

When does aggregation reduce risk aversion?*

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Abstract

We study the problem of risk sharing within a household facing subjective uncertainty. A household shares uncertain prospects using a social welfare function. We characterize the social welfare functions such that the household is collectively less risk averse than each member, and satisfies the Pareto principle and an independence axiom. We single out the sum of certainty equivalents as the unique member of this family which provides quasiconcave rankings over riskless allocations.

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1 Introduction

This paper is devoted to the sum of certainty equivalents as a method for aggregating individual welfare. The sum of certainty equivalent is a common and intuitive method of aggregating individual welfare judgments into a collective welfare in an environment of idiosyncratic risk. Each agent in a household¹ faces individual risk over monetary consumption. The sum of individual certainty equivalents for a given allocation of risky prospects is the sure amount that the household would need to be paid in order to give up the allocation (without first reallocating).

We show that the sum of individual certainty equivalents possesses a property singling it out from other methods of aggregating welfare. That is, with this rule, a household of risk averse individuals will behave in the aggregate in a fashion that is *less risk averse* than each member of the household. The notion that a group is, or should be, less risk averse than its members is a very familiar one in economics. Early arguments for the notion are in Samuelson (1964), Vickrey (1964) and Arrow and Lind (1970).

The environment is simple. Each member of a household of individuals faces some aggregate subjective uncertainty in the consumption of a single good, say money. Members of the household have differing attitudes toward uncertainty, but all believe uncertainty is probabilistic with a common prior; hence we speak of risk. The household seeks to allocate the risk to its members to maximize some notion of social welfare.

Our main result can be roughly stated as follows: Suppose that the household ranks allocations using a social welfare function $\succeq^0(\mathbf{R})$, which depends on individual preferences $\mathbf{R} = (R_i)_{i \in N}$. The *sum of individual certainty equivalents* represents the only social welfare function (SWF) that

¹We use “household” throughout to refer generically to a group of agents engaged in a risk sharing arrangement. Wilson (1968) uses the term “syndicate.”

1. generates households less risk averse than its members (for all individual preference profiles);
2. ignores risk preferences whenever it compares riskless allocations;
3. is quasiconcave over riskless allocations.

This result is a simple consequence of a theorem stating that (1) and (2) are equivalent to ranking allocations with a function which takes as input certainty equivalents, and when maximized over a simplex, has a solution at every corner. In particular, in any such social welfare function, a riskless amount should always all be given to one member of the household—but which member does not matter. This theorem, in turn, uses some classical results on aggregation from demand theory (concretely Samuelson (1956) and Chipman and Moore (1979)).

The sum of certainty equivalents has a simple economic interpretation: it is the certain monetary sum that the household would demand for an allocation of risky prospects. In other words, it is the collective willingness to accept for an allocation. Myerson (2004), for example, recommends MBAs and applied decision makers use the sum of certainty equivalents to share risk. The recommendation is based on the idea of maximizing the collective willingness to accept for an allocation. Our paper provides a completely different justification for this recommendation.

Other rules are commonly used in welfare analysis; arguably on the grounds that they are more tractable. We show that, in fact, the sum of individual certainty equivalents yields very tractable results. In the specialized setting of convex homothetic preference profiles (such as CRRA preferences), it yields the most risk averse *convex* household preference, among those which are less risk averse than members' preferences. This result lends itself to a simple representation using basic convex duality.

We proceed to discuss our results in more detail.

We suppose a finite set of states of the world. Agents' preferences are over state-contingent monetary payoffs, which we call acts. We impose little structure on the preferences of agents other than monotonicity and a common prior assumption; in particular, agents need not be subjective expected utility maximizers.

A SWF recommends a preference over allocations for any given list of individual preferences. We impose the Pareto principle: the SWF must be increasing in the welfare of individuals.

We present two new axioms. The first axiom is an independence axiom: the ranking of any two riskless allocations should be independent of the individuals' risk preferences. An allocation is riskless when each agent is allocated a constant act—an act whose payoff does not depend on the state. Note that acts involve monetary payoffs; hence comparing two riskless allocations involves a tradeoff in which agents receive more or less money. The axiom states that the tradeoff should be resolved without regard for the agents' risk preferences.

Our final and main axiom is reduction of risk aversion. It says that the household is less risk averse than its members. The axiom requires defining a comparative notion of risk aversion. Following Yaari (1969), we say that a preference R_1 is less risk averse than R_2 if, for every constant act c and every act x , when $x R_2 c$ then $x R_1 c$. The idea is that if the risk involved in choosing the risky act x over the certain act c is acceptable for R_2 , then it must also be acceptable for R_1 .

The social welfare function guides the household's decisions on how to share risky prospects. In a classic paper discussing the representative consumer problem in demand theory, Samuelson (1956) (see also Graaff (1957), p. 49) shows that a household which allocates aggregate bundles optimally according to some SWF behaves as if it is an individual (that is, it has a complete and transitive preference). Our social welfare function generates such

a preference for each list of individual preferences—this is what we call the *household preference*. Our main axiom requires that this household preference be less risk averse than the preferences of each member of the household.

The assumption that a government or a firm behaves in a risk neutral fashion is often justified on the grounds that large groups of agents will tend to behave in a risk neutral fashion. It is understood that the second order effects of risk can be mitigated by properly sharing risk. However, it is equally clear that not *all* methods of risk sharing will have this effect; for a simple counterexample consider a dictatorship. Our requirement of reduction of risk aversion is a simple fixed-population notion capturing the intuition that societies should tend toward risk neutrality.

We characterize the family of SWF's satisfying these properties. The sum of individual certainty equivalents represents the unique one which is *quasiconcave over riskless allocations*. That is, for each individual's state-contingent consumption, the rule finds the certain amount that the individual would need to be given in compensation, then adds these across individuals.

More generally, we refer to the only rules which satisfy our axioms as *anyone can take all* (ACTA) rules. These rules are characterized by a kind of social utility function. The social utility has the property that for any allocation, social utility is a function only of the certainty equivalents of that allocation. Further, the rule is called an ACTA rule as for any riskless amount, it is deemed socially optimal to give the entire amount to any one of the individuals in the household.

Section 2 provides the model; Section 3 has the main results; Section 4 presents results for homothetic preferences, and examples of familiar special cases. Section 5 provides discussion and related literature.

2 The model

Let Ω be a finite set of **states** of the world. **Acts** are state-contingent elements of \mathbb{R}_+ ; that is, the set of acts is $X = \mathbb{R}_+^\Omega$. Let $N = \{1, \dots, n\}$ be a finite set of agents. An **allocation** is an element of X^N . An allocation of $x \in X$ is a vector $\mathbf{x} = (x_1, \dots, x_n) \in X^N$ for which $\sum_{i \in N} x_i = x$.

A **preference relation** R is a complete, transitive, continuous, and monotonic² binary relation on X . The set of preferences is denoted \mathcal{R} . A **preference profile** is a vector $\mathbf{R} = (R_1, \dots, R_n) \in \mathcal{R}^N$. N -vectors are written in boldface throughout.

