Homework–due 10/23

The Kreps exercises are from Chapter 3
1. Kreps #1
2. Kreps #6(e)

3. Given is X, and a function $u : X \to \mathbb{R}^n$, where $n \in \mathbb{N}$. Define xRy if $u(x) \geq u(y)$ (here, $u(x) \geq u(y)$ means $u_i(x) \geq u_i(y)$ for all $i = 1, ..., n$). Show that R is reflexive and transitive.

Now suppose that $|X| < +\infty$, and that R on X is reflexive and transitive. Show that there exists $n \in \mathbb{N}$ and $u : X \to \mathbb{R}^n$ such that $xRy \Longleftrightarrow u(x) \geq u(y)$.

4. Suppose that R is reflexive and transitive on X. Prove that there exists $u : X \to \mathbb{R}$ for which for all $x, y \in X$, $xRy \Longrightarrow u(x) \geq u(y)$ and $xPy \Longrightarrow u(x) > u(y)$. How does this type of representation differ from that in question 2?

5. Suppose that $\psi : X \times X \to \mathbb{R}$ is a function, and define xRy if $\psi(x, y) \geq 0$. When are the following properties true?

i) Reflexive?
ii) Complete?
iii) Symmetric?
iv) Transitive?

Now show that for any binary relation R, there exists $\psi : X \times X \to \mathbb{R}$ for which $\psi(x, y) \geq 0 \Longleftrightarrow xRy$. Is it necessarily the case that $\psi(x, y) > 0$ whenever xPy?