Ec101: Behavioral Economics

Answer Key to Homework #1 – 14th May 2007

Question One (i) – Bayesian updating

Let $P(L) = p$. We can assume (following Kahneman & Tversky 1973, Psychological Review) that the only two choices are lawyers or engineers, thus $P(E) = 1 - p$.

a) Using Bayes' rule, write down the posterior probability $P_p(L \mid \text{Tom})$.

$$P_p(L \mid \text{Tom}) = \frac{P(\text{Tom} \mid L)P(L)}{P(\text{Tom} \mid L)P(L) + P(\text{Tom} \mid E)P(E)}$$

$$= \frac{P(\text{Tom} \mid L)p}{P(\text{Tom} \mid L)p + P(\text{Tom} \mid E)(1-p)}$$

(5 points)

b) Graph the relationship between $P_{.7}(L \mid \text{Tom})$ and $P_{.3}(L \mid \text{Tom})$, assuming the likelihood ratios do not depend on the priors.

Substituting for *p* in our equation from (a) gives us

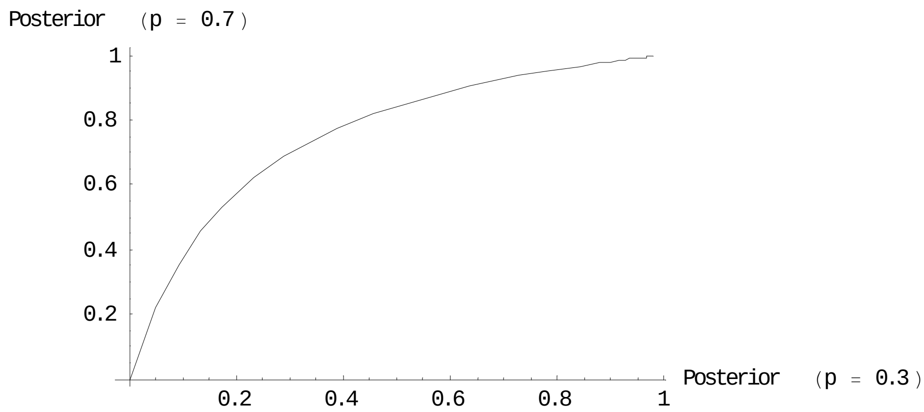
$$P_{0.7}(L \mid \text{Tom}) = \frac{P(\text{Tom} \mid L)}{P(\text{Tom} \mid L) + P(\text{Tom} \mid E)(0.3/0.7)} = \frac{x}{x + \frac{3}{7}}$$

$$P_{0.3}(L \mid \text{Tom}) = \frac{P(\text{Tom} \mid L)}{P(\text{Tom} \mid L) + P(\text{Tom} \mid E)(0.7/0.3)} = \frac{x}{x + \frac{7}{3}}$$

Where

$$x = \frac{P(\text{Tom} \mid L)}{P(\text{Tom} \mid E)}$$ is not a function of *p*, by assumption.

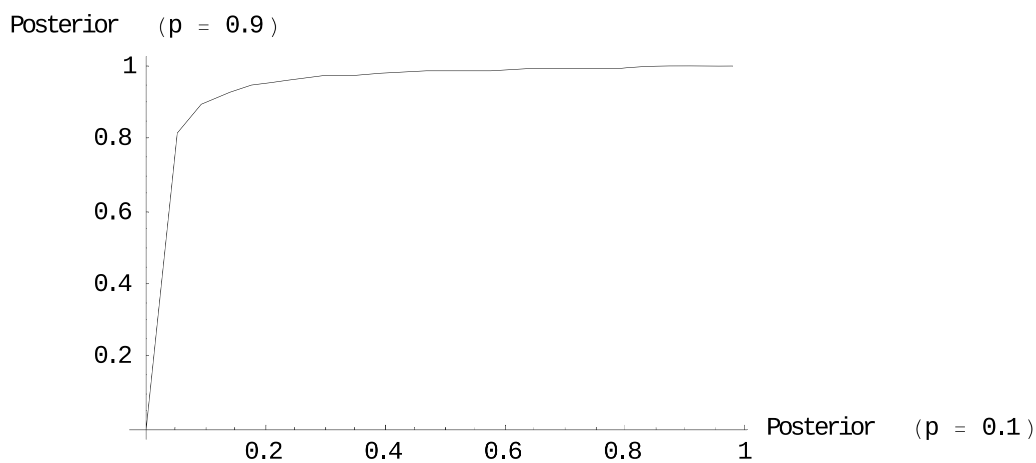
We can plot the two posteriors parametrically by varying *x* from 0 to ∞ (300 is sufficient for graphing purposes). As *x* is a ratio of probabilities, it cannot fall below 0. Alternatively you can solve one equation for *x* and substitute back into the other to get a Cartesian equation, and plot that directly. Here is my graph, using Mathematica, where $P_{.7}(L \mid \text{Tom})$ is on the *y*-axis and $P_{.3}(L \mid \text{Tom})$ is on the *x*-axis.



