

Homework 1: due 10/9/08

1. Austen Smith & Banks, exercise 1.2
2. Austen Smith & Banks, exercise 1.4
3. Suppose X is a finite set, and that R and R' are both weak orders on X . Define the choice function $c : 2^X \rightarrow 2^X$ by

$$c(A) \equiv \arg \max_R \left\{ \arg \max_{R'} A \right\}.$$

a) Interpret this choice function, in the case in which R and R' are intended to be preferences of individuals.

b) Is c generally rationalizable by a weak order? If no, provide a counterexample. If yes, provide a proof *and* explain how this weak order relates to R and R' .

4. On dominant and maximal elements.

Given R , a binary relation, define the following two sets:

$$\begin{aligned} D(A, R) &= \{x \in A : \text{for all } y \in A, xRy\} \\ M(A, R) &= \{x \in A : \text{for all } y \in A, yRx \implies xRy\}. \end{aligned}$$

(Note: Austen-Smith and Banks define $D(A, R)$ by $M(A, R)$, so be careful here).

D is the set of dominant elements; M the set of maximal elements.

- a) Show that in general, either of these sets can be empty
- b) Prove that $D(A, R) \subset M(A, R)$
- c) Show that the statement $D(A, R) = M(A, R)$ is false in general
- d) Show that if R is complete, then $D(A, R) = M(A, R)$
- e) Show that, for a given choice function $c : 2^X \setminus \{\emptyset\} \rightarrow 2^X \setminus \{\emptyset\}$, there exists R for which $c(A) \equiv D(A, R)$ iff there exists R' for which $c(A) \equiv M(A, R')$ (that is, a choice function is dominant rationalizable iff it is maximal rationalizable). (Hint: if $c(A) = M(A, R')$, define R as $xRy \iff \neg yP'x$).

5. Suppose that instead of observing all possible choice sets $2^X \setminus \{\emptyset\}$, we observe only a subset $\mathcal{B} \subset 2^X \setminus \{\emptyset\}$. That is, $c : \mathcal{B} \rightarrow 2^X \setminus \{\emptyset\}$, where $c(B) \subset B$ for all $B \in \mathcal{B}$.

a) Show that in general, the weak axiom is not equivalent to rationalization by a weak order in this case (that is, provide an example of a choice function satisfying the weak axiom but which is not rationalizable by a weak order).

b) Show that the base relation may be empty in such an environment.

c) Explain why the result in 4e no longer holds in this more general environment. This requires an example. (Hint: Anything which is maximal rationalizable remains dominant rationalizable by the same argument as in 4e. So, note that $D(B, R)$ can be nonempty for all $B \in \mathcal{B}$ even if R is not complete and establish the existence of c which is dominant rationalizable but not maximal rationalizable).

6. The testable implications of standard choice theory.

In general, we may not observe all possible choices an agent might make. Instead, we typically only observe a subset of choices they might make. To this end, say that a choice function c is *subrationalizable* if there exists a binary relation R for which for all A , $c(A) \subset D(R, A)$.

- a) Show that any choice function is subrationalizable by a weak order.
- b) When is it the case that c is subrationalizable by a linear order?
- c) Suppose we have given another function, $e : 2^X \setminus \{\emptyset\} \rightarrow 2^X \setminus \{\emptyset\}$ for which $e(A) \subset A$. We interpret e as being a set of alternatives which could never be chosen. Thus, assume that for all A , $c(A) \cap e(A) = \emptyset$. We now ask for the existence of a weak order R for which

$$\begin{aligned}c(A) &\subset D(R, A) \\ D(R, A) \cap e(A) &= \emptyset.\end{aligned}$$

Provide some sufficient conditions on the pair (c, e) so that there exists R for which the preceding two equalities hold. (There are some obvious such conditions; try to make them as general as possible. If you can make them necessary as well that would be ideal).