

Notational conventions for strategies

Let X be a finite set. Write $\Delta(X)$ for the set of all probability distributions on X . That is, $\Delta(X) = \{p \in \mathbf{R}_+^X : \sum_{x \in X} p_x = 1\}$. We use $\mathbb{E}_p f(x)$ to denote the mathematical expectation of a function $f : X \rightarrow \mathbf{R}$.

Let $(N, \{S_i : i \in N\}, \{u_i : i \in N\})$ be a normal-form game, with $N = \{1, \dots, n\}$. We write $S = \times_{i=1}^n S_i$ for the set of all *strategy profiles*.

Fix a player i . We represent a strategy profile in the following way:

$$s = (s_1, \dots, s_{i-1}, s_i, s_{i+1}, \dots, s_n) = (s_i, s_{-i}).$$

So, given s , s_{-i} is a profile of strategies for all players but i . That way we can change the strategy for i to, say $s'_i \in S_i$, and compare i 's change in utility from $u_i(s) = u_i(s_i, s_{-i})$ to $u_i(s'_i, s_{-i})$.

We also write $S_{-i} = \times_{j \neq i} S_j$ for the set of all profiles of strategies for players other than i .

A *mixed strategy* for player i is a probability distribution $\sigma_i \in \Delta(S_i)$.

If $\sigma = (\sigma_1, \dots, \sigma_n)$ is a profile of mixed strategies, then i 's expected payoff is

$$U_i(\sigma) = \mathbb{E}_\sigma u(s) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \dots \sum_{s_n \in S_n} u_i(s_1, s_2, \dots, s_n) = \sum_{s \in S} \sigma(s) u_i(s).$$

We abuse notation, and use lower-case u for expected utility. So we write $u_i(\sigma)$ for $U_i(\sigma)$; this is never a cause for confusion.

Using some notational simplifications, we can then write

$$u_i(\sigma) = \mathbb{E}_{\sigma_i} u(s_i, \sigma_{-i}) = \sum_{s_i \in S_i} \sigma_i(s_i) u_i(s_i, \sigma_{-i}),$$

where

$$u_i(s_i, \sigma_{-i}) = \mathbb{E}_{\sigma_{-i}} u_i(s_i, s_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) u_i(s_i, s_{-i}),$$

and $\sigma_{-i}(s_{-i})$ is the product of $\sigma_j(s_j)$ over $j \neq i$.

Similarly,

$$u_i(\sigma) = u_i(\sigma_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \sigma_{-i}(s_{-i}) u_i(\sigma_i, s_{-i}) = \sum_{s \in S} \sigma_i(s_i) \sigma_{-i}(s_{-i}) u_i(s_i, s_{-i}).$$