that HT always assigns chance 1 to T. Thus, to the extent that it is plausible and intuitive to think that laws never have any chance of failing to hold—and even the Humean should agree that, even if ultimately untenable, this claim *is* plausible and intuitive—OP itself is plausible and intuitive.

Less obviously, NP neatly avoids the problem of undermining futures. For suppose that HT assigns F some positive chance, even though FHT is a contradiction. Then $C(F/HT) = P(F/T) = P(F/HT) = 0$, exactly as desired. More generally, suppose that E is built up out of HTs, X is the proposition that the chance of A is x (where $x > 0$), and AEX is a contradiction. Then $C(A/EX) = 0$, *contra* OP together with the assumptions about what's admissible. To apply NP, first note that $C(A/EX)$ is the expectation of $C(A/H_{t_W}T_wEX)$. Since E is built up out of HTs, any HT compatible with it implies it; similarly, any HT compatible with X implies *it*. In that case, $C(A/HTEX) = C(A/HT)$. If A contradicts EX then A contradicts anything which implies EX, so A contradicts each HT compatible with EX. By NP, $C(A/HT) = P(A/T) = P(A/HT) = 0$ (since $P(H) = 1$). Since $C(A/EX)$ is the expectation of a quantity which is always zero, $C(A/EX) = 0$, as desired.

A still less obvious virtue of the New Principle is that it neatly handles at least some crystal ball cases. But before turning to this topic, I think it wise to dispel a serious worry: NP can't possibly be right, since it is manifestly too complicated and at any rate doesn't capture how we ordinarily use the concept of chance. It is the business of the next section to rebut these objections. Doing so will further clarify the relationship between the Old and New Principles.

5. *Utility*

The Old Principle was user-friendly, requiring for its everyday application only that we deal with simple quantities such as the chance that a coin will land heads. It also seems intuitively correct, at least when restricted to those everyday applications. But the revisionary New Principle seems user-hostile, since it requires us to evaluate such esoteric quantities as the *conditional* chance that the coin lands heads, given some complete law-proposition. Trade in OP for NP, and you're

forced to change your expectations about ordinary chance outcomes—without any guidance about how to do so!

These are fair and serious objections. Fortunately, they can be met.

The objections have force only on the assumption that there are undermining futures, since if there are not, then NP is trivially equivalent to OP. So assume, with the Humean, that there are. And assume—both for the sake of argument and because it seems right—that OP (plus the admissibility assumptions) describes our ordinary uses of the concept of chance reasonably accurately, even though it is not the correct credence-chance principle. Then here are the objections, rephrased: the switch from OP to NP had better not make more than a negligible difference in these ordinary cases; otherwise, believing in undermining futures will force us into a ridiculously revisionist stance toward everyday reasoning about chance, and leave us with the impossible burden of calculating the revision.

Response: If there are undermining futures, then, in general, $P(A) \neq P(A/T)$. But the difference between the quantities need not be great, at least when A describes some ordinary, localized chance outcome. For, according to the Humean story as told by Lewis,[8] a future which "undermines" some *HT* will typically describe some large and complicated pattern of chance outcomes, such that if the future went *that* way, T would not be the correct law-proposition. Perhaps such patterns are large and complicated enough that, according to the chances as fixed by *HT*, the proposition that such a pattern fails to obtain—equivalently, that T is true—is almost completely probabilistically independent of any proposition A describing an ordinary, localized chance outcome. Perhaps, that is, $P(A/T) \approx P(A)$ in all such cases. One assumption we can make, consistent with a commitment to undermining futures, is that all *HT*s have just this feature.

With this assumption, the New Principle yields the Old as an excellent approximation in all ordinary cases. Let A describe some ordinary outcome, and consider the credence $C(A/EX)$, where X is the proposition that the chance at t of A is x, and E is "admissible" in the old sense that it is built up out of *HT*s. Recall that $C(A/EX)$ is equal to the expectation of $C(A/H_{tw}T_wEX)$. Each $H_{tw}T_w$ compatible with EX implies EX, whence $C(A/H_{tw}T_wEX) = C(A/H_{tw}T_w) = P_{tw}(A/T_w)$, by NP. Therefore, $C(A/EX)$ is the expectation of $P_{tw}(A/T_w)$. By our assumption, $P_{tw}(A/T_w) \approx P_{tw}(A)$. Then, since $P_{tw}(A) = x$ whenever $H_{tw}T_w$ is compatible with X, $P_{tw}(A/T_w)\ x$, whence $C(A/EX) \approx x$, as desired.

In fact, our assumption is stronger than necessary. We can allow that there are "abnormal" *HT*s according to which $P(A)$ diverges widely from $P(A/T)$ even in some ordinary cases—and should, for reasons given in the next section. We need only require that they receive negligible credence compared to the rest of the *HT*s. Let w now range not over all worlds, but only over worlds such that $H_{tw}T_w$ is not "abnormal"; let w^* range over worlds such that $H_{tw^*}T_{w^*}$ is "abnormal". Then we can express $C(A/EX)$ as the sum of two terms: the expectation of $C(A/H_{tw}T_wEX)$ and the expectation of $C(A/H_{tw^*}T_{w^*}EX)$. If the "abnormal" *HT*s receive negligible credence, then the second term is close to zero, whence

[8] See, e.g., the Introduction to Lewis (1986), Lewis (1980 and 1994).