(5 points)

c) Now, graph the relationship between $P_{.9}(L \mid \text{Tom})$ and $P_{.1}(L \mid \text{Tom})$, with the same assumptions as for question (b).

Here is the same graph with the probabilities changed to .9 and .1 respectively. The curve bows out more.



(5 points)

d) If there is complete base rate neglect, what is the relationship between the two posteriors?

Complete base rate neglect means that the prior probabilities add no information. Mathematically this is accomplished by assuming $P(L) = P(E) = 0.5$. Under this assumption, the ratio of the two posteriors is equal to the likelihood ratio i.e.

$$P(L \mid \text{Tom}) = \frac{P(\text{Tom} \mid L)}{P(\text{Tom} \mid E)} P(E \mid \text{Tom}).$$

(5 points)

Question One (ii) – More Bayesian updating

e) A disease has an overall base rate $P(D) = 0.001$. A test for the disease has sensitivity S, (i.e. $P(\text{test positive} \mid D) = S$) and specificity Q (i.e. $P(\text{test negative} \mid D^C) = Q$). If a patient tests positive, what is the posterior $P(D \mid \text{test postive})$?

$$P(D \mid \text{test postive}) = \frac{P(\text{test positive} \mid D)P(D)}{P(\text{test positive} \mid D)P(D) + P(\text{test positive} \mid D^C)P(D^C)}$$

$$= \frac{0.001S}{0.001S + .999(1-Q)}$$

(5 points)

f) (Equivalent to (b) in the question sheet). Numerical examples.

If $S = Q = .99$ then the posterior is $11/122$ or 0.0901639.

If $S = .99$ and $Q = .90$ then the posterior is 0.009812667.

(5 points)

Question Two – Intertemporal Choice

a) When will a naïve hyperbolic discounter incur cost *c*?

Begin by finding out what a naïve hyperbolic discounter (call him Naïf) will choose at time $t = 0$. Naïf must pay the cost in one of the four periods, and will receive the benefit in the following period.

Naïf takes the cost in period	Notation	Discounted utility (according to Naïf's perception at time 0)
0	$U_0^0 =$	$-c + \beta\delta B(1) = -c + \beta\delta B$
1	$U_1^0 =$	$\beta\delta(-c) + \beta\delta^2 B(2) = \beta\delta(-c + \delta(B - e))$
2	$U_2^0 =$	$\beta\delta^2(-c) + \beta\delta^3 B(3) = \beta\delta^2(-c + \delta(B - 2e))$
3	$U_3^0 =$	$\beta\delta^3(-c) + \beta\delta^4 B(4) = \beta\delta^3(-c + \delta(B - 3e))$

Naïf wants to maximize his utility. He will choose which period to take the cost with this goal in mind. Notice immediately that Naïf will not choose to take the cost in periods 2 or 3, because $\delta < 1$ and because $B(t)$ is decreasing in *t*, $U_3^0 < U_2^0 < U_1^0$. Thus we need only compare utility between period 0 and period 1. Which is optimal will depend on the exact values of the parameters. $U_0^0 \geq U_1^0$ if and only if

$$\Leftrightarrow -c + \beta\delta B \geq -c\beta\delta + \beta\delta^2 (B - e)$$

$$\Leftrightarrow c \leq \frac{\beta\delta(\delta e + B(1-\delta))}{1-\beta\delta} \qquad [1]$$

If equation [1] holds true, then Naïf will take the cost immediately at time 0 and the problem is solved.

To solve the rest of the question, now suppose equation [1] is not true. Now Naïf decides at period 0 to take the cost in time 1. Since Naïf is naïve, his preferences change at time 1, so he could possibly change his decision.

