Homework #2 Psychology 101
Spr 03
Prof Colin Camerer

This is available Monday 28 April at 130 (in class or from Karen in Baxter 332, or on web) and due Wednesday 7 May at 130 (in class or to Karen). Collaboration policy: Open book. Work by yourself. After you complete answers you may check them with others, but then continue to work alone. Also show your work when it is useful for us to see that you are working through algebraic steps correctly. (Also there is partial credit for a correct sequence of steps even if you get the wrong answer after an algebra slip; but there is even *more* credit if you check your work and find the slips yourself!).

Most of these questions involve Fehr-Schmidt type utilities. Assume throughout that $0 \leq$, α, β <1 and $\alpha>\beta$unless otherwise stated.

1. Here is a PD game

	C	D
C	3,3	0,5
D	5,0	1,1

(a) Suppose there are players whose utilities are given by the Fehr-Schmidt formula (Camerer, chapter 2) and regular players (with $\alpha=\beta=0$). They play the PD simultaneously. What do regulars do? Suppose that there is a fraction q of Fehr-Schmidt players who all have the same α and have the same β (both strictly positive and weakly less then one). Find conditions on q, α, β so that there are two pure-strategy equilibria: One in which all players choose D; and one in which Fehr-Schmidt players choose C and regulars choose D.

(b) Find a condition on q, α, β such that for Fehr-Schmidt players, the equilibrium in which they play C (and regulars play D) is better in utility terms than the all-D equilibrium.

(c) When q is large enough to satisfy the two-equilibrium condition in (a), how does this game—played in *utilities*-- compare to games in chapter 7?

2. Income variation. Consider a situation with three players and potential earnings (x, x+d, x-d) (for players 1-3 respectively, d>0). Express the utility for this vector of the first player (i.e., the player who earns x) under various theories:

(a) Pure self-interest
(b) Bolton-Ockenfels
(c) Fehr-Schmidt
(d) Charness-Rabin "me-min-us" theory (utility is a weighted sum of own, minimum, and total payoffs)
(e) In many societies voters vote for progressive taxation (i.e., the *marginal tax rate*-- the rate at which one more dollar of income is taxed—goes up with income). This can be crudely be described as whether reducing the degree of inequality d in the example above is preferred by player 1 (who is "middle class"). Can any of the theories (a-d) explain middle-class voters' desire for progressive taxation? Explain briefly.

(f) Suppose you "clone" the second and third players so there is a fourth and fifth player with the same payoffs as the second and third (i.e. two players with payoffs x-d and two with payoffs x+d). Which of your answers to (a-d) change and which don't? Are any of the four types of players happier living in a "group" with five players rather than 3?

3. Price-matching with loyalty. Consider the price-matching game we discussed in class: Two firms, labelled 1 and 2, choose prices P_1 and P_2 simultaneously from an interval [100,200] (L,H>0) in integer increments. Both firms earn min(P_1,P_2). The firm naming the lower price also earns a reward R>1 and the firm naming the higher price suffers a penalty R. (If they name the same price the reward and penalty cancel out and they just earn the minimum price.)

(a) Suppose the firms have Fehr-Schmidt utilities and all have the same value of α, β. First show that if $\alpha=\beta$, the unique equilibrium is for both firms to choose 100.
(b) Now show conditions (α, β values for a given R) under which you can get tacit collusion at a price above 100. What prices are equilibria?
(c) Suppose there are many firms and each firm can have either a large enough value of β to support collusion (as in part b) or $\beta=0$, and each value is equally likely. Suppose the $\beta=0$ firms always choose 100. Sketch an intuitive argument about why, as the number of firms increases, the probability that the low price will be 100 converges to 1. (You don't have to solve this formally; too messy since as the number of firms grows there is a combinatorial explosion.)

4. Trust game. Assume Fehr-Schmidt preferences again. Assume an investor invests x which becomes 3x. Explain why the trustee would never repay more than half (i.e., 1.5x). Explain how much the trustees would repay various values of α, β.

5. A player is participating in one of two games (decisions actually) denoted 1 and 2

	Petunia's payoff	game 1 Elmer's payoff	game 2 Elmer's payoff
A	6	5	1
B	5	0	5

Petunia must pick A or B; she gets 6 from A and 5 from B regardless of which game she plays. If the game is #1, Elmer gets 5 from A and 0 from B; if the game is #2, Elmer gets 1 and 5 respectively. The two games are equally likely.

(a) Suppose Petunia knows which game she is playing (i.e., she knows how much Elmer will get from her Dictator decision) and is a Fehr-Schmidt player. What will she choose in game 1? What will she choose in game 2? (Your answers can depend on values of α, β, of course but for this example assume they are strictly between zero and one).