$C(A/EX)$ approximates the first term, which, by the argument given above, approximates x.

So—if we are content with good approximations—undermining futures do not make the New Principle intractable. The Humean need not grant that *HT* must always give T a chance of 1—or even a particularly *high* chance. In order to ensure the adequacy of OP, as applied to ordinary cases, he need only require that for any *HT* which deserves more than a negligible share of credence, *HT* makes T (approximately) probabilistically independent of localized outcomes, or reasonably restrained patterns of outcomes, of chance processes.

Now, it is the Humean's job, and not ours, to show how he can meet this reasonable requirement.[9] He cannot avoid doing so, as long as he remains committed to undermining futures. But it turns out—surprisingly—that in this respect the Humean's position is not so peculiar. For it turns out that if crystal balls are possible, then so are undermining futures.

6. *Crystal balls*

If crystal balls are possible at all, then they should be possible in the way described by the following scenario: at t_1, a genuinely chancy process takes place—say, the measurement of the spin of some electron. At t_0—a time *earlier* than t_1—a crystal ball reveals the outcome of this very chance process. And it does so in a perfectly reliable way: the laws, together with the facts of the ball's construction, together perhaps with other bits of auxiliary information (the ball is "tuned" to *this* chance process, and not some other, etc.), guarantee that the actual outcome of the chance process is revealed in the ball. My tasks in this section are, first, to show that the scenario involves undermining futures, and, second, to show how the New Principle applies to such a case.

Some will object that my scenario doesn't cover all the possibilities, since there are other ways crystal balls could work quite unlike the way I have described. Agreed. For example, the link between outcome and crystal ball might be chancy, in the sense that a particular outcome only makes it very likely (at a time just before t_0) that the ball will reveal that outcome—leaving some residual chance that it will show no outcome, or perhaps even the wrong outcome. More speculatively, there might be examples in which the backwards effect of the outcome on the ball makes it the case that the past is chancy. Finally, there might be cases of pseudo-crystal balls, in which the ball's revelation causes the chance of the revealed outcome to shoot up to one, so that the process taking place at t_1 is not genuinely chancy after all. But while these alternatives are certainly worth investigating, it is not my intention to cover all possible cases of what we might

[9] Some of that work has already been done: Lewis (1994) discusses a case simple enough to be amenable to a frequentist analysis of chance. It turns out that such an analysis meets the requirement quite easily.

call crystal balls. I prefer to sacrifice generality for ease of exposition—hoping that the lessons to be learned from my simple scenario will carry over to more complicated examples.

Still, there are others who will object that my scenario isn't a possibility at all.[10] Couldn't we arrange matters so as to guarantee that if the ball reveals *any* outcome, then the chance process never takes place? (E.g., let any image appearing in the ball trigger a bomb, which destroys the measuring apparatus.) But then we should have to say that the ball's revelation is brought about by an outcome which never happens—impossible. Myself, I think that arguments such as this show *not* that crystal ball scenarios are inconsistent, but rather that they are inconsistent with various other features which we might have thought it unproblematic to add. But that's a battle I don't wish to join, and I think I can avoid it in either of two ways. First, I can add to my scenario the stipulation that by the time the ball reveals the outcome, it is settled that the chance process will take place, and indeed settled what its chances are (i.e., there is no chance at t_0 that it will fail to take place, and no chance that its chances will be other than they actually are). Second, even if you remain sceptical about the possibility of the scenario, you are still free to treat the curious consequence I will derive from it as providing independent reason for thinking it impossible. This consequence will still have interest, even if it serves the purpose of such a *reductio*.

Let the actual outcome of the spin measurement be "up"; let the only other outcome with any chance be "down". Then in fact the crystal ball shows "up". Let T be the laws; let H be history up until time t_1; let P specify the chances as fixed by HT. For convenience, let the chance at t_1 of "up" be $1/2$. Then T, together with some auxiliary information (hereafter: "Aux") describing the construction of the ball, etc., implies that if the outcome is "up", then the ball shows "up", and if the outcome is "down", then the ball shows "down". In fact, we won't need this implication, but only a weaker consequence of it: T, Aux, and the fact that the ball shows "up" jointly contradict the claim that the outcome is "down". Now, the fact that the ball shows "up" is *historical*; so H implies it. Therefore, T, H, and Aux jointly contradict the claim that the outcome is "down".

