

Homework 4: due 10/30/08

1. Kreps, Chapter 5, #3 (prove independence of four axioms: completeness, transitivity, continuity, and independence).

2. Suppose that $|X|$ is finite, and that there is some set Λ for which for all $\lambda \in \Lambda$, $U_\lambda : \Delta(X) \rightarrow \mathbb{R}$ is an expected utility functional (that is $U_\lambda(\alpha p + (1 - \alpha)q) = \alpha U_\lambda(p) + (1 - \alpha)U_\lambda(q)$ for all λ, p , and q).

Define the binary relation R by

$$pRq \iff \sup_{\lambda \in \Lambda} U_\lambda(p) \geq \sup_{\lambda \in \Lambda} U_\lambda(q).$$

a) Suppose that Λ is some exogenous set and we rewrite $U_\lambda(p)$ as $U(\lambda, p)$. Explain now how $\sup_{\lambda \in \Lambda} U_\lambda(\lambda, p)$ is the utility of choosing p if, after choosing p , we are to choose from Λ .

b) Show that any such preference satisfies the following weakening of the independence axiom:

If qRp , then $[\alpha q + (1 - \alpha)p]Rp$.

Interpret this axiom.

3. Suppose $X = \{1, 2, 3, 4, 5\}$, and define $U(p) = \prod_{x \in X} x^{p(x)}$.

a) Verify that U represents an expected utility preference.

b) Give an example of a utility index which represents this preference.

4. Suppose that $|X| < \infty$. Suppose that $U : \Delta(X) \rightarrow \mathbb{R}$ is affine (that is, for all α, p, q , $U(\alpha p + (1 - \alpha)q) = \alpha U(p) + (1 - \alpha)U(q)$).

Define $u : X \rightarrow \mathbb{R}$ by $u(x) = U(\delta_x)$.

Show that $U(p) = \sum_{x \in X} p(x)u(x)$.