Naïf takes the cost in period	Notation	Discounted utility (according to Naïf's perception at time 1)
1	$U_1^1 =$	$-c + \beta\delta B(2) = -c + \beta\delta(B - e)$
2	$U_2^1 =$	$\beta\delta(-c) + \beta\delta^2 B(3) = \beta\delta(-c + \delta(B - 2e))$
3	$U_3^1 =$	$\beta\delta^2(-c) + \beta\delta^3 B(3) = \beta\delta^2(-c + \delta(B - 3e))$

As before, Naïf will not choose to take the cost in period 3, because period 2 is always a better choice. Comparing the first two periods, we find $U_1^1 \geq U_2^1$

$$\Leftrightarrow -c + \beta\delta(B - e) \geq -c\beta\delta + \beta\delta^2 (B - 2e)$$

$$\Leftrightarrow c \leq \frac{\beta\delta((2\delta - 1)e + B(1-\delta))}{1-\beta\delta} \qquad [2]$$

The RHS of equation [2] is strictly smaller than the RHS of equation [1]. Given equation [1] does not hold, then the following is true

$$c > \frac{\beta\delta(\delta e + B(1-\delta))}{1-\beta\delta} > \frac{\beta\delta((2\delta - 1)e + B(1-\delta))}{1-\beta\delta}.$$

In other words, equation [2] does not hold either so at time 1, Naïf will choose to take the cost in period 2.

Now consider what Naïf will do in period 2.

Naïf takes the cost in period	Notation	Discounted utility (according to Naïf's perception at time 2)
2	$U_2^2 =$	$-c+\beta\delta B(3) = -c+\beta\delta(B-2e)$
3	$U_3^2 =$	$\beta\delta(-c)+\beta\delta^2 B(4) = \beta\delta(-c+\delta(B-3e))$

Note that $U_2^2 \geq U_3^2$ if and only if

$$\Leftrightarrow -c+\beta\delta(B-2e) \geq -c\beta\delta+\beta\delta^2(B-3e)$$

$$\Leftrightarrow c \leq \frac{\beta\delta((3\delta-2)e+B(1-\delta))}{1-\beta\delta} \qquad [3]$$

The RHS of equation [3] is also less than the RHS of equation [1]. Thus, if Naïf has to make a decision in period 2, he will choose to wait until period 3 (given that equation [1] must hold).

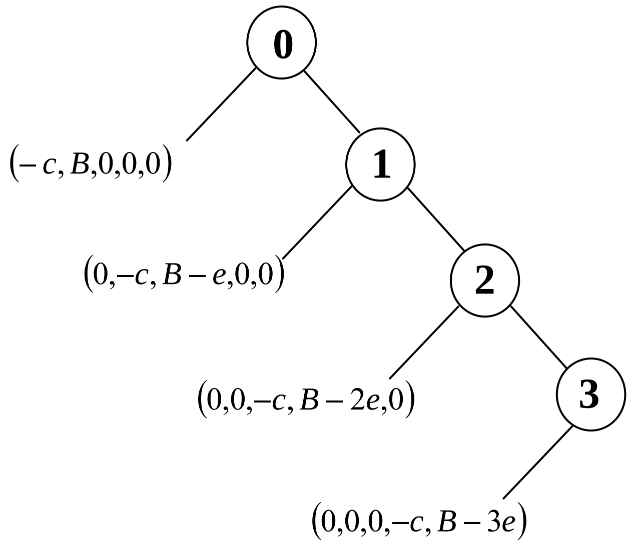
Thus if equation [1] holds true, naïve hyperbolic discounters takes the cost at period 0. Otherwise, they take the cost in period 3.

Note: if you numbered periods from 1-4, the solution will be the same (Naïf takes the cost in periods 1 or 4, but equation [1] will be slightly different.

[15 points – well deserved]

b) When will a sophisticated hyperbolic discounter incur cost *c*?

Again, we calculate the utility for a sophisticated hyperbolic discounter (call her Soph). The easiest way to find Soph's decision is to solve using backward induction. The game takes on the following extensive form:



In each period, Soph can either take the cost, or defer until the next period (take the cost in the future). The outcomes for each period of taking the cost in that period are depicted.

Working backwards, I find conditions under which Soph would take the cost at each period, assuming Soph understands her future behaviour.