Our aim in this study is to understand methods of aggregating preferences which reduce risk aversion. We imagine a set of agents who reside in a household and use some social welfare function to optimally distribute resources. Samuelson (1956) observed that such optimization leads to “rational” behavior in the aggregate. We ask when such household behavior is less risk averse than the behavior of each individual in the household.

To this end, we discuss a comparative notion of risk aversion and a domain of preferences on which this exercise becomes meaningful. For $c \in \mathbb{R}_+$, we abuse notation and identify c with the constant act whose outcome in every state is c . Let R' and R be two preference relations. As in Yaari (1969), we say that R is **more risk averse than** R' if for all $c \geq 0$, $\{x : x R c\} \subseteq \{x : x R' c\}$. Every uncertain prospect which is preferred to c by R is also preferred to c by R' .³

A **prior** for $R \in \mathcal{R}$ is a probability measure $\pi \in \Delta(\Omega)$ for which for all

²That is, if $x(\omega) > y(\omega)$ for all $\omega \in \Omega$, then x is strictly better than y for R ($x R y$ and not $y R x$).

³Epstein (1999) and Ghirardato and Marinacci (2002) provide modern adaptations of this comparative notion to general settings of Knightian uncertainty. Their definitions differ as to the benchmark of “uncertainty neutral” acts, but coincide with Yaari’s when uncertainty neutral acts are taken to be the constant acts.

$x \in X$, $\pi \cdot x R x$. In other words, a prior is a probability measure with respect to which R is risk averse.⁴ Denote the set of priors for R by $\Pi(R)$. We say that $\mathbf{R} \in \mathcal{R}^N$ is a **common prior profile** if $\bigcap_{i \in N} \Pi(R_i) \neq \emptyset$. We denote the set of common prior profiles by \mathcal{CP} .⁵ Subjective expected utility profiles with a common prior are in \mathcal{CP} , as are many other examples.

Geometrically, a common prior profile is one in which all individuals have a common marginal rate of substitution everywhere along the ray of equal coordinates. Consequently, common prior profiles are exactly those profiles for which all riskless non-wasteful allocations of a riskless amount are efficient.

A **domain** \mathcal{D} is a nonempty subset of \mathcal{R}^N . A social welfare function is a mapping which carries \mathcal{D} into binary relations over X^N , the space of allocations. Formally, we denote the set of binary relations over X^N by \mathcal{R}_N . Then a **social welfare function** is a function $\succeq^0: \mathcal{D} \rightarrow \mathcal{R}_N$. We write $\succeq^0(\mathbf{R})$ for the binary relation over allocations obtained when individual preferences are $\mathbf{R} = (R_i)_{i \in N}$.

Example 1: A classical domain of preferences is the domain of risk averse expected utility profiles with a common prior. We denote this domain by \mathcal{EU} . Formally, $\mathbf{R} \in \mathcal{EU}$ if there exists $\pi \in \Delta(\Omega)$ and for all $i \in N$, there exists $u_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ which is strictly increasing, concave, and continuous for which for all $i \in N$ and all $x, y \in X$, $x R_i y$ if and only if

$$\sum_{\omega \in \Omega} \pi(\omega) u_i(x(\omega)) \geq \sum_{\omega \in \Omega} \pi(\omega) u_i(y(\omega)).$$

A standard example of a social welfare function $\succeq^0: \mathcal{EU} \rightarrow \mathcal{R}_N$ used in the theory of risk sharing (for example, Borch (1962) and Wilson (1968)) is the **utilitarian rule**. For any $\mathbf{R} \in \mathcal{EU}$, there exists for all $i \in N$ a unique $u_{R_i}: \mathbb{R}_+ \rightarrow \mathbb{R}$ which represents R_i and which is normalized so that

⁴The measure π can be either objective or subjective.

⁵Obviously, this set is not a product set.

$u_{R_i}(0) = 0$ and $u_{R_i}(1) = 1$. We then require $\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y}$ if and only if

$$\sum_{i \in N} \sum_{\omega \in \Omega} \pi(\omega) u_{R_i}(x_i(\omega)) \geq \sum_{i \in N} \sum_{\omega \in \Omega} \pi(\omega) u_{R_i}(y_i(\omega)).$$

Note that, while we have restricted elements of \mathcal{R} significantly, elements of \mathcal{R}_N obey no restrictions whatsoever (we will later make assumptions on these elements in the form of axioms). This is because elements of \mathcal{R} are understood to be descriptive, whereas elements of \mathcal{R}_N are normative recommendations for a society.

3 Results

We proceed to describe the four axioms. The axioms will be equivalent to a certain class of SWF's. Coupled with quasiconcavity over riskless allocations, we characterize the SWF which is represented as the sum of certainty equivalents.

Our first axiom states that household preferences over allocations should be minimally "rational."

Rationality: *For all $\mathbf{R} \in \mathcal{D}$, $\succeq^0(\mathbf{R})$ is continuous, complete, and transitive.*

Our second axiom is natural and requires that the SWF comply with the (weak) Pareto property.

Pareto: *For all $\mathbf{R} \in \mathcal{D}$ and all $\mathbf{x}, \mathbf{y} \in X^N$, if $x_i R_i y_i$ for all $i \in N$, then $\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y}$ (with strict preference if all individual preferences are strict).*

Our next axiom is the first that deals specifically with the interpretation of uncertainty. It requires that in ranking profiles of certain (risk-free)

prospects, the social welfare function should ignore risk attitudes. A **constant act** $c \in X$ is an act which takes only one value. A constant allocation is an allocation of constant acts. Acts are monetary payoffs, and individual preferences are monotonic, so all individual preferences coincide over constant acts: more is better. The independence axiom says that, when comparing constant acts, \succeq^0 should not depend on individual risk preferences. These do not differ in the comparison of constant acts anyhow.

Independence of risk attitudes for constant acts: For all $\mathbf{R}, \mathbf{R}' \in \mathcal{D}$ and all constant $\mathbf{c}, \mathbf{d} \in X^N$, $\mathbf{c} \succeq^0(\mathbf{R}) \mathbf{d} \iff \mathbf{c} \succeq^0(\mathbf{R}') \mathbf{d}$.

We often refer to the axiom simply as *independence*. Mathematically, together with the Pareto property, the axiom allows us to work with a ranking over vectors of certainty equivalents (defined below). This ranking over vectors of certainty equivalents is independent of the preference profile in question.

To understand our last axiom, we need to discuss the notion of risk sharing. It is the standard notion of risk sharing, in an environment of possibly non-expected-utility maximizers (see e.g. Epstein (2001) and Rigotti and Shannon (2005)).

The individuals in N are all members of a household. Household members entertain different attitudes toward risk. We imagine that the household uses a SWF to allocate an aggregate bundle x among its members. That is, given individual preferences \mathbf{R} , they maximize $\succeq^0(\mathbf{R})$ across $\{\mathbf{x} \in X^N : \sum_{i \in N} x_i \leq x\}$. Under our continuity assumptions, this risk-sharing maximization problem is well-defined. Risk sharing generates a well-defined household preference over acts: a “household preference.” This aggregation results from a well-known aggregation result in classical demand theory (see Samuelson (1956) and Chipman and Moore (1979)).

