

PSY101: The Psychology of Strategic Thinking
Solutions for Homework Set 1

- 1a) The pure strategy Nash equilibria are {(Dare, Back Down), (Back Down, Dare)}. For a mixed strategy Nash equilibrium, a player must be indifferent between any strategies he chooses to play, because if this were not the case our concept of equilibrium would be violated as he could earn a higher payoff by choosing one strategy all time. That means we must find $p = \text{prob}(\text{Dare})$ such that

$$?(\text{Dare}) = ?(\text{Backdown})$$

which implies

$$-3p + 2(1-p) = 0p + 1(1-p)$$

so $p = .25$ and our mixed strategy equilibria is

$$(.25 \text{ Dare} + .75 \text{ Back Down}, .25 \text{ Dare} + .75 \text{ Back Down}).$$

- 1b) The pure strategy Nash equilibria are {(Dare, Back Down), (Back Down, Dare)}. Only (Dare, Back Down) is subgame perfect.

- 2) Here we assume that C , B , and F are positive. If $C > 4 + F$ then a pure strategy Nash equilibria exists (Trust, Cheat). Otherwise the solution is a mixed strategy equilibria. The probabilities can be determined using the same method as above, namely finding the probability of Audit such that the taxpayer is indifferent between Cheat and Obey, and the probability of Cheat such that the IRS is indifference between Audit and Trust. We obtain the equilibria

$$(B/(F+B)\text{Audit} + F/(F+B)\text{Trust}, C/(F+4)\text{Cheat}, (4+F-C)/(F+4)\text{Obey})$$

So the probability of cheating depends on the fine F but not the benefits B . This is an unusual property of mixed strategy equilibrium that the probability of choosing a particular action is determined only by the payoffs of the other player.

- 3a) To determine the Nash Equilibrium we to find the optimal level of output for each firm given the choices of the other firm. Since in our model prices are a pure function of a quantity produced, we maximize the firm's profit with respect to quantity. The profit function is

$$?(Q_a) = Q_a(120 - Q_a - Q_b)$$

By differentiating with respect to Q_a we get the first order conditions

$$120 - 2Q_a - Q_b = 0$$

which implies

$$Q^*_a = 60 - .5Q_b$$

And similarly

$$Q^*_b = 60 - .5Q_a$$

Then solving this simultaneous system of equations yields the results

$$Q^*_a = Q^*_b = 40 ; p^* = 120 - 40 - 40 = 40 ; \pi(A) = \pi(B) = 1600$$

- 3b) Without loss of generality we assume Firm A goes first. Firm A knows Firm B's optimal choice of output given Q_a , shown above, and uses this to find its optimal level output.

$$\begin{aligned} \pi(Q_a) &= Q_a(120 - Q_a - [Q^*_b \text{ given } Q_a]) \\ \pi(Q_a) &= Q_a(120 - Q_a - [60 - .5Q_a]) \\ \pi(Q_a) &= Q_a(60 - .5Q_a) \end{aligned}$$

Now we after differentiation we obtain the first order conditions

$$60 - Q^*_a = 0$$

which tells us

$$Q^*_a = 60 ; Q^*_b = 30 ; p^* = 30 ; \pi(A) = 1800 ; \pi(B) = 900$$

So it does pay to move first. This is a Stackelberg model.

- 3c) Here the cartel is able to control the total output $Q = Q_a + Q_b$ and thus maximizes the equation

$$\pi(Q) = Q(120 - Q)$$

which leads to

$$Q^* = 60 ; Q^*_a = Q^*_b = 30, p^* = 60, \pi(A) = \pi(B) = 1800$$

So clearly it pays to join a cartel as the profits are greater than under Cournot competition. To determine if the firm should cheat we want to determine the optimal level of output for firm A given that Firm B produces 30 units.

Thus we maximize

$$\pi(Q_a) = Q_a(120 - 30 - Q_a) = Q_a(90 - Q_a)$$

So

$$Q^*_a = 45 > 30$$

So there is an incentive to expand production and “cheat” on the cartel.

