

On managerial risk-taking incentives when compensation may be hedged against

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Abstract

When the compensation risk can be hedged away completely, the manager will try to maximize the market value of the compensation. When the risk can be hedged partially, numerical examples for a CARA manager illustrate how incentive effects depend on the relative size of systematic and specific risk premia of the output and of the hedging asset. When the specific risk can be modified, higher specific risk premium leads to more incentive contracts. In most cases call and put options induce higher risk taking than shares, but put options may induce lowering of the specific risk. There are cases in which the hedging manager may be less aggressive than the non-hedging manager, for example when the output's risk premium is low.

Key words: compensation, incentives, hedging, specific and systematic risk.

JEL classification: J33; G30; G32; G34; G11

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1 Introduction

In this note we examine incentive effects of managerial compensation on managerial risk taking when that compensation can be partially hedged against. Ross (2004) performs a detailed analysis of incentive effects of nonlinear contracts on risk-averse managers when there is no possibility of hedging. In particular, he identifies three effects of the compensation form on the incentives: convexity, magnification and translation effect. The combination of the three determines whether the manager will act more or less aggressively. That paper complements the work of Carpenter (2000), who also noted, in a dynamic setting, that convex payoffs need not increase managerial appetite for risk.¹ Also in a dynamic model, we focus on another dimension influencing the manager's risk preferences, namely the possibility of hedging.² In particular, it is of interest to study how much higher risk-taking the possibility of hedging will induce, if at all. Most of our analysis is restricted to the case of CARA preferences, for tractability reasons, but also because it eliminates the translation effect, thereby enabling us to concentrate better on the hedging effect.³

We first find optimal contracts from the firm's point of view, in the first-best case of symmetric information, in a setting in which either the total volatility can be controlled cost-free, or the specific and the systematic risk may be controlled separately and cost-free. This extends (for the CARA case and cost-free effort) results of Ou Yang (2003) and Cadenillas, Cvitanić and Zapatero (2007) (henceforth CCZ 2007), to the presence of compensation hedging.

A number of other papers in recent years have studied the effect of managerial hedging on incentives. We mention here the works of Jin (2002), Garvey and Milbourn (2003), Acharya and Bisin (2005), Bisin et al. (2006), Ozerturk (2006) and Gao (2008). Not unlike this paper, in those studies the firm and the manager have CARA preferences. However, unlike this article, most of those papers are set in a traditional principal/agent theory setting in which the agent controls only the return, and not the risk/volatility of the output, and consider only linear contracts. In such a framework, they focus on the dependence of the pay-per-performance sensitivity (PPS) on the underlying risks. We are, on the other hand, interested in how the possibility of hedging and the form of compensation contracts, possibly nonlinear, influence the manager's choice of risk/volatility.

Optimal contracts may depend on the returns of the assets available for hedging, the type

¹Recently, Panageas and Westerfield (2008) show that even the risk-neutral managers need not behave aggressively when paid with high water marks contracts, if the time-horizon of their compensation is not fixed.

²It has been well documented in the literature that managers and executives engage in a substantial way in the practice of hedging their compensation contracts; e.g. Bettis, Bizjak, and Lemmon (2001).

³Carpenter, Stanton and Wallace (2008) study optimal exercise of executive stock options for a CRRA executive who can partially hedge the compensation.

of compensation usually referred to as relative performance evaluation, or RPE. Moreover, in the first-best framework of observable managerial actions, and if the manager has either CARA preferences or devotes zero initial capital to hedging (say, using futures or swaps, as in Ozerturk 2006), we find that there is a contract which is optimal regardless of whether the manager can hedge or not, and which will, in fact, induce the manager not to hedge. This is accomplished even without monitoring the manager's hedging activity, unlike in Bisin et al. (2006), where the manager does not hedge due to costly monitoring by the firm.

The RPE feature is partially due to the desire to remove the “systematic risk” component from the contract, a theoretical prediction that has been known since the work of Holmstrom (1982). Since our firm is also risk averse, RPE is also motivated by hedging needs of the firm. Moreover, as mentioned above, the firm may want to offer a contract which preempts managerial hedging (by performing hedging on her account), and this also leads to the dependence of the payoff on the risk factors driving the hedging opportunities. This is in agreement with findings of Jin (2002) and Garvey and Milbourn (2003), who find that RPE should be used when the managers are constrained in their hedging opportunities, while there is no need for RPE if the managers have low cost of hedging. In relation to this, it should be mentioned that in our first-best world there may be more than one optimal contract, and we only consider the contracts which induce the manager not to hedge, which we call hedge-neutral contracts, which may then include the RPE component even when managerial hedging is costless.

In a two-factor dynamic model with CARA preferences, we find that the optimal hedge-neutral contract is linear in the output value and in the log-returns of the risky asset used for hedging, the latter being the RPE component. However, because of the linearity and costless hedging, there is actually no need for the RPE component – the firm is indifferent with respect to the size of the RPE in the contract. Moreover, if the specific risk of the output can be modified, we find that PPS gets higher with higher specific risk premium. In other words, the higher the benefits of managing the specific risk, the higher the incentives provided to the manager.

In the second part of the paper, we examine numerically the incentive effects of compensating a hedging CARA manager with shares, call options and put options. Other papers considering the effect of specific contracts on portfolio managers, typically without possibility of hedging, include Basak, Pavlova and Shapiro (2008), Basak, Shapiro and Tepla (2006), Cuoco and Kaniel (2007), Hugonnier and Kaniel (2008). In our setup, we have in mind a high-level executive of the firm, rather than a portfolio manager, and her behavior depends on whether she controls the overall volatility, or separately the specific and the systematic risk. Moreover, it depends on the size of the corresponding risk premia. In particular, when the output's risk premium is low, she will choose low total volatility when paid with shares, and high when paid with puts, while choosing it in the middle when paid with calls. As the

output’s risk premium gets higher, she becomes less conservative when paid with shares. If she can influence only the systematic risk, when the systematic risk premium is equal to the hedging asset’s risk premium the manager is indifferent with respect to the size of systematic risk, while calls and puts make her go for high systematic risk. With low systematic risk premium, calls and puts make the manager go for high systematic risk, while shares make her conservative. With high systematic risk premium all three compensation instruments increase the appetite for risk. When the manager can control only the specific risk, in most cases she picks it low when compensated by puts, and is less conservative with calls and shares. On the other hand, this gets reversed if the specific risk premium is sufficiently low relative to the systematic risk premium.