Household preferences over acts are given by $x \succeq^h(\mathbf{R}) y$ if and only if for all

$\mathbf{y} \in X^N$ such that $\sum_{i \in N} y_i \leq y$, there exists $\mathbf{x} \in X^N$ such that $\sum_{i \in N} x_i \leq x$ and $\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y}$. This binary relation is the **household preference**. In particular, $x \succeq^h(\mathbf{R}) y$ whenever the maximal allocation of x according to $\succeq^0(\mathbf{R})$ is better than *every* allocation of y .

We are now ready to state our next axiom.

Reduction of risk aversion: For all $\mathbf{R} \in \mathcal{D}$, $\succeq^h(\mathbf{R})$ is less risk averse than R_i for all $i \in N$.

One reason for the common prior assumption is now apparent. When priors may differ, there may not in general *exist* a risk-averse, monotonic preference which is less risk-averse than each individual preference.

Example 2: Suppose $\mathcal{D} = \mathcal{EU}$ and consider the utilitarian rule defined in Example 1. In general, for any $\mathbf{R} \in \mathcal{EU}$, the household preference $\succeq^h(\mathbf{R})$ is expected utility with von Neumann Morgenstern utility index given by

$$u(x) = \sup_{\sum_{i \in N} x_i = x} \sum_{i \in N} u_{R_i}(x_i).$$

The function u is referred to as the “sup-convolution” of the functions u_{R_i} . It is easily verified that the resulting \succeq^h need not have any relation to the individual preferences in terms of attitudes toward risk.

Figure 1 presents the problem geometrically. One preference R_1 is less risk averse than R_2 if, when we compare their upper contour sets through any riskless act, the upper contour set of R_1 contains that of R_2 . Figure 1(a) illustrates how the preferences having the $\bar{U}_1 - \bar{U}_1$ indifference curve are less risk averse than those having the $\bar{U}_2 - \bar{U}_2$ curve. Note that there is a common prior, indicated by the tangent line. In Figure 1(a), any household preference satisfying the reduction of risk aversion axiom will need to have indifference curves “below” $\bar{U}_1 - \bar{U}_1$.

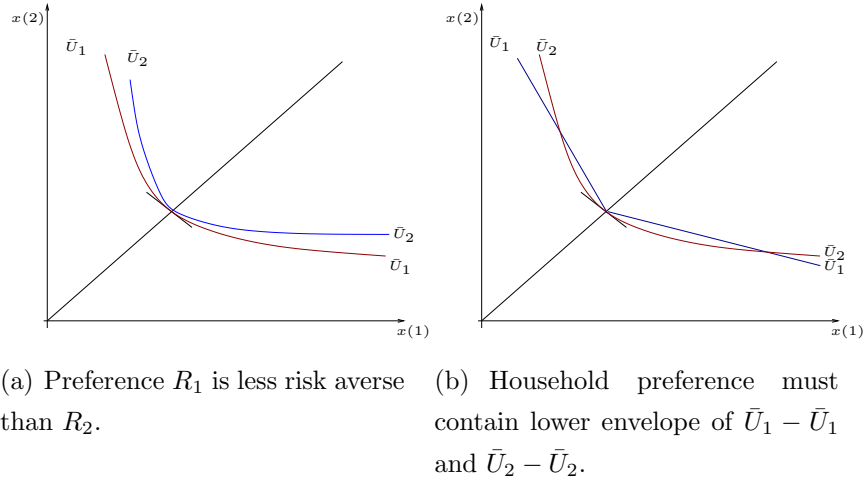


Figure 1: Comparisons in risk aversion.

Figure 1(b) presents a case where the preferences do not have comparable risk aversion. In the figure, the most risk averse household preference which is less risk averse than each individual agent is given by the lower envelope of the two indifference curves (i.e. by the curve which goes from \bar{U}_1 to the intersection of the two indifference curves, then coincides with $\bar{U}_2 - \bar{U}_2$ until the second intersection, then coincides with $\bar{U}_1 - \bar{U}_1$). Note that this household preference is not convex, and any household preference satisfying reduction of risk aversion must have indifference curves below this lower envelope of $\bar{U}_1 - \bar{U}_1$ and $\bar{U}_2 - \bar{U}_2$.

For a preference $R \in \mathcal{R}$, the **certainty equivalent** $ce_R : X \rightarrow \mathbb{R}$ is defined by

$$ce_R(x) = \inf \{c : c R x\}.$$

It is the value of the unique constant act which is indifferent to x ; that is, by monotonicity and continuity, $ce_R(x) I x$.⁶ Critically for us, for a given R , $ce_R : X \rightarrow \mathbb{R}$ is a continuous utility representation of R .

⁶We write $x I y$ for $x R y$ and $y R x$.

For an allocation $\mathbf{x} \in X^N$ and preference profile $\mathbf{R} \in \mathcal{R}^N$, $\mathbf{ce}_{\mathbf{R}}(\mathbf{x}) = (ce_{R_1}(x_1), \dots, ce_{R_n}(x_n))$.

Remark: For two preferences $R, R' \in \mathcal{R}$, R is more risk averse than R' if and only if for all $x \in X$, $ce_R(x) \leq ce_{R'}(x)$.

Proposition 3: A social welfare function on \mathcal{D} satisfies rationality, Pareto, and independence if and only if there exists a strictly monotonic and continuous function $W : \mathbb{R}_+^N \rightarrow \mathbb{R}$ for which for all $\mathbf{R} \in \mathcal{D}$ and all $\mathbf{x}, \mathbf{y} \in X^N$,

$$\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y} \iff W(\mathbf{ce}_{\mathbf{R}}(\mathbf{x})) \geq W(\mathbf{ce}_{\mathbf{R}}(\mathbf{y})).^7$$

PROOF: Let $\mathbf{R}' \in \mathcal{D}$. Define $W : \mathbb{R}^N \rightarrow \mathbb{R}$ so that $W(\mathbf{d}) \geq W(\mathbf{c})$ if and only if

$$\mathbf{d} \succeq^0(\mathbf{R}') \mathbf{c}.$$

Such a W exists as $\succeq^0(\mathbf{R}')$ is continuous (Debreu, 1964). W is strictly monotonic by the Pareto property. Now, let $\mathbf{R} \in \mathcal{D}$ be arbitrary. Let $\mathbf{x}, \mathbf{y} \in X^N$. Then by Pareto and rationality, $\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y}$ if and only if $\mathbf{ce}_{\mathbf{R}}(\mathbf{x}) \succeq^0(\mathbf{R}) \mathbf{ce}_{\mathbf{R}}(\mathbf{y})$. By independence $\mathbf{ce}_{\mathbf{R}}(\mathbf{x}) \succeq^0(\mathbf{R}) \mathbf{ce}_{\mathbf{R}}(\mathbf{y})$ if and only if $\mathbf{ce}_{\mathbf{R}}(\mathbf{x}) \succeq^0(\mathbf{R}') \mathbf{ce}_{\mathbf{R}}(\mathbf{y})$. By definition of W , $\mathbf{ce}_{\mathbf{R}}(\mathbf{x}) \succeq^0(\mathbf{R}') \mathbf{ce}_{\mathbf{R}}(\mathbf{y})$ if and only if $W(\mathbf{ce}_{\mathbf{R}}(\mathbf{x})) \geq W(\mathbf{ce}_{\mathbf{R}}(\mathbf{y}))$. \square

Our aim from this point on is to characterize those functions W which reduce risk aversion.