At time $t = 3$, Soph has no choice. She must take the cost and receive the outcome shown.

At time $t = 2$, Soph can choose between taking the cost now, with utility $U_2^2 = -c + \beta\delta(B - 2e)$, or taking the cost at time 3, with utility $U_3^2 = -c\beta\delta + \beta\delta^2(B - 3e)$. She will choose whichever is greater.

$$U_2^2 \geq U_3^2 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta) + e(3\delta - 2))}{1 - \beta\delta} \qquad [4]$$

So if equation [4] is true, Soph will take the cost in period 2. If not, she will defer to the next period.

Now we have two cases to consider.

Case 1: Assume equation [4] holds – i.e. Soph chooses 2 over 3 at time 2.

At time $t=1$ Soph faces the choice of taking the cost now, and receiving utility $U_1^1 = -c + \beta\delta(B-e)$ or deferring to the next period – where she knows (because she is sophisticated) that her future self will take the cost in period 2, with utility (in time 1) of $U_2^1 = -c\beta\delta + \beta\delta^2(B-2e)$. She will choose whichever is greater.

$$U_1^1 \geq U_2^1 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta)+e(2\delta-1))}{1-\beta\delta} \qquad [5]$$

It is fairly easy to show that if equation [4] is true, so too is equation [5]. Thus at time $t=1$ Soph will choose to take the cost in period 1 (knowing that if she delayed, she would take the cost in period 2).

At time $t=0$ Soph faces the choice of taking the cost now and receiving utility $U_0^0 = -c + \beta\delta(B)$ or deferring to the next period, where she knows she will take the cost for sure because that is what is best for her future self, with utility $U_1^0 = -c\beta\delta + \beta\delta^2(B-e)$.

$$U_0^0 \geq U_1^0 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta)+e\delta)}{1-\beta\delta} \qquad [6]$$

Again note that if equation [4] is true, so too is equation [6]. Thus Soph will choose to take the cost at time 0.

So we can summarize case 1 by saying if equation [4] holds, Soph chooses to take the cost at time 0.

Case 2: Assume equation [4] does not hold – i.e. Soph chooses 3 over 2 at time 2.

As before, now consider Soph's decision at time $t=1$. Soph faces the choice of taking the cost now, and receiving utility $U_1^1 = -c + \beta\delta(B-e)$ or deferring to the next period –

where she knows that her future self will take the cost in period 3, with utility (at time 1) of $U_3^1 = -c\beta\delta^2 + \beta\delta^3(B - 3e)$.

$$U_1^1 \geq U_3^1 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta) + e(3\delta - 1))}{1 - \beta\delta} \qquad [7]$$

So if equation [7] holds, Soph will choose to take the cost at time 1. Otherwise Soph will delay, and eventually take the cost at time 3.

Now we can evaluate Soph's decision at time $t = 0$, having calculated her decisions further down the game (assuming of course that equation [4] does not hold).

Case 2a: Assume equation [7] holds – i.e. Soph chooses 1 over 3 at time 1.

Soph knows her choice her is between time 0 and time 1. Her utility, measured at time 0 of taking the cost at time 1 is $U_1^0 = -c\beta\delta + \beta\delta^2(B - e)$. Alternatively, she can take the cost now, with utility $U_0^0 = -c + \beta\delta B$.

$$U_0^0 \geq U_1^0 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta) + \delta e)}{1 - \beta\delta} \qquad [8]$$

She will take the cost at time 0 if equation [8] holds. Otherwise, she will take the cost at time 1.

Case 2b: Assume equation [7] does not hold – i.e. Soph chooses 3 over 1 at time 1.

Here Soph has a choice between time 0 and time 3, with utilities $U_0^0 = -c + \beta\delta B$ and $U_3^0 = -c\beta\delta^3 + \beta\delta^4(B - 3e)$ respectively.

$$U_0^0 \geq U_3^0 \Leftrightarrow c \leq \frac{\beta\delta(B(1-\delta^3) + e(4\delta^3 - 1))}{1 - \beta\delta} \qquad [9]$$

She will take the cost at time 0 if equation [9] holds. Otherwise, she will take the cost at time 3.

