Homework 8: due 12/2/08

1. Recall a simple rule is a rule for which there exists a proper and monotonic collection $\mathcal{D}$ for which for all $\theta \in \Theta$, and all $x, y \in X$,

$$xP(\theta)\,y \Longleftrightarrow \text{there exists } G \in \mathcal{D} \text{ for which } G \subset P(x, y|\theta).$$

Recall that this can equivalently be defined as

$$P(\theta) = \bigcup_{G \in \mathcal{D}} \bigcap_{i \in G} P_i(\theta).$$

Suppose that $N \in \mathcal{D}$.

a) Show that

$$R(\theta) = \bigcap_{G \in \mathcal{D}} \bigcup_{i \in G} R_i(\theta).$$

b) Let $\mathcal{B}$ be the set of coalitions B for which for all $G \in \mathcal{D}$, $B \cap G \neq \varnothing$. Show that

$$R(\theta) = \bigcup_{B \in \mathcal{B}} \bigcap_{i \in B} R_i(\theta).$$

c) The set $\mathcal{B}$ is called the set of *blocking* coalitions. Show that the set of blocking coalitions is monotonic and strong.

d) Show that $B \in \mathcal{B} \Longleftrightarrow N \backslash G \notin \mathcal{D}$

e) What are the blocking coalitions for simple majority? Dictatorship? Pareto extension rule?

2. A rule is a q rule if and only if there exists some integer q for which the decisive coalitions are $\mathcal{D}_q = \{G : |G| \geq q\}$. (note that $q > \frac{|N|}{2}$ is necessary for this to be a simple PAR). Provide a generalization of May's theorem axiomatizing the family of simple q rules.

3. Suppose we have two simple PAR's. Associated with each simple PAR is a set of decisive coalitions (satisfying properness and monotonicity): say $\mathcal{D}_1$ and $\mathcal{D}_2$ in this case.

a) Can $\mathcal{D}_1 \cap \mathcal{D}_2$ be the set of decisive coalitions for some simple PAR? If so, what is this PAR?

b) Can $\mathcal{D}_1 \cup \mathcal{D}_2$ be the set of decisive coalitions for some simple PAR? If so, what is this PAR?