⁷We say a function $W : \mathbb{R}_+^N \rightarrow \mathbb{R}$ is strictly monotonic if $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^N$ and $\mathbf{x} \geq \mathbf{y}$ implies $W(\mathbf{x}) \geq W(\mathbf{y})$, and $\mathbf{x} \gg \mathbf{y}$ ($x_i > y_i$ for all $i \in N$) implies $W(\mathbf{x}) > W(\mathbf{y})$.

Example 4: One of the simplest examples of a W which reduces risk aversion is given by

$$W(d_1, \dots, d_n) = \max_{i \in N} \{d_i\}.$$

This function illustrates some of the properties of the reduction of risk aversion. The household rule generated by this function reduces risk aversion on the domain \mathcal{CP} , but the function W constitutes a very unfair rule. Moreover, the induced household preferences are typically not convex, even when the individual preferences are convex. If we denote by $U_i(c)$ the upper contour set of R_i at c , then it is easily verified that $U^*(c) = \bigcup_{i \in N} U_i(c)$ (this also verifies that this household preference is the most risk averse preference which is less risk averse than each individual preference in the household).

The following proposition illustrates that for any preference profile in \mathcal{CP} , any prior which is common to all agents is also a household prior. It holds because under profiles in \mathcal{CP} , constant allocations are always efficient, and constant allocations are always supported by the common prior. The intuition for the result is similar to results appearing in Billot, Chateauneuf, Gilboa, and Tallon (2000), Dana (2002) and Rigotti, Shannon, and Strzalecki (2008).

Proposition 5: *Suppose that the conditions in Proposition 3 are satisfied. Let $\mathbf{R} \in \mathcal{CP}$. Then*

$$\bigcap_{i \in N} \Pi(R_i) \subseteq \Pi(\succeq^h(\mathbf{R})).$$

PROOF: Let $\pi \in \bigcap_{i \in N} \Pi(R_i)$. We wish to show that $\pi \in \Pi(\succeq^h(\mathbf{R}))$. So, let $x \in X$ and let $\mathbf{x} \in X^N$ such that $\sum_{i \in N} x_i = x$. Then, for all $i \in N$, as $\pi \in \Pi(R_i)$, $\pi \cdot x_i R_i x_i$. Clearly, $\sum_{i \in N} \pi \cdot x_i = \pi \cdot \sum_{i \in N} x_i = \pi \cdot x$. By

Pareto, $(\pi \cdot x_1, \dots, \pi \cdot x_n) \succeq^0 (\mathbf{R}) \mathbf{x}$. Consequently, by definition of $\succeq^h (\mathbf{R})$, $\pi \cdot x \succeq^h (\mathbf{R}) x$. Therefore, $\pi \in \Pi (\succeq^h (\mathbf{R}))$. \square

The following result is our main characterization theorem. It tells us that under our axioms, a social welfare function reduces risk aversion if and only if it is associated with a function W which is maximized on any c -simplex at the vertices. This property of W means that in allocating a constant act, all allocations which give one agent all consumption are socially optimal.

For $c \in \mathbb{R}_+$, the c -simplex $\Delta_c = \{\mathbf{d} \in \mathbb{R}_+^N : \sum_{i \in N} d_i = c\}$ is the set of nonnegative vectors summing to c .

Formally, define an **anyone can take all (ACTA) rule** as a SWF for which there exists some strictly monotonic, continuous $W : \mathbb{R}_+^N \rightarrow \mathbb{R}$ for which for all $c \in \mathbb{R}_+$, for all $i \in N$,

$$c1_i \in \arg \max_{\mathbf{d} \in \Delta_c} W(\mathbf{d})$$

such that for all $\mathbf{x}, \mathbf{y} \in X^N$ and all $\mathbf{R} \in \mathcal{R}^N$, $\mathbf{x} \succeq^0 (\mathbf{R}) \mathbf{y}$ if and only if $W(\mathbf{c}_{\mathbf{R}}(\mathbf{x})) \geq W(\mathbf{c}_{\mathbf{R}}(\mathbf{y}))$. The terminology ACTA refers to the fact that, for any riskless amount c , an ACTA rule finds it optimal to distribute *all* of c to some individual in the household—but the rule is completely indifferent as to *which* individual is to receive c .

Examples of W functions generating ACTA rules are the max function and the sum (utilitarian) function as discussed in Examples 1 and 4. Other examples include the functions $W(\mathbf{u}) = (\sum_{i \in N} u_i^p)^{1/p}$ for $p \geq 1$.

Theorem 6: *Suppose that $\mathcal{EU} \subseteq \mathcal{D} \subseteq \mathcal{CP}$. A social welfare function satisfies rationality, Pareto, independence, and reduction of risk aversion on \mathcal{D} if and only if it is an ACTA rule.*

Figure 2 shows the level curves of a W corresponding to an ACTA rule. It should be clear that requiring W to be quasiconcave pins down the sum

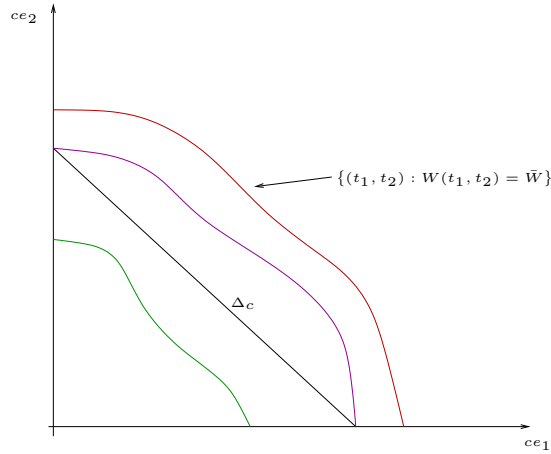


Figure 2: Level curves of W in the hypotheses of Theorem 6.

of certainty equivalents. The following axiom contains the quasiconcavity restriction. It has the interpretation that in an “divide the dollar” environment, without risk, social preferences should be “fair.”

Quasiconcavity: Let \mathbf{c} and \mathbf{d} be constant allocations. Let $\mathbf{R} \in \mathcal{R}^N$. Suppose $\mathbf{c} \succeq^0(\mathbf{R}) \mathbf{d}$. Then for all $\alpha \in [0, 1]$, $\alpha \mathbf{c} + (1 - \alpha) \mathbf{d} \succeq^0(\mathbf{R}) \mathbf{d}$.