The hedging manager is less conservative than the non-hedging manager, in general. However, if compensated with shares, the hedging manager may be more conservative than the non-hedging manager when modifying the total volatility or the systematic risk, if the corresponding output’s risk premium is low; the same is true if compensated with put options when modifying the output’s specific risk.

We describe the general setting in Section 2, analyze optimal contracts in Section 3, and discuss incentives of particular contracts in Section 4. Section 5 concludes, while technical proofs are provided in Appendix.

2 The Setting

Suppose a manager is promised a compensation with payoff C_T at future time T . However, the manager also trades on her own account, with value H_t at time t . She may be able to influence an output process (e.g., the stock price of a company, a portfolio value, and so on), by exerting effort and/or by choosing different “projects” (say, picking different stocks to invest in). Altogether, she is maximizing the value of

$$E[U_A(C_T - G_T + H_T)] \tag{2.1}$$

where U_A is her utility function (A stands for agent), and G_T is a cost of exerting effort, if any.

Let us now fix the compensation C_T and the cost G_T (implicitly also fixing the actions of the manager related to the firm). Then, in the standard models, the optimal hedging strategy of the manager will result in the marginal utility of the manager being proportional to a stochastic discount factor (henceforth SDF) in the market in which she is trading, and which includes the claim $C_T - G_T$. More precisely, if we introduce the inverse of the marginal utility

$$I_A(x) = (U'_A)^{-1}$$

the optimal hedging strategy results in

$$C_T - G_T + H_T = I_A(zZ_T^{C,G}) \quad (2.2)$$

where $Z_T^{C,G}$ is an appropriate SDF, and the constant z is determined so that the budget constraint is satisfied:

$$H_0 = E[Z_T^{C,G} H_T] = E \left[Z_T \left(I_A(zZ_T^{C,G}) - C_T + G_T \right) \right] \quad (2.3)$$

Here, H_0 is the initial capital devoted to hedging. Thus, from (2.1) and (2.2), we see that the manager would like to choose her actions so as to maximize the value

$$E[U_A(I_A(zZ_T^{C,G}))] \quad (2.4)$$

under the budget constraint (2.3).

2.1 Complete Market

Suppose the manager has access to a complete market, which, in particular, means that the claim $C_T - G_T$ is attainable. In that case Z_T is the unique SDF, independent of the contract C_T and cost G_T . Then we conclude from (2.4) that the manager wants to minimize the value of z . From (2.3), we obtain the following familiar result:

Proposition 2.1 *Suppose that the manager has access to a complete market with the SDF Z_T . In particular, suppose that for any feasible payoff C_T and cost G_T there is a hedging strategy which accomplishes (2.2), with (2.3) satisfied. Then the manager will choose her actions so as to maximize the no-arbitrage value $E[Z_T(C_T - G_T)]$ of her compensation minus the costs.*

For example, if $G_T = 0$ and if C_T is a call option, and the manager's actions influence the volatility of the option's underlying, then the manager would choose the effort so as to increase the volatility as much as possible. This is in contrast with the case in which the manager cannot hedge at all. For example, Ross (2004) shows that a manager who cannot hedge, and has a DARA utility function, would not necessarily become less risk averse when call options are added to her compensation, and may, in fact, become more risk averse. Similarly, Carpenter (2000) also pointed out that convex payments do not necessarily induce risk seeking behavior. In our present setting, on the contrary, the possibility of complete hedging always induces less risk aversion when compensation is in call options.

If the market for hedging is incomplete, so that the manager can only partially hedge the risk of the compensation payoff, the situation is much more complicated. In that case there are many risk-neutral densities, hence many SDF's, and the one in the manager's

optimization problem may depend in a complex way on the payoff C_T and cost G_T . In any case, the manager no longer maximizes solely the market price of $C_T - G_T$, and her risk aversion would come into the picture. We consider some tractable cases in following sections.

3 Optimal Contracts with Symmetric Information

In this section we assume the first-best world of symmetric information, in which the firm can force the manager to apply actions of the firm's choosing. In that case, the firm is definitely not worse off if the manager can hedge, because it is easier for the firm to meet the manager's "participation constraint", that is, to meet her reservation wage, while there is no need to worry about incentives. Moreover, if we assume either CARA preferences for the manager, or that the manager starts hedging with zero initial capital, we show now that there is a contract which is optimal whether the manager can hedge or not.

We prove the latter through the following steps: (i) we find a representation for optimal contracts when the manager does not hedge; (ii) we show that given such a contract the manager would not hedge even if she could, but would deposit all the hedging capital into the risk-free account – we call such contracts "hedge neutral"; (iii) we show that even when the manager can hedge, the search for optimal contracts can be restricted to the family of hedge-neutral contracts.

Step (i): a representation of optimal contracts. Denote by X_T the output influenced by (observable) manager's actions and by G_T the cumulative cost of manager's actions. With symmetric information and in absence of managerial hedging, but allowing the firm to hedge, the firm would be maximizing, for some constant weight L depending on the reservation wage, the expression

$$E[U_P(X_T - C_T + F_T)] + LE[U_A(C_T - G_T)] \quad (3.5)$$

over the contract payoff C_T , the actions influencing X_T and the firm's hedging strategy with T -value F_T . Here U_P is the utility function of the firm (P stands for principal). We assume that the firm has access to a market of assets for the purpose of hedging, and we consider the family \mathcal{Z} of SDFs in that market, in which the information structure is given by the random factors driving the traded assets, plus the random factors driving the manager's output process. Using standard martingale methods it follows that under a wide range of conditions the optimal hedging strategy will be such that

$$U'_P(X_T - C_T + F_T) = z'Z_T$$

for some SDF $Z \in \mathcal{Z}$ and some constant z' . Then, taking a derivative inside the expectation

in (3.5) with respect to C_T , we see that the first-order condition is

$$C_T = G_T + I_A(zZ_T) \quad (3.6)$$

for $z = z'/L$. If this is also a sufficient condition, the first-best contract will be of this form. We now argue that given such a contract, the manager will not hedge, but will deposit all her hedging money into the bank account.