- 4) The subgame perfect Nash equilibrium is (Enter, Give Up). The strategy set (Stay Out, p Fight + $(1-p)$ Give Up) is a Nash equilibrium for $p > 10/11$, but is not subgame perfect.

In problem 6, mixed strategy equilibria can be calculated using the same methods as in problem 1 and problem 2. Pure strategy equilibria can be determined by inspection.

- 5a) This solution is very close to the one given by Dylan Simon. Denote the location of the first firm by x and the location of the second firm by y . Without loss of generality we can assume $x \leq y$. In equilibrium, the third firm will choose a location z which maximizes its earnings π_z given x and y .

- $z = x - \frac{1}{2}$ if $y = 3x$ and $x + y = 100$

after which $\pi_x = .5(y - x + \frac{1}{2})$, $\pi_y = .5(200 - x - y)$, $\pi_z = x - .5$

- $z = .5(x + y)$ if $y = 3x$ and $x + 200 = 3y$

after which $\pi_x = .25(3x + y)$, $\pi_y = .25(400 - 3y - x)$, $\pi_z = .5(y - x)$

- $z = y + \frac{1}{2}$ if $x + 200 = 3y$ and $x + y = 100$

after which $\pi_x = .5(x + y)$, $\pi_y = .5(y + \frac{1}{2} - x)$, $\pi_z = 100 - y - .5$

Then, using these best responses for firm 3 and given that the first firm has chosen location x , the second firm maximizes its earning by choosing

- $y = (200 + x)/3$ if $x \leq 25$

after which $\pi_x = (200 + 10x)/12$, $\pi_y = (100 - x)/2$

- $y = 100 - x$ (or something fractionally larger) if $x > 25$

after which $\pi_x = .5(100 - 2x + \frac{1}{2})$, $\pi_y = .5(200 - x - (100 - x)) = 50$

So we can see firm 1 maximizes its profits by choosing $x = 25$, after which firm 2 maximizes its profits by choosing $y = 75$, after which firm 3 weakly maximizes its profits by choosing $z = 50$. Therefore $(25, 75, 50)$ is an equilibrium.

- 5b) The outcome come from collusion is still $(25, 75, 50)$, the same result we obtained in part a. We can prove this result as following. The total earnings of the firms in

this economy must always add up to 100. This means the first two firms can maximize their profit by minimizing the profit of the third firm. Above we showed that under the location triplet (25, 75, 50), the third firm will earn 25. For the third firm to be worse off under collusion, it must earn less than 25. This means firms 1 and 2 must be located at locations $x = 25$ and $y = 75$, if they chose other locations firm 3 could position itself at $z = 25$ or $z = 75$ and earn more than 25. However, if $x < 25$ and $y > 75$ then firm 3 can earn more than 25 by moving anywhere in the middle. Therefore the outcome (25,75,50) is an equilibrium even under collusive behavior by the first and second firms.

- 6a) There are no pure strategy equilibria. A mixed strategy equilibrium exists and is given by

$$(155/293T + 101/586M + 175/586B, 155/293T + 101/586M + 175/586B)$$

- 6b) The pure strategy equilibria are {(B,B), (T,M), (M,T)}. The mixed strategy equilibrium is

$$(2/7T + 5/7M, 2/7T + 5/7M)$$

- 6c) The pure strategy equilibria are {(B,B), (B,T), (T,B)}. No mixed strategy equilibrium exists.

- 6d) The pure strategy equilibria are {(T,T), (B,M), (M,B)}. No mixed strategy equilibrium exists.

- 6e) No pure strategy equilibrium exists. A mixed strategy equilibrium exists and is given by

$$(450/1231T + 493/3693M + 1850/3693B, 450/1231T + 493/3693M + 1850/3693B)$$

- 6f) The only equilibrium is the pure strategy equilibrium (T,T).