Corollary 7: Suppose that $\mathcal{EU} \subseteq \mathcal{D} \subseteq \mathcal{CP}$. A social welfare function \succeq^0 satisfies rationality, Pareto, independence, reduction of risk aversion, and quasiconcavity if and only if for all $\mathbf{R} \in \mathcal{D}$ and all $\mathbf{x}, \mathbf{y} \in X^N$,

$$\mathbf{x} \succeq^0(\mathbf{R}) \mathbf{y} \iff \sum_{i \in N} ce_{R_i}(x_i) \geq \sum_{i \in N} ce_{R_i}(y_i).$$

Corollary 7 tells us that the only “fair” SWF to reduce risk aversion is the one which ranks allocations according to the sum of its certainty equivalents. When allocating a constant act by the sum of certainty equivalents, all constant allocations are socially optimal.

We end the section with a proof of Theorem 6.

PROOF (PROOF OF THEOREM 6): First, suppose there exists a W as in the statement of the theorem. We shall prove that it satisfies the reduction of risk aversion axiom. That it satisfies the remaining axioms is immediate. Let $\mathbf{R} \in \mathcal{CP}$, and let $\pi \in \bigcap_{i \in N} \Pi(R_i)$. Let c be a constant act. We wish to show that for all $i \in N$, $\{x : x R_i c\} \subseteq \{x : x \succeq^h(\mathbf{R}) c\}$.

As a first step, we show that for all $i \in N$, $(c, 0_{-i}) \in \arg \max_{\mathbf{x} \in A_c} W(\mathbf{ce}_{\mathbf{R}}(\mathbf{x}))$, where A_c is the set of allocations of the constant act c . That is, $A_c = \{\mathbf{y} \in X^N : \sum_{i \in N} y_i = c\}$.

Let $\mathbf{y} \in A_c$. For all $i \in N$, $\pi \cdot y_i R_i y_i$, and $(\pi \cdot y_1, \dots, \pi \cdot y_n) \in \Delta_c$. So, for any $\mathbf{y} \in A_c$ there is $\mathbf{d} \in \Delta_c$ such that $W(\mathbf{d}) \geq W(\mathbf{ce}_{\mathbf{R}}(\mathbf{x}))$. Continuity of W and compactness of Δ_c implies there exists an optimal allocation for W in Δ_c . There therefore exists a constant allocation $\mathbf{d}^* \in \arg \max_{\mathbf{x} \in A_c} W(x)$; $\mathbf{d}^* \in \Delta_c$. By the hypothesis on W , $W(c, 0_{-i}) \geq W(\mathbf{d}^*)$; we therefore establish that $(c, 0_{-i}) \in \arg \max_{A_c} W$.

Now, let $x \in X$ and suppose that $x R_i c$. So $ce_{R_i}(x) \geq c$. Then for all $\mathbf{y} \in X^N$ for which $\sum_{i \in N} y_i = c$,

$$W(ce_{R_i}(x), 0_{-i}) \geq W(c, 0_{-i}) \geq W(y_1, \dots, y_n).$$

Therefore, for every allocation \mathbf{y} of c , $(x, 0_{-i}) \succeq^0(\mathbf{R}) \mathbf{y}$. By definition of $\succeq^h(\mathbf{R})$, $x \succeq^h(\mathbf{R}) c$.

Conversely, suppose that \succeq^0 satisfies the axioms. W exists from Proposition 3; we will show that the vertices of every simplex maximize W on the simplex.

Define $\Delta_{++}(\Omega) = \{q \in \mathbb{R}_{++}^\Omega : \sum_{\omega} q(\omega) = 1\}$ (the set of full support probability measures). Let $q \in \Delta_{++}(\Omega)$. We shall consider a profile $\mathbf{R} \in \mathcal{EU}$ with common prior q . Fix an arbitrary $j \in N$. Let R_j be defined by

$$ce_{R_j}(x) = q \cdot x$$

and for all $i \neq j$, choose some strictly increasing, concave, and differentiable $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ for which $\lim_{x \rightarrow 0^+} u'_i(x) = +\infty$ and $\lim_{x \rightarrow +\infty} u'_i(x) = 0$ (the Inada conditions) and define R_i by

$$x R_i y \iff \sum_{\omega} q(\omega) u_i(x(\omega)) \geq \sum_{\omega} q(\omega) u_i(y(\omega)).$$

Note that $\mathbf{R} \in \mathcal{EU}$ and that q is a common prior for \mathbf{R} . Importantly for what follows, each R_i is a convex preference relation.

By Proposition 5, q is a prior for $\succeq^h(\mathbf{R})$. We shall prove that, by reduction of risk aversion, $\succeq^h(\mathbf{R}) = R_j$. To see this, let $c \in X$ be a constant act. Then as q is a prior for $\succeq^h(\mathbf{R})$, if $x \succeq^h(\mathbf{R}) c$, then $q \cdot x \geq c$; consequently $x R_j c$ by the definition of R_j . Hence, $\{x : x \succeq^h(\mathbf{R}) c\} \subseteq \{x : x R_j c\}$. By reduction of risk aversion, we obtain that $\{x : x R_j c\} = \{x : x \succeq^h(\mathbf{R}) c\}$. This implies that $ce_{R_j} = ce_{\succeq^h(\mathbf{R})}$; thus $R_j = \succeq^h(\mathbf{R})$.

For each $i \in N$, define the **indirect utility function** $v_{R_i} : \Delta_{++}(\Omega) \times \mathbb{R}_+$ by

$$v_{R_i}(\pi, m) = \max_{\pi \cdot x \leq m} ce_{R_i}(x).$$

Define $U_{\succeq^h(\mathbf{R})} : X \rightarrow \mathbb{R}$ by

$$U_{\succeq^h(\mathbf{R})}(x) = \sup_{\sum x_i = x} W(\mathbf{ce}_{\mathbf{R}}(\mathbf{x})).$$

Similarly, define the household indirect utility function by

$$V_{\succeq^h(\mathbf{R})}(\pi, m) = \max_{\pi \cdot x \leq m} U_{\succeq^h(\mathbf{R})}(x).$$

By Chipman and Moore (1979), Theorem 3.9,

$$V_{\succeq^h(\mathbf{R})}(\pi, m) = \max_{\mathbf{d} \in \Delta(N)} W((v_{R_i}(\pi, d_i m))_{i \in N}).$$

By the Maximum Theorem, the correspondence $\delta : \Delta(\Omega) \times \mathbb{R}_+$ defined by

$$\delta(\pi, m) = \arg \max_{\mathbf{d} \in \Delta(N)} W((v_{R_i}(\pi, d_i m))_{i \in N})$$

is well-defined and upper semi-continuous. Define the demand correspondence $x_R(\pi, m)$ as those allocations which are R -maximal in the set $\{x : \pi \cdot x \leq m\}$.