Step (ii): hedge-neutral contracts. Assume the manager can hedge, starting with H_0 in initial capital, and resulting in the hedging portfolio value of H_T at time T . For fixed C_T, G_T , introduce the value function of the manager:

$$V(H_0) = \max E[U_A(C_T + H_T - G_T)]$$

where the maximum is taken over hedging strategies, and it is assumed to be attained. We have the following result:

Proposition 3.2 *Suppose either that the manager has CARA preferences, or that her initial hedging capital H_0 is zero. Also assume that the interest rate is deterministic.⁴ If a contract C_T is of the form*

$$C_T = G_T + I_A(zZ_T) \quad (3.7)$$

where Z_T is an SDF in \mathcal{Z} , then C_T is hedge-neutral. Conversely, if the dual problem (6.18) in Appendix, with value $\bar{V}(H_0)$, has a solution for a given contract C_T and if we have $V(H_0) = \bar{V}(H_0)$,⁵ then, in order for C_T to be hedge-neutral it has to be of the form (3.7).

The proposition tells us that a hedge-neutral contract is such that the manager's marginal utility is proportional to an SDF. This is because that is exactly what the manager would like to accomplish by hedging.

We provide a proof of the proposition in Appendix, while giving here an argument in a simple single-period market, with a risky asset S and risk-free rate r : The manager needs to maximize over the number of shares δ

$$\sum_i p_i U_A(I_A(zZ_T) + \delta S_T^i + (H_0 - \delta S_0)(1 + r))$$

Under our assumptions, the maximizer of this expression does not change if we delete the term $H_0(1 + r)$. The first-order condition is then

$$0 = \sum_i p_i U'_A(I_A(zZ_T) + \delta(S_T - S_0(1 + r))) (S_T - S_0(1 + r))$$

⁴Alternatively, we could assume that the firm and the manager enjoy utility from discounted values.

⁵These technical conditions are typically either satisfied, or they are satisfied if instead of \mathcal{Z} we consider its closure in an appropriate topology; for details see the references mentioned in the proof.

Since $U'_A(I_A(x)) = x$, and since Z_T is an SDF, if $\delta = 0$ this condition will be satisfied.

- **Step (iii): it is enough for the firm to consider the hedge-neutral contracts.**

Indeed, suppose that the firm gives a contract C_T which induces the manager to hedge optimally in a way that results in the final amount of H_T , starting with initial capital H_0 . Instead, the firm could borrow H_0 at the risk-free rate, invest it in the same way the manager would, and thus have H_T at time T as a result of that investment, and pay $C_T + H_T - H_0B_T$ to the manager at time T , where B_t is the value process of the risk-free account. This would result in the same utility for the firm as the contract with payoff C_T , because the firm would be able to return the debt of H_0B_T , while it would forward the remaining profit/loss $H_T - H_0B_T$ to the manager. It would also result in the manager's utility of $E[U_A(C_T + H_T - G_T)]$, if she invests all of the hedging capital H_0 in the risk-free asset. This is exactly what is optimal for the manager. Thus, to recap, the manager would not hedge with this contract, and both the firm and the manager are equally well-off as with the original contract.

Combining all of the above, we get the conclusion announced in the first paragraph of this section, which we now state as

Theorem 3.1 *Under the above conditions, in particular assuming the company can observe the manager's actions, if the manager has CARA preferences or starts hedging with zero initial capital, there is a hedge-neutral contract which is optimal both in the presence and in the absence of hedging opportunities.*

Note that the contracts of the form (3.7) will not, in general, be based solely on the random factors driving the manager's performance, but also on those driving the assets in the hedging market, leading to a relative performance evaluation (RPE) component of the contract. We illustrate this fact with an example in the following subsection. The RPE component is present for the purpose of removing the non-specific risk from the manager's payoff (as in Holmstrom 1982), as well as from the firm's payoff as the firm is also risk averse. At the same time RPE also preempts the managerial need to hedge.

In general, the optimal contract we found requires full monitoring of the manager's actions by the firm. However, we show in the following section that there may exist an equivalent contract which requires no such monitoring.

3.1 A Two-Factor Model and CARA preferences

In CCZ (2207) it was shown, in the absence of hedging and with zero cost function, that when the output is of the "portfolio value" form $dX_t = \alpha_t v_t dt + v_t dm_t$ for some martingale m , where v can be modified freely so that all possible random outcomes can be replicated by X ("complete market"), there exists a contract of the form $f(X_T)$ which is optimal and

does not require that the firm monitor v_t , even with non-CARA preferences. We present here a model of the similar type, but assuming CARA preferences. It turns out that there are optimal contracts which are linear in the manager's output and in the log-value of the risky asset she is hedging with, and the firm is indifferent with respect to whether to include the hedging asset into the compensation package.

Consider the model

$$\begin{aligned} dX_t &= [\alpha_x x_t + \alpha_y y_t] dt + x_t dW_t + y_t dM_t \\ dS_t/S_t &= \mu dt + \sigma dW_t \end{aligned}$$

where W and M are independent Brownian motions, α_i, μ, σ are constants, and x_t, y_t are adapted processes interpreted as the systematic risk and the specific risk. Then, α_x is interpreted as the systematic risk premium, and α_y is the specific risk premium.

Both the manager and the firm can hedge by trading in the risky asset S and a risk-free asset. For simplicity, we set the risk-free rate to zero. Introduce the process

$$dZ_t^\theta = -Z_t^\theta [\lambda dW_t + \theta_t dM_t]$$

where $\lambda = \mu/\sigma$ is the risk premium of S , and θ_t is an arbitrary adapted process. Then, processes Z^θ represent SDFs for the incomplete market in which only S is traded, but the information structure includes, in addition, the process M .

Suppose there is no cost of effort, so $G_T = 0$. We assume that the preferences are of CARA type:

$$U_i(x) = -\frac{1}{\gamma_i} e^{-\gamma_i x}$$

Then,

$$I_i(y) = -\frac{1}{\gamma_i} \log(y), \quad U_i(I_i(y)) = -\frac{1}{\gamma_i} y$$

Thus, $E[U_A(I_A(zZ_T^\theta))] = -z/\gamma_A$, and z is fixed by the manager's reservation value. We have shown above that an optimal contract for the firm is of the form $C_T = I_A(zZ_T^\theta)$. Given such contract, the firm is maximizing the value $E[U_P(X_T - I_A(zZ_T^\theta) + F_T)]$, where F_T is the value of the firm's hedging portfolio. That value can also be written as

$$E \left[U_P \left(X_T + F_T - \frac{1}{\gamma_A} \left[\frac{1}{2} \int_0^T (\lambda^2 + \theta_t^2) dt + \int_0^T \lambda dW_t + \int_0^T \theta_t dM_t \right] \right) \right] \quad (3.8)$$

Straightforward computations, some of which are provided in Appendix, now lead to the following conclusions:

- (i) Assume deterministic x_t, y_t . Optimal choice for the contract parameter process θ_t is

$$\theta_t \equiv y_t \frac{\gamma_A \gamma_P}{\gamma_A + \gamma_P} \quad (3.9)$$

Optimal amount π invested in S by the firm satisfies

$$\sigma\pi_t + x_t \equiv \lambda \frac{\gamma_A + \gamma_P}{\gamma_A \gamma_P} \quad (3.10)$$

- (ii) Suppose the specific risk y_t can be modified, while $x_t \equiv x$ is constant and fixed. Then, it is optimal for the firm to have

$$y_t \equiv \alpha_y \left(\frac{1}{\gamma_A} + \frac{1}{\gamma_P} \right) \quad (3.11)$$

In other words, the specific risk will match the specific risk premium adjusted by the sum of inverted risk aversions. If either the firm or the manager are almost risk-neutral, the optimal specific risk becomes huge. More interestingly, it can also be easily checked that the contract is ex-post linear,⁶

$$C_T = aX_T + b \log(S_T) + c$$

where we have

$$a = \frac{\gamma_P}{\gamma_A + \gamma_P}, \quad b = \frac{\lambda}{\sigma \gamma_A} - \frac{a}{\sigma} x$$

Actually, the firm is indifferent with respect to which b it uses, due to the possibility of hedging with asset S ; different choice of b will induce a different choice of c , and the above choice of b is the one with which the manager would not hedge. Moreover, if, instead of being promised $C_T = I_A(zZ_T^\theta)$, the manager is offered the contract in the form $C_T = aX_T + \tilde{b} \log(S_T) + c$ ex-ante, for a as above and any \tilde{b} , she will use the above value of y_t , even without being dictated by the firm to do so, and thus, such linear contracts are optimal. When there are no hedging possibilities, it is well known that a contract of the same linear form is optimal, with the same a , and with $b = 0$ (e.g., CCZ 2007).

- (iii) Suppose the systematic risk x_t can be modified, while y_t is deterministic and fixed. It now has to be the case that the corresponding risk premia are equal, $\lambda = \alpha_x$, otherwise there is arbitrage for the firm by appropriate choice of x and π . Assuming $\lambda = \alpha_x$, it is optimal for the firm to have

$$x_t \equiv \alpha_x \left(\frac{1}{\gamma_A} + \frac{1}{\gamma_P} \right) \quad (3.12)$$

The manager would not hedge if given a contract $C_T = aX_T + c$, with a as above. Moreover, if the manager was given the contract $C_T = aX_T + \tilde{b} \log S_T + c$ for any \tilde{b} , instead of being promised $C_T = I_A(zZ_T^\theta)$, she would be indifferent between various choices for the systematic risk x .

- (iv) Suppose both the systematic risk x_t and the specific risk y_t can be controlled. We again need to have $\lambda = \alpha_x$. Then, it is optimal for the firm to have the same x_t as in (iii),

⁶If we modeled S as a Brownian motion with drift rather than a geometric Brownian motion, the optimal contract would be linear in S , not in $\log(S)$, in the case of CARA preferences.

and the same y_t as in (ii). We again have that $C_T = aX_T + c$ induces the manager not to hedge, with the same a as in (iii), and a similar discussion applies, with the following modification: if the manager was offered $C_T = aX_T + \tilde{b} \log(S_T) + c$ for any \tilde{b} , she would be indifferent between various choices for the systematic risk x , and she would choose the specific risk y_t which is optimal for the firm, too.

- (v) A manager who cannot hedge and is offered $C_T = aX_T + \tilde{b} \log(S_T) + c$ would choose optimally the values also optimal for the firm, $x_t \equiv \alpha_x / (a\gamma_A)$, $y_t \equiv \alpha_y / (a\gamma_A)$.

The main message of this discussion is that with CARA preferences simple linear contracts are optimal even in the presence of hedging opportunities, and they do not require the firm to monitor the manager's choice of risk. Furthermore, the firm is indifferent with respect to the contract's level of dependence on the value of hedging assets (the RPE level).

3.2 Unobservable effort

Extend now the above model to

$$dX_t = [\delta u_t + \alpha_x x_t + \alpha_y y_t] dt + x_t dW_t + y_t dM_t \quad (3.13)$$

where u_t is manager's effort, unobservable by the firm. The manager suffers cumulative cost of effort $G_T = E \int_0^T g(u_t) dt$. We have the following result, proved in Appendix:

Proposition 3.3 *Assume CARA preferences for the manager and the firm, that the cost $g(u)$ is quadratic, and that the specific risk y can be controlled. Among the contracts of the form $C_T = aX_T + b \log(S_T) + c$, the firm is indifferent with respect to the value of b . Moreover, the optimal value of a depends only on risk aversions γ_A , γ_P and specific risk premium α_y , and is increasing in α_y .*

The papers Jin (2002), Garvey and Milbourn (2003) and Gao (2008) consider similar models, but with fixed systematic risk x and specific risk y , and with hedging which is costly. The main conclusion is that the optimal contract does not depend on the systematic risk x , and that the PPS value a is decreasing in specific risk y . In our framework y can be modified in a costless manner, and the prediction is that, in such a case, firms whose specific risk premium is high will offer high-power contracts. Put differently, higher benefits of controlling the specific risk lead to higher managerial incentives. Moreover, as seen in the previous section, if the managers are compensated only for the mean-variance trade-off (that is, only for modifying x and y , but not u), the optimal PPS (the value of a) depends only on the risk aversions, and not on the underlying risks.

4 Performance Based Contracts

The above-discussed optimality of linear contracts for CARA managers hinges on several assumptions: the firm also has CARA preferences; the firm and the manager have the same hedging opportunities and that fact is known by the firm; there are no limited liability constraints (the contract payoff may take negative values). In reality, these assumptions may not be satisfied, and the firm may offer nonlinear contracts, and, if the hedging opportunities of the manager are not known, the contracts may depend only on the manager's output. We therefore study now the effect of several such contracts on the manager's actions, and in particular, we study compensation in shares, call options and put options.

