

Homework #2: due 10/16

1. Suppose X is a finite set, and let R be a complete, P -acyclic relation on this set. Prove the following results.

a) R is transitive if and only if for all $A \in 2^X \setminus \{\emptyset\}$, for all $y \notin M(R, A)$, for all $x \in M(R, A)$, we have xPy .

b) R is quasitransitive if and only if for all $A \in 2^X \setminus \{\emptyset\}$, for all $y \notin M(R, A)$, there exists $x \in M(R, A)$ for which xPy .

c) Find an example of a complete, P -acyclic R and some A for which there exists $y \notin M(R, A)$ such that for all $x \in M(R, A)$, xIy .

2. Suppose $X = C([0, 1])$, the set of continuous, real-valued functions on the unit interval. Define fRg if there exists $x \in [0, 1]$ for which $f(x) \geq g(x)$.

a) What is the asymmetric part of R ?

b) Show that R is not transitive but that it is quasi-transitive.

c) What is the transitive closure of R ?

3. Given R , demonstrate that the asymmetric part of the transitive closure is acyclic; that is, show that P_{T_R} is acyclic. What about T_{P_R} (the transitive closure of the asymmetric part)?

4. Given is $u : X \rightarrow \mathbb{R}$. We define a binary relation R on X by xRy if $u(x) \geq u(y) + 1$. Which of the properties hold in general?

a) completeness

b) reflexivity

c) transitivity

5. Show that if R is complete and transitive, then for all $x, y, z \in X$, xRy and yPz imply xPz . Does this statement hold without completeness?

6. Suppose that R is given, and suppose that I and P are both transitive. Does it follow that R is transitive? What about if R is complete?

7. Suppose R is complete and transitive. Define a "price function" $\pi : X \rightarrow \mathbb{R}$ such that $\pi(x) > 0$ for all $x \in X$. Define $u(x) = \inf \{\pi(y) : yRx\}$. Show that xRy implies $u(x) \geq u(y)$. Is it necessarily true that xPy implies $u(x) > u(y)$?