By Chipman and Moore (1979), Corollary 3.5,

$$x_{\succeq^h(\mathbf{R})}(\pi, m) = \bigcup_{\mathbf{d} \in \delta(\pi, m)} \sum_{i \in N} x_{R_i}(\pi, d_i m).$$

Now, let $\pi \in \Delta_{++}(\Omega), \pi \neq q, \pi \gg 0$. Since $\succeq^h(\mathbf{R})$ coincides with R_j , if $x \in x_{\succeq^h(\mathbf{R})}(\pi, m)$, then if $\frac{\pi(\omega)}{q(\omega)} > \frac{\pi(\omega')}{q(\omega')}$, $x(\omega) = 0$. Therefore, there exists ω for which $x(\omega) = 0$. Moreover, for all $i \neq j$, if $m > 0$, $x_{R_i}(\pi, m) \gg 0$. Consequently, we conclude that for all $\mathbf{d} \in \delta(\pi, m)$, $d_i = 0$ for $i \neq j$. By upper semicontinuity of δ , conclude that $(1_j, 0_{-j}) \in \delta(q, m)$. Note that for all $i \in N$, $v_{R_i}(q, d_i m) = d_i m$ (as v_{R_i} was defined using the certainty equivalent utility representation of R_i). Consequently $(v_{R_i}(q, d_i m))_{i \in N}$ lies on Δ_m . As $(1_j, 0_{-j}) \in \delta(q, m)$, we therefore conclude that $W(m_j, 0_{-j}) \geq W(u)$, for all $u \in \Delta_m$.

As j and m were arbitrary, the proof is complete. \square

4 Application: homothetic preferences

Theorem 6 gives a family of functions that reduce risk aversion. We singled out the sum of certainty equivalents based on quasiconcavity. Here we provide another justification, one that holds for profiles of homothetic preferences.

The maximum function in Example 4 is the most risk averse preference which is less risk averse than all individual preferences. As we remarked, this rule may in general induce non-convex household preferences even when all

individual preferences are convex: see Figure 1(b).

Here we study the most risk averse *convex* preference which is less risk averse than all individual preferences for convex individual preferences. We show that, for profiles of convex and homothetic preferences, the sum of certainty equivalents gives the most risk averse convex preference that is less risk averse than members' preferences.

In Figure 1(b), the most risk averse convex household preference relation is given by the convex hull of the two upper contour sets. If we denote the upper contour set of agent i 's preference at c as $U_i(c)$, the upper contour set of the household preference at c is

$$\overline{\text{co} \bigcup_{i \in N} U_i(c)},$$

the closed convex hull of the union of the individual upper contour sets. We shall prove this below.

Say a preference $R \in \mathcal{R}$ is **homothetic** if for all $x, y \in X$ and all $\alpha > 0$, $x R y \implies (\alpha x) R (\alpha y)$. Denote the set of homothetic and convex preferences by \mathcal{H} .

Example 8: Homothetic preferences are an important family of preferences in economic analysis. In particular, a large family of such preferences are given by the CRRA multiple priors agents, who have certainty equivalent utility representations $ce : X \rightarrow \mathbb{R}$ given by

$$ce(x) = \left(\min_{\pi \in \Pi \subseteq \Delta(\Omega)} \int_{\Omega} [x(\omega)]^{\rho} d\pi(\omega) \right)^{\frac{1}{\rho}},$$

where $\rho \leq 1$. Given a set of such agents with indices (Π_i, ρ_i) , the condition that the set of preferences has a common prior is equivalent to the condition that $\bigcap_{i \in N} \Pi_i \neq \emptyset$.

Theorem 9: *Suppose that $\mathbf{R} \in \mathcal{H}^N \cap \mathcal{CP}$. Consider the SWF represented by the sum of certainty equivalents. Then the household preference $\succeq^h(\mathbf{R})$ is homothetic, and is the most risk averse convex preference which is less risk averse than R_i for all $i \in N$.*

The theorem demonstrates that at any constant act, the upper contour set of the household preference is the closed convex hull of the union of the individual upper contour sets. This means that the household preference generated by the sum of certainty equivalents is both tractable and geometrically simple.⁸

The proof of Theorem 9 exploits the fact that every profile of common prior homothetic preferences has, for each agent, a representation as: $ce_{R_i}(x) = \inf_{y \in C_i} x \cdot y$, where the common prior π minimizes $\sum_{\omega \in \Omega} y(\omega)$ in C_i . In particular, this set C_i can be explicitly calculated as

$$C_i = \{y : x R_i 1 \implies x \cdot y \geq_i 1\}.$$

Using this representation, it is easy to explicitly calculate household preference: it is given by $U_{\succeq^h(\mathbf{R})}(x) = \inf_{y \in \bigcap_{i \in N} C_i} x \cdot y$, verifying the tractability of the sum of individual certainty equivalents as an instrument of applied analysis.

Example 10: Multiple priors: Suppose that for all $i \in N$, $ce_{R_i}(x) = \min_{\pi \in \Pi_i \subseteq \Delta(\Omega)} \pi \cdot x$. Then $x \succeq^h(\mathbf{R}) y \iff \min_{\pi \in \bigcap \Pi_i + \mathbb{R}_+^\Omega} \pi \cdot x \geq \min_{\pi \in \bigcap \Pi_i + \mathbb{R}_+^\Omega} \pi \cdot y$.

⁸There is a similar result in the theory of international trade, on the maximization of profits under constant returns to scale and more than one industry. Lerner (1934) and Chipman (1966) present a “diagrammatic” argument.

Example 11: CRRA expected utility maximizers: Suppose that for all $i \in N$, $ce_{R_i}(x) = \left(\int_{\Omega} [x(\omega)]^{\rho_i} d\pi(\omega)\right)^{\frac{1}{\rho_i}}$ for $\rho_i \in [0, 1]$. Then

$$x \succeq^h(\mathbf{R})y \iff \left(\int_{\Omega} [x(\omega)]^{\max_{i \in N} \rho_i} d\pi(\omega)\right)^{1/\max_{i \in N} \rho_i} \geq \left(\int_{\Omega} [y(\omega)]^{\max_{i \in N} \rho_i} d\pi(\omega)\right)^{1/\max_{i \in N} \rho_i}.$$

We end this section with a proof of Theorem 9

PROOF (PROOF OF THEOREM 9): The following two lemmas are well-known, but we reproduce them here for completeness.