4.1 Manager's Actions for a Given Contract

Let us assume that the expected value of all SDF's is independent of the manager's actions. For example, if the market in which the manager hedges trades a zero-coupon default-free bond with face value of one unit of currency, then the price of this bond is $E[Z_T^{C,G}]$, and this value is independent of which SDF $Z^{C,G}$ is used.⁷

Assume also that the manager/agent has exponential utility $U_A(x) = -\frac{1}{\gamma_A}e^{-\gamma_A x}$. Thus, using the fact that $E[Z_T^{C,G}]$ is constant independent of the manager's actions, from (2.4) we see that the manager wants to minimize the value of z . Using (2.3), we get the following

Proposition 4.4 *If the expected value of all SDF's is independent of the manager's actions, and the manager has CARA preferences, then she will choose her actions so as to maximize*

$$\gamma_A E[Z_T^{C,G}(C_T - G_T)] + E[Z_T^{C,G} \log(Z_T^{C,G})] \quad (4.14)$$

In other words, the manager maximizes a weighted sum of the value of compensation minus cost, obtained using $Z^{C,G}$ as the pricing kernel, and of the "entropy" of that pricing kernel.

4.2 Managerial hedging in a two-factor model

We consider a specific model next to study the effect of standard options in compensation. The model is analogous to the ones in Sections 3.1 and 3.2.

Consider Brownian Motions W and \tilde{W} with correlation ρ . The manager can trade in a risky asset which satisfies

$$dS_t/S_t = \mu_t dt + \sigma_t dW_t$$

⁷Alternatively, we could assume that the market trades a risk-free asset with the interest rate independent of all the risk-neutral densities in the market.

and the bank account satisfying

$$dB_t = B_t r_t dt, \quad B_0 = 1$$

She can influence the output process given by

$$dX_t = [\delta u_t + p_t X_t + \alpha_t v_t] dt + v_t d\tilde{W}_t$$

by modifying the values of u_t and v_t . For example, this includes the case when X is a value of a managed portfolio when investing in a risky asset with risk-premium α_t and a risk-free asset with interest rate p_t , according to a portfolio strategy v_t . In addition, there is an extra infusion of funds into, or a consumption of money out of the portfolio, at rate δu_t .

Denote by $\tilde{F} = \{\tilde{\mathcal{F}}_t\}$ the filtration generated by \tilde{W} and introduce the risk-premium process

$$\lambda_t = \frac{\mu_t - r_t}{\sigma_t}$$

We assume that $u, p, v, \alpha, r, \lambda$ are \tilde{F} -adapted, and that r is constant. In this section we set $G_T \equiv 0$ and we consider only the contract payoffs C_T which are measurable with respect to $\tilde{\mathcal{F}}_T$. For example, C_T can be a functional of the output process X , such as an option written on that process. However, it cannot depend on the hedging asset S . We recall now a result by Tehranchi (2004), also obtained in Brendle and Carmona (2004):

Proposition 4.5 (Tehranchi 2004) *The optimal value of the expected utility for the CARA manager given payment C_T , is*

$$V_A := E[U_A(C_T + H_T)] = -\frac{1}{\gamma_A} e^{-\gamma_A H_0 e^{rT}} \left(E^Q e^{-\gamma_A(1-\rho^2)C_T - \frac{1-\rho^2}{2} \int_0^T \lambda_s^2 ds} \right)^{\frac{1}{1-\rho^2}} \quad (4.15)$$

where Q is the probability measure under which

$$W_t^Q = \tilde{W}_t + \rho \int_0^t \lambda_s ds$$

and W are Brownian motions.⁸

⁸In order to interpret this, write now

$$\tilde{W} = \rho W + \sqrt{1-\rho^2} M$$

for a Brownian Motion M independent of W . Note that we have

$$dS/S = [r + \sigma\lambda]dt + \sigma/\rho d\tilde{W}_t - \sigma\sqrt{1-\rho^2}/\rho dM_t = rdt + \sigma/\rho dW^Q - \sqrt{1-\rho^2}/\rho dM_t$$

Thus, Q is a risk-neutral measure for S . It is actually the projection on \tilde{F} of the measure which would be the risk-neutral measure if S was the only traded asset.

In particular, we see from (4.15) that when the hedging asset's risk premium λ is independent of the contract compensation C_T under measure Q , *the CARA manager will choose her actions as would the manager who cannot hedge, but who has risk aversion reduced by a factor of $1 - \rho^2$, and whose subjective probability is Q* . The reduction of risk aversion would make the hedging manager more aggressive, but the change of the subjective probability does not have unambiguous effects. Effectively, the latter reduces the output's drift by the amount $\rho\lambda$, as in (4.16) below, and thus may lead to the manager becoming more conservative. The combination of the two effects will determine whether the manager is less or more aggressive when she can hedge.

The above result will enable us to compute such incentives effects when offering to the manager various types of contracts. We specialize the model to the case

$$dX_t = [\delta u + (\alpha - \lambda\rho)v]dt + v dW_t^Q \quad (4.16)$$

where we assume that all the parameters are constant. For the computations, it will be useful to denote

$$\begin{aligned} b(v, \rho) &= X_0 + \delta u T + v T (\alpha - \rho\lambda) \\ c(\rho) &= n\gamma_A (1 - \rho^2) \end{aligned}$$

where n is the number of contracts issued. We also normalize H_0 to be zero.

We will consider two settings, one in which the manager can control the total volatility v , and another in which she may possibly be able to control separately the specific risk and the systematic risk, as in Section 3.1.

4.2.1 Shares contract

Assume the manager is given n shares,

$$C_T = nX_T$$

Then, it is straightforward from (4.15) to compute the manager's utility

$$V_A = -\frac{1}{\gamma_A} e^{-T\lambda^2/2 - \gamma_A n b(v, \rho) + \frac{1}{2} T \gamma_A^2 n^2 (1 - \rho^2) v^2}$$

Suppose the manager can control the total volatility v . Recall that we showed above that in the case of hedging in a complete market the manager compensated by stock shares is indifferent with respect to which volatility to choose, as the market value of the shares is fixed. This is not the case here, where the optimal level of volatility is

$$v(\rho) = \frac{1}{n\gamma_A} \frac{\alpha - \lambda\rho}{1 - \rho^2}$$

With $\rho = 0$ the manager cannot hedge the exposure to the compensation risk. In that case the optimal total volatility is $v(0) = \frac{\alpha}{n\gamma_A}$. The difference is

$$v(\rho) - v(0) = \frac{\rho(\alpha\rho - \lambda)}{(1 - \rho^2)n\gamma_A}$$

Thus, we have the following conclusions:

- If the correlation is positive and the output's risk premium adjusted by correlation, $\alpha\rho$ is larger than the hedging asset's risk premium λ , then the hedging manager prefers higher volatility than the non-hedging manager. And vice versa when $\alpha\rho < \lambda$. The difference in preferred volatilities becomes small as the number of shares or the risk aversion become large, or as the correlation gets closer to 1 or -1.