To prove that we cannot rule out any of the options in Case 2, let $B=\delta=0.5$, $B=10$, and $e=1$. Under these parameters, the following is true.

c	Equation [4]	Equation [7]	Equation [8]	Equation[9]	Outcome
1.5	False	True	True		Time 0
1.8	False	True	False		Time 1
2	False	False		True	Time 0
2.4	False	False		False	Time 3

Thus a sophisticated hyperbolic discounter will take the cost in either periods 0, 1, or 3, with the exact time being specified by the following conditions.

Equation [4]	Equation [7]	Equation [8]	Equation[9]	Outcome
True	N/A	N/A	N/A	Time 0
False	True	True	N/A	Time 1
False	True	False	N/A	Time 1
False	False	N/A	True	Time 0
False	False	N/A	False	Time 3

[15 points – also well deserved]

Question Three – Endowment Effects

a) Solve for P_c^* such that the consumer is indifferent between the money and a pen.

The utility from receiving a single pen is

$$u(1,0;1,0)=b+\varpi .b\,,$$

and the utility from receiving \$ P_c is

$$u(0,P_c,0,P_c)=P_c+\varpi .P_c\,.$$

Setting the two equations equal and solving gives us $P_c^*=b$.

(5 points)

b) Find the value P_b^* which is the consumer's maximum willingness to pay.

The utility from receiving a single pen *and* giving up \$ P_b is

$$u(1,-P_b,1,-P_b) = b - P_b + \varpi b - \mu\varpi P_b$$

To find out the maximum price the consumer would be willing to pay to receive the pen, you must consider the alternative. What is the consumer's utility if she pays nothing and receives nothing? Clearly, 0. The consumer will pay \$ P_b only up to the point her utility from the transaction is 0. Setting the equation above equal to 0 and solving gives us

$$P_b = \frac{b(1+\varpi)}{1+\mu\varpi}.$$

(5 points)

c) Find the value P_s^* which is the consumer's minimum willingness to accept (the selling price).

The utility from giving up a pen and receiving \$ P_s for it is

$$u(0,P_s,-1,P_s) = P_s - \mu\varpi b + \varpi P_s$$

The alternative (if you didn't engage in the transaction) is that you would just have a pen

$$u(1,0,0,0) = b$$

Setting the two equations equal and solving gives

$$P_s = \frac{b(1+\mu\varpi)}{1+\varpi}.$$

(5 points)

d) Show the parameter values which yield an endowment effect ($P_s^* > P_b^*$).

$$P_s^* > P_b^* \Leftrightarrow \frac{b(1+\mu\varpi)}{1+\varpi} > \frac{b(1+\varpi)}{1+\mu\varpi}$$

$$\Leftrightarrow (1+\mu\varpi)^2 > (1+\varpi)^2$$

$$\Leftrightarrow 1+\mu\varpi > 1+\varpi$$

$$\Leftrightarrow \mu > 1$$

[Note I assume $\varpi > 0$ because the model does not make sense otherwise.]

(2.5 points)

e) Where does the choosing value P_c^* lie compared to buying and selling prices?

Since $\mu \geq 1$, we have $1+\mu\varpi \geq 1+\varpi$. Thus

$$P_b = b\frac{1+\varpi}{1+\mu\varpi} < b = P_c = b < b\frac{1+\mu\varpi}{1+\varpi} = P_s.$$

(2.5 points)

Question Four – Neuroeconomics

This question has infinite possible answers.

(10 points)

(90 POINTS TOTAL)

Some Notes on Grading

1(b) – If you graphed the posteriors against the likelihood ratio, instead of against each other, you lost 2 points. (Same for (c)) If you graphed against other things (there were quite a variety) you generally lost 3 points or more per graph.

1(d) – 'No relationship' got just 1 point. Most other answers got 3 points or more.

2(a) – You could achieve a maximum of 7 points if you only analysed Naïf's utilities at time 0 (or time 1 depending on which scale you used). You lost one point for doing an analysis assuming no benefit if you take the cost in the last period. (Mainly because this assumption simplified the analysis.)

If you did not identify that Naif would never take periods 1 & 2 (or 2 & 3) then I subtracted 2.5 points each (i.e. 5 points if you missed both).

2(b) – You could achieve a maximum of 8 points if you just examined payoffs at time 0 (or 1) without thinking about Soph's future tradeoffs. You could get up to 14 points for any reasonable backward induction analysis, but 15 points was only given for positing conditions under which periods 0, 1 and 3 (or 1, 2 & 4) could exist (three people achieved this).

3(a) -