Lemma 12: *If $R \in \mathcal{H}$, then the function $ce_R : X \rightarrow \mathbb{R}$ is a utility representation for R which is homogeneous of degree one.*

PROOF: Recall

$$ce_R(x) = \inf \{c : c R x\}.$$

To see that the certainty equivalent is homogeneous, let $x \in X$ and $\alpha > 0$. Then

$$\begin{aligned} ce_R(\alpha x) &= \inf \{\alpha c : \alpha c R \alpha x\} \\ &= \alpha \inf \{c : \alpha c R \alpha x\} \\ &= \alpha \inf \{c : c R x\} \\ &= \alpha ce_R(x), \end{aligned}$$

where the second to last equality holds by homotheticity. \square

Lemma 13: *If u is monotone, homogeneous of degree one, and quasi-concave, then it is concave.*

PROOF: Let $x, y \in X$ and $\alpha \in [0, 1]$. Suppose without loss of generality that $u(y) \geq u(x)$. If $u(x) = 0$, then by monotonicity,

$$\begin{aligned} u(\alpha x + (1 - \alpha)y) &\geq u((1 - \alpha)y) \\ &= (1 - \alpha)u(y) = \alpha u(x) + (1 - \alpha)u(y), \end{aligned}$$

verifying concavity. Otherwise, suppose $u(x) > 0$. Then

$$\begin{aligned} &\alpha u(x) + (1 - \alpha)u(y) \\ &= \left[\left(\frac{\alpha}{\alpha + (1 - \alpha)\frac{u(y)}{u(x)}} \right) u(x) + \frac{(1 - \alpha)\frac{u(y)}{u(x)}}{\alpha + (1 - \alpha)\frac{u(y)}{u(x)}} u\left(\frac{u(x)}{u(y)}y\right) \right] \\ &\quad \times \left[\alpha + (1 - \alpha)\frac{u(y)}{u(x)} \right]. \end{aligned}$$

Note now that $u(x) = \frac{u(x)}{u(y)}u(y) = u\left(\frac{u(x)}{u(y)}y\right)$, so that by quasiconcavity,

$$\leq u(x) \left[\alpha + (1 - \alpha)\frac{u(y)}{u(x)} \right] = \alpha u(x) + (1 - \alpha)u(y),$$

verifying concavity. □

Let $\mathbf{R} \in \mathcal{H}^N \cap \mathcal{CP}$. By Lemmas 12 and 13, the certainty equivalent function $ce_{R_i} : X \rightarrow \mathbb{R}$ is homogeneous and concave. Moreover, for all constant acts c , $ce_{R_i}(c) = c$. Extend ce_{R_i} to all of \mathbb{R}^Ω by defining

$$ce'_{R_i}(x) = \begin{cases} ce_{R_i}(x) & \text{if } x \geq 0 \\ -\infty & \text{otherwise} \end{cases}.$$

The function ce'_{R_i} is concave, monotonic, and upper semicontinuous. Its conjugate, $(ce'_{R_i})^* : \mathbb{R}^N \rightarrow \mathbb{R}$ is defined by

$$(ce'_{R_i})^*(x) = \inf_{y \in \mathbb{R}^\Omega} x \cdot y - ce'_{R_i}(y).$$

It is well-known that this function is itself concave and that there is a nonempty, closed, convex, upper comprehensive⁹ set $C_i \subseteq \mathbb{R}_+^N$ for which

$$(ce'_{R_i})^*(x) = \begin{cases} 0 & \text{if } x \in C_i \\ -\infty & \text{otherwise.} \end{cases}.$$

Moreover,

$$ce'_{R_i}(x) = \inf_{y \in C_i} x \cdot y.$$

(See, Rockafellar (1970) Theorem 12.2, Theorem 13.2 and Corollary 13.2.1). Now, let π be a common prior for the profile \mathbf{R} . We claim that $\pi \in \bigcap_{i \in N} C_i$ and moreover that π lies on the boundary (has minimal sum) of each C_i . To see this, note that for each i and each constant act c , $ce'_{R_i}(c) = c$, so $1 = ce'_{R_i}(1) = \inf_{y \in C_i} 1 \cdot y = \inf_{y \in C_i} \sum_{\omega} y(\omega)$. Now, suppose that $\pi \notin C_i$ for some C_i . In particular, by a standard separation argument, there exists $x \in \mathbb{R}_+^N \setminus \{0\}$ for which $\pi \cdot x < \inf_{y \in C_i} y \cdot x$. Let c be a real number for which $\pi \cdot x < c < \inf_{y \in C_i} y \cdot x = ce'_{R_i}(x)$. But then $x R_i c$, while $\pi \cdot x < c$, contradicting the fact that π is a prior for R_i .

Now consider the function defined on X for which

$$U_{\succeq h(\mathbf{R})}(x) = \max_{\sum x_i = x} \sum ce_{R_i}(x_i).$$

Clearly, this function can also be defined on all of \mathbb{R}^Ω , so that

$$U'_{\succeq h(\mathbf{R})}(x) = \max_{\sum x_i = x} \sum ce'_{R_i}(x_i).$$

Moreover, it is easy to see, that since $U'_{\succeq h(\mathbf{R})}$ takes infinite values outside of X , for $x \in X$,

$$U_{\succeq h(\mathbf{R})}(x) = U'_{\succeq h(\mathbf{R})}(x).$$

⁹That is, if $x \in C$ and $y \geq x$, then $y \in C$.

Finally, as $U'_{\succeq^h(\mathbf{R})}$ is the sup-convolution of the functions $(ce'_{R_i})_{i \in N}$, we conclude that the conjugate

$$\left(U'_{\succeq^h(\mathbf{R})}\right)^*(x) = \inf_{y \in \mathbb{R}^N} x \cdot y - U'_{\succeq^h(\mathbf{R})}(y)$$

is given by

$$\begin{aligned} \left(U'_{\succeq^h(\mathbf{R})}\right)^*(x) &= \sum_{i \in N} (ce'_{R_i}(x))^* \\ &= \left\{ \begin{array}{l} 0 \text{ if } x \in \bigcap_{i \in N} C_i \\ -\infty \text{ otherwise.} \end{array} \right\}. \end{aligned}$$

See Rockafellar (1970), Theorem 16.4 and Corollary 16.4.1. Consequently,

$$U'_{\succeq^h(\mathbf{R})}(x) = \inf_{y \in \bigcap_{i \in N} C_i} x \cdot y.$$

Importantly for these arguments, $\bigcap_{i \in N} C_i \neq \emptyset$, as each C_i is upper comprehensive and contains π . Hence, we conclude that household preference $\succeq^h(\mathbf{R})$ is represented by

$$x \succeq^h(\mathbf{R}) z \iff \inf_{y \in \bigcap_{i \in N} C_i} x \cdot y \geq \inf_{y \in \bigcap_{i \in N} C_i} z \cdot y,$$

where for all $i \in N$,

$$x R_i z \iff \inf_{y \in C_i} x \cdot y \geq \inf_{y \in C_i} z \cdot y.$$

Clearly, then, $\succeq^h(\mathbf{R})$ is homothetic. To see that it is the most risk averse convex preference which is less risk averse than each individual preference, let c be a constant act. Note that $\pi \in \bigcap_{i \in N} C_i$ and also lies on the boundary

of $\bigcap_{i \in N} C_i$ (it minimizes $\sum y(\omega)$ across $y \in \bigcap_{i \in N} C_i$). Consequently for any constant act c ,

$$\begin{aligned} c &= c \sum_{\omega \in \Omega} \pi(\omega) \\ &= c \inf_{y \in \bigcap_{i \in N} C_i} \sum_{\omega \in \Omega} y(\omega) \\ &= \inf_{y \in \bigcap_{i \in N} C_i} c \cdot y \end{aligned}$$