- It may very well happen that the manager who hedges would prefer lower volatility even in absolute value, than the manager who does not hedge. For example, for fixed positive ρ , because of diversification the hedging manager wants volatility that is decreasing in the hedging asset's risk premium λ , and for the values of λ close to α/ρ she will want volatility close to zero.

- Similar results would hold in the case of CRRA utility functions, if X was a geometric Brownian motion process instead of arithmetic Brownian motion.

Suppose now we fix v . The value of the manager's utility depends on the value of ρ in a quadratic fashion (up to a monotone transformation), and it is highest for $\rho = -1$, if $\lambda v > 0$. When $\rho = -1$, the highest utility is attained for the highest value of v , if $\alpha > \rho\lambda$. In other words, under the latter condition, if the manager can find an asset fully negatively correlated with the stock compensation, she will go for highest volatility. On the other hand, in case $\alpha < \rho\lambda$, $\rho = -1$, the manager wants as low volatility as possible. In the boundary case when $\alpha = \rho\lambda$, the manager wants to make the specific risk term $(1 - \rho^2)v^2$ small, and doesn't care about the systematic risk term ρ^2v^2 .

We next consider the case when the manager can control the systematic risk x and the specific risk y separately, where

$$x = v\rho, y = v\sqrt{1 - \rho^2}$$

We also assume that we can write

$$\alpha = \rho\alpha_x + \sqrt{1 - \rho^2}\alpha_y$$

for some constants α_x and α_y . In other words, the systematic risk premium α_x and the specific risk premium α_y are constant. We get the following conclusions, as in section 3.1:

- If the risky asset's risk premium is equal to the corresponding systematic risk premium of the output, that is, if $\lambda = \alpha_x$, the manager is indifferent between various choices for the systematic risk x .

- If $\alpha_x < \lambda$, then the manager likes as small systematic risk as possible, while she likes it to be high when $\alpha_x > \lambda$.

- The optimal value for the specific risk y is

$$y = \frac{1}{n\gamma_A} \alpha_y$$

That is, if the specific risk premium α_y is positive, the manager would like to choose lower specific risk with higher number of shares paid, and higher risk aversion.

A manager who cannot hedge would choose optimally the value $x_t \equiv \alpha_x / (n\gamma_A)$, and, the same as the hedging manager, the value $y_t \equiv \alpha_y / (n\gamma_A)$.

4.2.2 Call options contract

Assume

$$C_T = n(X_T - K)^+$$

Then we can compute

$$-\gamma_A V_A = e^{-T\lambda^2/2} (I_1 + I_2)^{\frac{1}{1-\rho^2}}$$

where

$$I_2 = Q(X_T < K) = N\left(\frac{1}{|v|\sqrt{T}} [K - b(v, \rho)]\right)$$

and

$$I_1 = e^{c(\rho)[K-b(v,\rho)]} E^Q \left[e^{-c(\rho)vW_T^Q} \mathbf{1}\{X_T > K\} \right] = e^{c(\rho)[K-b(v,\rho)] + \frac{1}{2}c^2(\rho)v^2T} \hat{Q} \left(b(v, \rho) - c(\rho)v^2T + v\hat{W}_T > K \right)$$

where \hat{Q} is the measure under which $\hat{W}_t := W_t^Q + c(\rho)vt$ is a Brownian motion. Thus, we can compute

$$I_1 = e^{c(\rho)[K-b(v,\rho)] + \frac{1}{2}c^2(\rho)v^2T} N\left(\frac{1}{|v|\sqrt{T}} [b(v, \rho) - c(\rho)v^2T - K]\right)$$

4.2.3 Put options contract

Assume

$$C_T = n(K - X_T)^+$$

Then we can compute

$$-\gamma_A V_A = e^{-T\lambda^2/2} (J_1 + 1 - I_2)^{\frac{1}{1-\rho^2}}$$

where

$$J_1 = e^{c(\rho)[b(v,\rho)-K] + \frac{1}{2}c^2(\rho)v^2T} Q^* \left(b(v, \rho) + c(\rho)v^2T + vW_T^* < K \right)$$

where Q^* is the measure under which $W_t^* := W_t^Q - c(\rho)vt$ is a Brownian motion. Thus, we can compute

$$J_1 = e^{c(\rho)[b(v,\rho)-K] + \frac{1}{2}c^2(\rho)v^2T} N\left(\frac{1}{|v|\sqrt{T}} [K - b(v) - c(\rho)v^2T]\right)$$

4.2.4 Comparative Statics

We now present comparative statics results obtained numerically in Excel using the above expressions, with the Excel spreadsheet available from the author by request. We use the following parameters:

$$u = 0, r = 0, \mu = 0.1, \sigma = 0.4, X_0 = 1, K = 1, T = 5.$$

Moreover, we choose γ_A so that $\lambda/(\gamma_A\sigma) = 0.7nX_0$. This means that the manager who maximizes her utility from investing in S and the risk-free account, disregarding the hedging of the compensation package, and starts with nX_0 , would invest 70% of that amount in S . In addition, we use $n = n_L = 0.001$ for the linear contract, $n_C = 10n$ for the call contract, and $n_P = 10n$ for the put contract, in order to make the manager's expected utility of the same order of magnitude across different contracts.

(i) Changing total volatility v . We only consider positive values for v . We set the correlation to $\rho = 0.5$, but the results are robust to this choice.

- For low values of the output's risk premium, $0 < \alpha < \lambda\rho$, the hedging manager's utility with the shares contract is decreasing in volatility on its positive domain, while with the call and the put contract it is increasing. However, as the strike price gets lower, the call utility becomes decreasing. For high values of the output's risk premium, $\alpha > \lambda\rho$, with the shares and call contracts it has a maximum, attained at a higher value for the calls than with the shares. The put contract utility is still increasing for the values considered. Calls make the utility increase more steeply when their downside protection is more useful, that is, when α is low.

Shares utility of the manager who does not hedge always has a maximum and thus she is less conservative than the hedging manager for low values of α . Options utilities are increasing, but less steeply than for the hedging manager, for all values of α .

(ii) Changing correlation ρ .

- With shares and call compensation the utility is maximized at $\rho = -1$, with put at 1.

(iii) Changing systematic risk $x = \rho v$. We set $y = 0.1$, but the results are robust to its value. Other parameters are the same as before. The range of x we consider is $[-0.25, 0.25]$.