Aux describes certain conditions which are needed, along with the laws, to secure the link between the chance outcome and the contents of the crystal ball. As an easy case, let us assume not only that Aux is true, but that its truth is settled by t_1, in the sense that H implies Aux. In that case, HT contradicts the claim that the outcome is "down"—even though $P(\text{"down"}) = 1/2$. Then "down" is an undermining future: a course of future history with some chance of coming about, but which contradicts HT. Next, even if the truth of Aux is not settled by t_1, it's still reasonable to require that the conjunction of Aux and "down" has, at t_1, some chance of coming about. In that case, this conjunction is our undermining future, since it, too, contradicts HT. Notice, moreover, that

[10] Here I am indebted to David Lewis for reminding me of these others, and for helpful suggestions about how to respond to them.

we've made no mention of Humeanism. The Humean has not, after all, cornered the market on undermining futures, since their existence is a by-product of crystal balls.[11]

Recall that crystal ball scenarios posed trouble for the Old Principle: if my evidence makes it likely that the past carries news from the future, then that evidence may not be admissible—even if it is purely historical. So it seems that the Old Principle cannot be used when I have evidence that there are crystal balls, and evidence of what they say. (Never mind, for now, whether crystal balls are possible, and hence whether anything could count as evidence for them.)

But the New Principle can be used. To see how, let X be the proposition that the chance at t_1 of "down" is $1/2$. Let E contain crystal ball information to the effect that the outcome "up" will be obtained. We can assume that E is purely historical. Thus, E might include a description of the enormous number of trial runs in which our favourite crystal hall has revealed information about the future of exceptional accuracy and precision, plus the ball's latest revelation that the outcome "up" is about to happen. Then we need to show that according to NP, $C(\text{"down"}/EX) \approx 0$.

For simplicity, assume that E is evidence for a crystal ball scenario of the type discussed above. Then $C(\text{-}/EX)$ must assign the lion's share of credence to certain sorts of hypotheses about history, law, and Aux: namely, to those conjunctions (HT&Aux) according to which the ball shows "up" only if the outcome is in fact "up". In that case, $C(\text{"down"}/EX)$ approximates a weighted sum of the quantities $C(\text{"down"}/EXHT\&\text{Aux})$, where we sum over the different conjunctions (HT&Aux) compatible with EX, and where the weights are the corresponding conditional credences $C(HT\&\text{Aux}/EX)$. Since E is historical, any such conjunction will in fact imply EX; hence $C(\text{"down"}/EX)$ is approximately equal to a weighted sum of the quantities $C(\text{"down"}/HT\&\text{Aux})$. Using NP, we will have, for any such conjunction (HT&Aux), $C(\text{"down"}/HT\&\text{Aux}) = P(\text{"down"}/HT\&\text{Aux})$, where P is the chance function picked out by HT.[12] But since (HT&Aux) contradicts "down", the right-hand side is zero (as it should be). Therefore, $C(\text{"down"}/EX)$ approximates a weighted sum of quantities, each of which is zero. So $C(\text{"down"}/EX) \approx 0$, as required.

It is instructive to simplify, and suppose that in each case Aux is included as part of H. Then $C(\text{-}/EX)$ must assign the lion's share of credence to those hypotheses about history and law according to which "down" (as opposed to the conjunction ("down" & Aux)) is an undermining future. Any such hypothesis HT will

[11] Of course, those who doubt the possibility of crystal ball scenarios will see things differently. If they are also happy to reject undermining futures, then, as noted above, they are invited to support their position on crystal balls by contraposing my argument: undermining futures are impossible, therefore so are crystal balls.

[12] Derivation: By the definition of conditional probability,

$$C(\text{"down"}/HT\,\&\text{Aux}) = \frac{C(\text{"down"}\&\text{Aux}/HT)}{C(\text{Aux}/HT)}$$

Using NP in both the numerator and denominator, we get

$$C(\text{"down"}/HT\,\&\text{Aux}) = \frac{P(\text{"down"}\&\text{Aux}\,/HT}{P(\text{Aux}/HT)}) = P(\text{"down"}/HT\,\&\text{Aux}).$$

imply that $P(\text{"down"}/T) = P(\text{"down"}/HT) = 0$, but that $P(\text{"down"}) = {}^1/_2$ (since HT must be compatible with X, the proposition that the chance of "down" is ${}^1/_2$). And this is so even though "down" describes a perfectly ordinary, localized chance outcome. In that case, the assumption used in the last section to derive the approximate equivalence of OP and NP (when applied to ordinary chance outcomes) fails if the credence is $C(\text{-}/EX)$. For that assumption said, in effect, that it is irrational to give more than negligible credence to any hypothesis according to which $P(A)$ diverges widely from $P(A/T)$, when A describes an ordinary chance outcome. It gets support from the idea that $P(A)$ can diverge widely from $P(A/T)$ only if either A or $\neg A$ describes an undermining future or something close to it, and that in order for a proposition to achieve *that* status it must describe some vast, complicated pattern of chance outcomes. But this idea is misguided, assuming crystal balls are possible: for it is exactly the lesson of crystal balls—a lesson which Humean and anti-Humean alike should heed—that situations can arise in which a single, highly localized, perfectly ordinary chance outcome is an undermining future. (The idea is misguided, but not horribly so, given that it is reasonable to require extraordinary evidence before believing in crystal balls!)

If E contains crystal ball information which (nearly) guarantees chance outcome A, E must make it (subjectively) likely that there are undermining futures. No wonder the Old Principle had trouble with crystal balls, even in the easy cases treated here—and no wonder the New Principle does not.[13]

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