We will show that for any c ,

$$\{x : x \succeq^h(\mathbf{R}) c\} = \overline{\text{co} \bigcup_{i \in N} \{x : x R_i c\}},$$

which will verify the result. So first, we show that for all $i \in N$, $\{x : x R_i c\} \subseteq \{x : x \succeq^h(\mathbf{R}) c\}$. Note that $x R_i c$ implies that for all $y \in C_i$, $x \cdot y \geq c$ which implies that for all $y \in \bigcap_{i \in N} C_i$, $x \cdot y \geq c$, which implies that $x \succeq^h(\mathbf{R}) c$. We therefore know that

$$\overline{\text{co} \bigcup_{i \in N} \{x : x R_i c\}} \subseteq \{x : x \succeq^h(\mathbf{R}) c\}$$

as $\succeq^h(\mathbf{R})$ is upper semicontinuous and convex. Suppose now that there exists $w \in X$ such that $w \succeq^h(\mathbf{R}) c$, and for which $w \notin \overline{\text{co} \bigcup_{i \in N} \{x : x R_i c\}}$. In particular, by a standard separation argument, there exists y for which, when normalized, $y \cdot w < c \leq y \cdot x$ for all i and all $x R_i c$. We claim that for all $i \in N$, $y \in C_i$; otherwise, there would exist a separating vector (again nonnegative and normalized) z for which $y \cdot z < c < \inf_{y' \in C_i} y' \cdot z$. But then $z R_i c$ and $y \cdot z < c$, contradicting $y \cdot x \geq c$ for all $x R_i c$. Consequently, $y \in \bigcap_{i \in N} C_i$.

Therefore, $\inf_{y \in \bigcap_{i \in N} C_i} y \cdot w < c$, so that $c \succ^h(\mathbf{R}) w$, a contradiction. \square

5 Discussion

5.1 Related literature

There is a vast literature on risk-sharing in economics. Seminal papers discussing optimal risk sharing include Borch (1962) and Wilson (1968) (see also Chateauneuf, Dana, and Tallon (2000)). Under the assumption that all agents are subjective expected utility maximizers, they determine that, under certain conditions (risk aversion or a continuum of states) all Pareto optimal allocations can be obtained by maximizing a weighted sum of subjective expected utilities.¹⁰ A central result of Wilson (1968) is that the risk tolerance of household preference is the sum of risk tolerances of each individual *at the optimal household consumption*.¹¹

We present our results in a framework with general “non-expected utility” preferences. Our theorem is general enough to apply to most decision theoretic models existing in the literature, including (but not limited to) Schmeidler (1989), Gilboa and Schmeidler (1989), Machina and Schmeidler (1992), Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), Siniscalchi (2007), Ergin and Gul (2008), Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2008), and Seo (2008). In

¹⁰In particular, under these assumptions, Pareto optimal allocations satisfy what Gollier (2001) terms the “mutuality” principle—consumption of each individual depends only on the aggregate amount in each state. As the sum of certainty equivalents satisfies the Pareto principle, any allocation it recommends is Pareto optimal and hence satisfies the mutuality principle.

¹¹Gollier (2001) builds on this result, showing that if all individuals have identical preferences, then a weighted utilitarian planner who optimizes social welfare given a constraint on *average* consumption results in a less risk-averse household preference if and only if the individual risk tolerance is convex. It should be noted that this is a fixed-profile result: the weighted utilitarian rules applied to arbitrary subjective expected utility profiles do not typically reduce risk aversion.

the paper, we have used the expression “risk aversion,” although probabilities here could obviously be taken to be subjective (and common). In this sense, we operate in a general framework of Knightian uncertainty.

The results hold when we restrict the domain to subjective expected utility preferences, preference profiles in \mathcal{EU} : this should be clear from the statements and proofs in Section 3. The results are also novel as results about \mathcal{EU} ; but no additional insights or simplifications are gained from presenting them as results about \mathcal{EU} .

Our work differs from previous studies concerning risk in that it is normative. Most previous studies seek to explain behavioral phenomena in markets, for example see Dow and Werlang (1992), Epstein and Wang (1994), Epstein (2001), Rigotti and Shannon (2005) and Rigotti, Shannon, and Strzalecki (2008). In contrast, we try to understand the SWF that satisfy normatively appealing axioms. In that sense, the exercise is closer to Wilson (1968).

5.2 Conclusion

We study household preferences in the context of sharing risk and uncertainty. We are especially interested in household preferences that are less risk averse than the members’ individual preferences.

Arguments for reduction in risk aversion are familiar in economics, and appear as early as in Samuelson (1964), Vickrey (1964) and Arrow and Lind (1970). These arguments are normative: a collective *should* behave in a less risk averse way. The arguments roughly say that less cautious collectives may reap the benefits of larger expected gains, and mitigate the risks by risk sharing. The idea that a household should be less risk averse is also strategically motivated. For example, often in strategic interactions, all else equal, an agent who is less risk averse will fare better according to all preferences. This is the case in Nash bargaining (Rubinstein, Safra, and Thomson

(1992)), as well as in many other game theoretic models of bargaining. So, by appropriately sharing risk, a household seeks to become more “competitive.”

Our result says much more than that ACTA rules yield a *local* reduction in risk aversion. The local statement would be that the household is locally less risk averse than each individual at each individual’s optimal consumption. Indeed, this would be true for *any* rule satisfying the Pareto property. Our result is much stronger: ACTA rules generate a household which is less risk averse than each of its members *at every level of consumption*.

We introduce two additional axioms: the Pareto criterion and that certain (sure) acts should be compared without regard for risk preferences. From the normative perspective, the Pareto criterion is obviously desirable, and the independence axiom should be appealing. Independence may not be appealing in a descriptive setting, in which any one agent can force a “breakdown” of negotiations; then risk attitudes play a role even when the “optimal” choices feature no risk (Rubinstein, Safra, and Thomson (1992) explain how attitudes toward risk are important in a Nash bargaining context).

We characterize the household SWF which respect the Pareto criterion, compare certain acts without regard for risk preferences, and which reduce risk aversion. The results single out the sum of certainty equivalents as the unique member of this class which is quasiconcave over certain allocations. Quasiconcavity, in turn, is a basic fairness requirement.¹²

Our paper thus provides a justification for using the sum of certainty equivalents as a guide in allocating risky prospects. The existing justification (Myerson, 2004) says that any allocation which does not maximize the

¹²Gorman (1959) argues that actual collectives may use a convex W : he believes that utility profiles that are not very unequal may be inherently stable. Any small advantage obtained by a group of agents will result in a political advantage, which will then reinforce the initially small advantage. The resulting collective will behave as if it used a convex social choice function.

sum of certainty equivalents could be improved upon by an allocation that each individual agent would be willing to accept more for. This is a simple consequence of interpreting certainty equivalents as willingness to accept. Our justification is entirely different, and depends on the desirability of the collective being less risk averse than household members.

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