- For the values in the low range of the systematic risk premium, $\alpha_x < \lambda$, shares utility for the hedging manager is decreasing in x , while put utility has a minimum, with the maximum attained at the highest value of x . For call, if α_y is not very large, the maximum is attained at the lowest negative value of x . However, if x is restricted to positive values, that is, the systematic risk cannot be shorted, call utility is also the highest at the highest value of x . On the other hand, if α_y is very large, the call utility becomes decreasing in systematic risk.

Compared to the previous, the manager who does not hedge, when compensated with shares, is less conservative in her choice of positive systematic risk (a maximum is attained

at $x = \alpha_x/(n\gamma_A)$), while with put options she still likes higher systematic risk in its positive domain, but her utility grows therewith less steeply than when she hedges. With call options, if the specific risk premium α_y is low, she still likes as high as possible systematic risk in its positive domain, but if α_y is moderately large, she likes it close to zero. The behavior can become completely different in the absence of hedging: for example, with $\alpha_x = 0.1$ and $\alpha_y = 0.5$, the call utility with hedging has a minimum close to zero, while without hedging it has a maximum close to zero.

- At the “no arbitrage” case $\alpha_x = \lambda$, shares utility for the hedging manager is constant in x , while call and put utilities are symmetric around zero and have a minimum at zero. The manager who does not hedge and receives shares likes the value $x = \alpha_x/(n\gamma_A)$, while with options compensation, with low values of α_y she likes high systematic risk in its positive domain, although with puts the increase in expected utility is minimal. The call utility is no longer decreasing in the negative domain of x , but increasing. Thus, the non-hedging manager may again be somewhat more conservative with options. Moreover, with high values of α_y the call utility has a maximum close to zero.

- In the range $\alpha_x > \lambda$, shares utility for the hedging manager is increasing in x , while call and put utility have a minimum. The maximum is attained at the highest value of x for call, and at the lowest for put. However, if x is restricted to positive values, put utility is also the highest at the highest value of x . Thus, all three types of compensation induce high systematic risk taking in the positive domain. For the non-hedging manager, the shares utility has again the same maximum point $x = \alpha_x/n\gamma_A$. The put utility has a minimum close to zero, and it increases slowly thereafter. With low α_y the call utility is increasing, less steeply than when hedging, while with high α_y it has a maximum close to zero. Thus, in all cases the non-hedging manager is more conservative.

(iii) Changing specific risk y . We set $x = 0.1$, and we comment below on other values. Other parameters are the same as before. The range of y we consider is $[0, 0.5]$.

- Case A: For most values of systematic risk premium α_x and specific risk premium α_y , the shares and the call utility for the hedging manager have a maximum, and the put utility is decreasing, the latter opposite to the case when total volatility or systematic risk is being modified. However, when systematic risk x is low enough, the put utility becomes increasing, as it should, because at $x = 0$ the specific risk becomes equal to the total volatility.

- Case B: As the values of α_y get sufficiently low relative to α_x , the call utility becomes decreasing in y , while the put utility has a minimum, attaining its maximum value at the highest value of specific risk y . However, when systematic risk x is low enough, the corresponding behavior remains the same as in Case A.

For the non-hedging manager, the shares utility has the same maximum point as with hedging. In the analogue of Case A the call utility also behaves similarly as with hedging, but the put utility may become increasing in the upper range of y values. Thus, in fact, the

manager compensated with puts may be more aggressive when not hedging. For example, this happens with $\alpha_x = 0.25$, $\alpha_y = 0.2$. On the other hand, this does not happen when x is close to zero, in which case the put utilities for the non-hedging and the hedging manager are similar. In the analogue of Case B, the put utility is always increasing, more steeply than for the hedging manager, again making the non-hedging manager at least as aggressive as the hedging one.



To summarize, in the presence of hedging opportunities, a CARA manager would have the following incentives:

- In terms of total volatility, when the output's risk premium is low, she is conservative when paid with shares, and aggressive when paid with calls and puts. When the output's risk premium is high, she is still most aggressive when paid with puts, but she also becomes less conservative when paid with shares. She is more conservative than the non-hedging manager when paid with shares if the output's risk premium is low, and less conservative when it is high. She is always less conservative than the non-hedging manager when paid in options.

- In terms of systematic risk, when the systematic risk premium is equal to the hedging asset's risk premium, shares provide no incentives, while calls and puts make the manager go for large systematic risk (in absolute value). With low systematic risk premium, puts make the manager aggressive, shares make her conservative, and with calls she likes highly negative systematic risk, or, if that is not possible, highly positive. With high systematic risk premium, shares and calls make the manager aggressive, and with puts she likes highly negative systematic risk, or, if that is not possible, highly positive. The comparison to the non-hedging manager is similar as in the case of changing total volatility – she is always less conservative except when paid in shares and the output's systematic risk premium is low.

- In terms of specific risk, for most cases the shares and the calls make the manager somewhat aggressive, while put contracts make her conservative (the latter only if systematic risk is not close to zero). However, when the specific risk premium is sufficiently low relative to the systematic risk premium, the incentives change in the opposite direction. Her level of aggressiveness is same or similar as for the non-hedging manager when paid with shares or calls, but she is typically more conservative than the non-hedging manager when paid with puts.

Overall, in most cases calls will induce higher risk taking than shares, and both will do that more so than in the absence of hedging, except when the output's risk premium is low. The puts also tend to induce higher risk taking than shares, except they might induce lowering of the specific risk; when paid with puts, the hedging manager is more aggressive with respect to total volatility and systematic risk, but tends to be less aggressive with

respect to specific risk.

5 Conclusions

We study risk-taking incentives when the manager can hedge her compensation payoff. When the manager assigns zero initial capital to hedging or has CARA preferences, there is a contract which is optimal regardless of whether the manager can hedge or not. The payoff of that contract may include the returns on the assets available for hedging, in order to preclude the manager from using them. We also compute incentive effects of compensating a CARA manager with shares, call options and put options. We find that options do, indeed, increase the appetite for risk in most cases, although put options may induce lowering of the specific risk. Moreover, the hedging manager is typically less conservative than the non-hedging manager, except when compensated with shares in case the output's risk premium is low, and when compensated with put options while affecting the output's specific risk.

In our analysis, we have assumed that the volatility choices of the manager are observable by the firm, or that we have CARA preferences and linear contracts, in which case observability of the volatility choices is irrelevant. It would be of interest, but not easy, to extend that framework to the moral hazard case of unobserved risk taking actions, with non-CARA preferences and nonlinear contracts.⁹ Moreover, as option compensation may be partly motivated by a need to distinguish between managers of varying abilities, one would like to see the incentive effects studied when the type of the manager is unknown, namely, the case of adverse selection. Similarly, the firm might not know what assets the manager has available for hedging, and in particular, it might not know the correlation between those assets and the firm's output.