(b) Now suppose Petunia does not know which game is being played; so she has to choose either A or B without knowing exactly what Elmer's payoff will be (but knowing that games 1 and 2 are equally likely). First calculate her expected utility (i.e., the Fehr-Schmidt utilities from each payoff allocation multiplied by the probability .5 that each game is the one which was played) from choosing A or B before knowing which game is being played.

(c) Now calculate her expected utility if she gets to find out which game is being played first (i.e., she does not have to make the same choice A or B in the two games).

(d) Comparing (b) and (c), for what parameter values should Petunia want to know which game she is playing before she makes her choice?

(e) Empirical fact(oid): In experiments (from Caltech Ph.D. Roberto Weber, now at Carnegie) 100% of subjects pick A in game 1, about 60% pick B in game 2, but 60% never want to know which game they are playing if they are uncertain (as in part b), and those players always pick A. How do you explain this pattern? What does it tell us about the combination of Fehr-Schmidt social utilities and "backward induction" (i.e. subgame perfection) as modelling principles (briefly speculate)?

5. Consider the game of matching pennies

	H	T
H	1,0	0,1
T	0,1	1,0

(a) Suppose the players have Fehr-Schmidt utilities. Write a game matrix in which the α, β terms modify the money payoffs in the table. Compute the mixed-strategy equilibrium with $\alpha=\beta=0$ and with α, β varying between 0 and 1.

(b) Suppose the players have Charness-Rabin "me-min-us" utilities (they maximize a weighted average of own payoff, minimum payoff, and total payoff). Does switching from pure self interest (maximizing own payoff only) to me-min-us make *any difference* at all for the mixed-strategy equilibrium? (Hint: This is deceptively simple. Don't overthink it.)

(c) Suppose a critic said, "I don't believe all this stuff about fairness, Fehr-Schmidt, Rabin-Charness etc. In our experiments on competitive games like matching pennies we never see any evidence that those kinds of psychobabble [actual term used in a paper by a famous theorist and colleagues] things matter". How would you—pithily [i.e., briefly]—reply to this criticism?

6. Cognitive hierarchy & betting. Recall the betting game from the first day of class, shown below. In each case a player has an "information set" (a set of possible states; e.g., if the state is A player 1 has the information set (A,B) which means they don't know whether the true state is A or B). If both players choose to bet the payoffs are as shown in the table.

	states			
	A	B	C	D
	(A,B)		(C,D)	
Player 1 payoff	+32	-24	+20	-16

Player 2 payoff	-32	+24	-20	+16
Player 2 info sets	A	(B, C)		D

(a) Show that the Nash equilibrium creates no state in which both players bet. (Apply dominance iteratedly).

(b) Take the cognitive hierarchy model of Camerer and Ho (get the paper from my website http://www.hss.caltech.edu/~camerer/camerer.html; it is "A cognitive hierarchy theory of one-shot games"..."Short version here" (you are welcome to read the long version but it has much more material than you need to do the homework) or from Karen Kerbs). The model recursively creates decisions by 0, 1,... types. 0's randomize. Higher level types use the true distribution of frequencies of types but truncates types at lower levels (e.g., 1's think there are only 0's, 2's think there are only 1's and 0's, etc.) and normalizes so the perceived frequencies add to one. Use a τ value of 1.5. Go only up to level 4 and then normalize the overall predicted betting frequencies (i.e., add up the frequencies for the types 0-4 and then divide by $1-\Sigma_{k=0}^{4} f(k)$.) Compute the betting rates for the various information sets that are predicted by the CH model. (Note: This is boring numerical computation once you grasp the basic concept. You can do it with a calculator or but feel free to use Mathematica or a simpler package like Excel to do the computations if you are a fast programmer; same for part (d) of problem 7 below.)

7. Cognitive hierarchy (CH) & "cloning" (inessential transformations).
Consider the modified matching pennies game below.

	L	R
T	0,1	1,0
B	1,0	0,1
B'	1,0	0,1

(a) First consider the truncated version in which there is no strategy B' (the only row strategies are T and B). What is the Nash equilibrium?

(b) In the truncated game (no B') what does the CH model predict for various τ? Does the prediction depend on the average thinking steps τ?

(c) Now consider the expanded game with the "cloned" strategy B'. What are the Nash equilibria? (Hint: There is a range of them which are equivalent in payoff implications but not exactly equivalent in which of the cloned strategies are played.)

(d) Compute the prediction of the CH model when τ=1.5 and (as in problem 6) you truncate the iteration at level 4.