

Homework assignment #6: due 11/13/08

1. Imagine an Anscombe-Aumann environment, where Z is the set of alternatives, $\Delta(Z)$ the set of lotteries, and suppose that the state space has a product structure, so that $\Omega = \Omega_1 \times \Omega_2$. Suppose R defined over $\Delta(Z)^\Omega$ satisfies the AA axioms. What additional axioms are necessary to imply that the probability measure associated with the representation of R satisfies statistical independence across Ω_1 and Ω_2 (that is for all $E \subset \Omega_1$ and $F \subset \Omega_2$, $p(E \times F) = p_1(E)p_2(F)$ for some p_1, p_2 ?)

2. Suppose a Savage environment: Ω and Z are given. Also given are n preference relations $\{R_i\}$, each satisfying the Savage axioms (and hence each possessing a probability measure p_i and a (cardinally unique) utility index u_i).

Suppose we consider the preference R_0 which is represented by the utility function $U_0 : \mathcal{F} \rightarrow \mathbb{R}$ given by

$$U_0(f) = \sum_{i=1}^n \left(\int_{\Omega} u_i(f(\omega)) dp_i(\omega) \right).$$

a) Which of the Savage axioms are generally satisfied by such a preference, and which are violated (provide examples for those which may be violated)

b) answer part a supposing that $p_i = p_j$ for all i, j

c) answer part a supposing that $u_i = u_j$ for all i, j .

3. Suppose a finite-states (anscombe aumann) environment, where $Z = \mathbb{R}$, and R on the set of acts conforms to the AA axioms. Further suppose that we know that the utility index in the representation is risk-neutral (that is, for any lottery $p \in \Delta(Z)$, $u(p) = E[p]$ (the expectation of p)).

We now wish to offer a menu of choices to the decision maker which reveal her probability measure. Specifically, we wish to imagine a function $g : \Delta(\Omega) \rightarrow \mathcal{F}$ for which for all $p, q \in \Delta(\Omega)$ for which $p \neq q$,

$$\int_{\Omega} g(p)(\omega) dp(\omega) > \int_{\Omega} g(q)(\omega) dp(\omega).$$

a) Interpret the preceding inequality and explain why a function g as discussed here allows us to recover the probability measure p .

b) Suppose there are only two states $\Omega = \{1, 2\}$. Can you give an example of such a function g ? How about a characterization of all such functions? (It's probably best to draw a picture here).

4. We talked before about conditional preference: note that for an event $E \subset \Omega$, a conditional preference (for a SEU decision maker) can meaningfully be defined as

$$fR_Eg \iff (fEh)R(gEh)$$

for any $h \in \mathcal{F}$.

- a) Show that if R satisfies the Savage axioms and that E is non-null with respect to R , then R_E satisfies the Savage axioms as well.
- b) What is the probability measure associated with R_E ?
- c) Can a conditional preference be meaningfully defined for a multiple priors decision maker in the same fashion? If yes, prove this, if not provide a counterexample.