It would also be of interest to test empirically implications of our analysis. In particular, our results predict that, when compensated by shares, the hedging managers have a similar attitude towards the specific risk as the non-hedging managers, while they may even prefer lower systematic risk, or, for more realistic values of risk premia, be indifferent with respect to systematic risk level. This is to be contrasted with Acharya and Bisin (2005), who find, in a CAPM equilibrium framework, that the managers would like to substitute the hedgeable systematic risk for the un-hedgeable specific risk. Another prediction is that, typically, the increased ability to hedge will induce higher managerial risk-taking when compensated with options than when compensated with shares. Finally, our results imply that in the presence of unobservable effort and modifiable specific risk, higher specific risk premium corresponds to higher incentives; on the other hand, if the manager is compensated only for the mean-

⁹Guo and Ou-Yang (2006) consider the case of non-CARA preferences with no hedging, and with linear contracts.

variance trade-off, her compensation's PPS depends neither on systematic nor on specific risk values nor on their premia.

6 Appendix

Proof of Proposition 3.2: The manager wants to maximize $E[U_A(C_T - G_T + H_T)]$, which, under our assumptions, is the same as maximizing $E[U_A(C_T - G_T + H_T - H_0B_T)]$. The standard martingale/duality approach to portfolio selection (see, e.g., Karatzas and Shreve 1997 for diffusion models, or Kramkov and Schachermayer (1999) for general semimartingale models) says that she will hedge so that

$$C_T - G_T + H_T - H_0B_T = I_A(yY_T) \quad (6.17)$$

where y, Y_T solves, over constant numbers y and SDFs $Y_T \in \mathcal{Z}$, the dual problem

$$\bar{V}(H_0) := \min_{y, Y} E[U_A(I_A(yY_T)) - yY_T I_A(yY_T) + yY_T(C_T - G_T) + yY_T(H_T - H_0B_T)] \quad (6.18)$$

if a solution exists, and if we have $V(H_0) = \bar{V}(H_0)$. The last term in (6.18) disappears, because $E[Y_T(H_T - H_0B_T)] = 0$. Then, if (3.7) is satisfied, it is easily seen that the above is minimized for $yY_T = zZ_T$. From (6.17) this implies $H_T = H_0B_T$. Conversely, if the optimal hedging strategy results in $H_T = H_0B_T$, we see from (6.17) that C_T has to be of the form (3.7).

■

Computations for Sections 3.1 and 3.2: Assume model (3.13) and that the manager is given a contract of the form $C_T = aX_T + b \log(S_T) + c$. Denote by \tilde{c} the certainty equivalent (CE) of c and with \tilde{R} the CE of the manager's reservation utility. Also denote by π_A the amount of capital manager invest in S , and by π_P the analogous hedging amount for the firm. Given that at the optimum π_A, π_P and effort u are constant in our framework, the manager's CE is then

$$\tilde{c} + a[X_0 + \alpha_y y T + \delta u T] + [\alpha_x a x + \lambda \sigma \pi_A - g(u)] T + b[\log S_0 + (\mu - \frac{\sigma^2}{2}) T] - \frac{\gamma_A T}{2} [(ax + \pi_A \sigma + b\sigma)^2 + a^2 y^2]$$

Doing maximization over u, π_A and y we get

$$ax + \sigma \pi_A + b\sigma = \frac{\lambda}{\gamma_A}$$

$$y = \frac{\alpha_y}{a\gamma_A}$$

$$g'(u) = a\delta$$

This means that the manager's CE is

$$\begin{aligned} \tilde{R} = \tilde{c} + a[X_0 + (\frac{\alpha_y^2}{a\gamma_A} + \delta u)T] - g(u)T + (\alpha_x - \lambda)ax + \lambda[\frac{\lambda}{\gamma_A} - b\sigma]T + b[\log S_0 + (\mu - \sigma^2/2)T] \\ - \frac{T}{2}\gamma_A[(\frac{\lambda}{\gamma_A})^2 + (\frac{\alpha_y}{\gamma_A})^2] \end{aligned}$$

The firm's CE is

$$\begin{aligned} -\tilde{c} + (1-a)[X_0 + (\frac{\alpha_y^2}{a\gamma_A} + \delta u)T] + [\alpha_x(1-a)x + \lambda\sigma\pi_P]T - b[\log S_0 + (\mu - \sigma^2/2)T] \\ - \frac{T}{2}\gamma_P[((1-a)x + \pi_P\sigma - b\sigma)^2 + (1-a)^2(\frac{\alpha_y}{a\gamma_A})^2] \end{aligned}$$

Fixing \tilde{R} and computing \tilde{c} from the previous expression we get the firm's CE as

$$\begin{aligned} -\tilde{R} - g(u)T + (\alpha_x - \lambda)ax + \lambda[\frac{\lambda}{\gamma_A} - b\sigma]T - \frac{T}{2}\gamma_A[(\frac{\lambda}{\gamma_A})^2 + (\frac{\alpha_y}{\gamma_A})^2] + [X_0 + (\frac{\alpha_y^2}{a\gamma_A} + \delta u)T] \\ + [\alpha_x(1-a)x + \lambda\sigma\pi_P]T - \frac{T}{2}\gamma_P[((1-a)x + \pi_P\sigma - b\sigma)^2 + (1-a)^2(\frac{\alpha_y}{a\gamma_A})^2] \end{aligned}$$

This means that the hedging portfolio π_P is chosen so that

$$\gamma_P[(1-a)x + \pi_P\sigma - b\sigma] = \lambda$$

and hence the firm needs to maximize

$$\delta u - g(u) - \lambda b\sigma + \frac{\alpha_y^2}{a\gamma_A} + (\alpha_x - \lambda)x + \lambda\sigma b - \frac{\gamma_P}{2}(1-a)^2(\frac{\alpha_y}{a\gamma_A})^2$$

We see that the principal is indifferent with respect to the choice of b . In case there is no effort u , $u = g(u) = 0$, we get

$$a = \frac{\gamma_P}{\gamma_A + \gamma_P}$$

In case $g(u)$ is quadratic, taking into account that $g'(u) = \delta a$, numerical computations show that the optimal a decreases as the squared specific risk premium α_y^2 decreases.

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