The Consistency Argument for Ranking Functions

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Introduction

Ranking theory is a normative theory of epistemic states and their dynamics.

Normative theories need to be justified: why should I obey those norms rather than not?

Means-End Justification: Norms are justified in relation to an end whenever obeying the norms is a means to attain that end.

The consistency argument provides a justification for ranking theory relative to the end of consistency and deductive closure.

I will show: you attain this end if and only if (iff) you obey the norms of ranking theory.

This is means-ends epistemology.

The Plan…

… is to present ranking theory in analogy to probability theory, and

the consistency argument for ranking functions in analogy to the “Dutch Book” argument for probability measures.

If you are not familiar with probability theory, just focus on the simpler ranking theory.

Probability Measures

A function P from a field of propositions $\mathbf{A}_W$ into $\{0\}\cup(0,1]$ is a probability measure iff for all disjoint propositions A, B in $\mathbf{A}_W$:

1. $P(W) = 1$
2. $P(A \text{ or } B) = P(A) + P(B)$

$P(A|B) = P(A \,\&\, B) \div P(B) \quad \text{if} \quad P(B) > 0$

P(*A*) is the degree to which an epistemic agent believes *A*.

P(*A*) can be measured by the agent's betting ratio for *A*, i.e.

the highest price she is willing to pay for a bet that gives her $ 1 if *A*, and $ 0 otherwise.

Probabilities	Ranks
belief	disbelief
1	0
0	∞
>	<
{0}∪(0,1]	$\mathbb{N}$∪{∞}
+	min
×	+
÷	–

Ranking Functions

A function R from a field of propositions $\mathbf{A}_W$ into $N \cup \{\infty\}$ is a ranking function iff for all propositions A, B in $\mathbf{A}_W$:

1. $R(W) = 0$
2. $R(A \text{ or } B) = \min\{R(A), R(B)\}$

$R(A|B) = R(A \,\&\, B) - R(B) \quad \text{if} \quad R(B) < \infty$

R(*A*) is the degree to which an epistemic agent disbelieves *A*.

R(*A*) can be measured by the agent's degree of entrenchment for *A*, i.e.

the number of (mp-reliable) information sources saying *A* that it takes for the agent to give up her disbelief that *A*.

Belief Sets

Bel_R is the belief set based on R:

$$Bel_R = \{A \text{ in } \mathbf{A}_W: R(\text{not-}A) > 0\}$$

Bel_R is consistent and deductively closed,
i.e. one cannot derive a contradiction from it.

The Dutch Book Argument

1. Degrees of belief are identical to or measured by betting ratios.

2. It is defective to accept a bet which guarantees a sure loss (a Dutch Book).

3. Dutch Book Theorem: An agent's betting ratios obey the probability calculus iff she never accepts a Dutch Book.

Conclusion:

It is defective to have degrees of belief that violate the probability calculus.

In Other Words

The probability calculus as a set of betting norms is justified relative to the end of never accepting a Dutch Book:

you attain this end iff your betting ratios obey those norms.

The Consistency Argument

1. Degrees of disbelief are identical to or measured by degrees of entremchment.

2. It is defective to have beliefs that are not both consistent and deductively closed.

3. Consistency Theorem: An agent's degrees of entrenchment obey the ranking calculus iff her beliefs are never inconsistent or not deductively closed.

Conclusion:

It is defective to have degrees of disbelief that violate the ranking calculus.

In Other Words

The ranking calculus as a set of entrench ment norms is justified relative to the end of consistency and deductive closure:

you attain this end iff your degrees of entrenchment obey those norms.

Degrees of Entrenchment

An agent's degree of entrenchment for a proposition *A* is the number of "independent and minimally positively reliable" information sources saying *A* that it takes for the agent to give up her disbelief that *A*.

If the agent does not disbelieve *A* to begin with, her degree of entrenchment for *A* is 0.

The Consistency Theorem

An agent's entrenchment function R is a ranking function if (and only if*) every possible current or future belief set based on R is consistent and deductively closed.

* Principle of Categorical Matching...

Conditionalization I

If evidence comes in form of a certainty *E*, then:

Strict Conditionalization: $P_{new}(A) = P_{old}(A|E)$

Plain Conditionalization: $R_{new}(A) = R_{old}(A|E)$

Conditionalization II

If evidence changes the agent's degrees of (dis)belief of E_i *to* p_i or to n_i, then:

Jeffrey Cond.: $\mathrm{Pr}_{Ei \rightarrow pi}(A) = \sum \mathrm{Pr}(A|E_i) \times p_i$

Spohn Cond.: $\mathrm{R}_{Ei \rightarrow ni}(A) = \min \{\mathrm{R}(A|E_i) + n_i\}$

Conditionalization III

If evidence changes the agent's degrees of (dis)belief of E_i *by* α_i or *by* n_i, then:

Field C.: $\Pr_{Ei\uparrow\alpha i}(A) = \sum \Pr(A\&E_i) \times e^{\alpha i} \div s$

Shenoy C.: $R_{Ei\uparrow ni}(A) = \min \{R(A\&E_i) + n_i - m\}$

Plain Conditionalization

Suppose evidence comes in form of a certainty *E*, and an agent updates her ranking function R to R*. Then

$R^*(\cdot) = R(\cdot|E)$

if and only if

Every current or future belief set is consistent and deductively closed.

Spohn Conditionalization

Suppose evidence changes the agent's ranks to n_i with min $\{n_i\} = 0$, and she updates her ranking function R to R*. Then

$R^* = R_{Ei \rightarrow ni}$

if and only if

Every current or future belief set is consistent and deductively closed.

Shenoy Conditionalization

Suppose evidence changes the agent's ranks by n_i with $\min\{n_i\} = 0$, and she updates her ranking functions R to R*. Then

$R^* = R_{Ei \uparrow ni}$

if and only if

Every current or future belief set is consistent and deductively closed.

Conclusion

Given a link between degrees of disbelief and entrenchment, the normative force of the consistency argument is proportional to how odd one views the possibility of having beliefs that are inconsistent or not deductively closed.

Thank you!