Homework #5: due 11/06

1. A binary relation that satisfies P2 on the set of acts admits a well defined notion of preference conditional on some event E obtaining, whereby for an event $E \subset \Omega$ and acts $f, g \in \mathcal{F}$, $f R_E g \iff (fEh)\, R\, (gEh)$ for some $h \in \mathcal{F}$.

Which axioms are necessary to establish the result that for all $E, F \subset \Omega$ for which $E \cap F = \varnothing$, $f R_E g$ and $f R_F g$ imply $f R_{E \cup F} g$?

2. Is there a way to define conditional preference if P2 is not satisfied?

3. A decision maker faces two choices. After choosing an act, she faces another choice from a set A. For any $a \in A$ that she chooses, her preference over acts $\mathcal{F}$ is subjective expected utility (with the same probability measure across all $a \in A$). When she chooses among acts, she knows that she will choose from A optimally. Consequently, she has a utility representation over acts of the form

$$U(f) = \sup_{a \in A} \int_{\Omega} u(f(\omega), a)\, dp(\omega).$$

All we observe in this formulation is a preference over acts. Is there a way to determine the measure p from such observations?

4. Given Savage's axioms, define the likelihood relation on events $\succeq^l$ by $E \succeq^l F$ if there exists $x, y \in Z$ for which xPy and $(xEy)\, R\, (xFy)$. Verify the following properties:

a) $\succeq^l$ is a weak order

b) For all $E \subset \Omega$, $E \succeq^l \varnothing$

c) $\Omega \succ^l \varnothing$

d) For all $A, B, C \subset \Omega$, if $A \cap C = B \cap C = \varnothing$, then $\left(A \succeq^l B\right) \iff \left(A \cup C \succeq^l B \cup C\right)$.

e) For all $A, B \subset \Omega$, if $B \succ^l A$, then there exists a finite partition $\{C_1, ..., C_m\}$ of Ω for which for all $i = 1, ..., m$, $B \succ^l A \cup C_i$.

(these conditions are sufficient for $\succeq^l$ to be represented by a probability measure–you do not need to prove this).