Veto Players and Policy Development

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Abstract

We analyze the effects of veto players when the set of available policies isn't exogenously fixed, but rather is determined by policy developers who work to craft new high-quality proposals. If veto players are moderate then there is active competition between policy developers on both sides of the political spectrum. However, more extreme veto players induce asymmetric activity, as one side disengages from policy development. With highly-extreme veto players, policy development ceases and gridlock results. We also analyze effects on centrists' utility. Moderate veto players dampen productive policy development and extreme ones eliminate it entirely, either of which is bad for centrists. But some effects are surprisingly positive. In particular, somewhat-extreme veto players can induce policy developers who dislike the status quo to craft moderate, high-quality proposals. Our model accounts for changing patterns of policymaking in the U.S. Senate and predicts that if polarization continues centrists will become increasingly inclined to eliminate the filibuster.

Many political organizations use decisionmaking procedures that empower *veto players* – individuals or groups who have the power to *block* policy change. For example, chief executives often have constitutionally-granted veto powers (Cameron, 2000); supermajority procedures in legislatures, parliaments, and commissions generate implicit veto pivots (Brady and Volden, 1997; Crombez, 1996; Diermeier and Myerson, 1999; Krehbiel, 1998; Tsebelis, 2002); and bureaucrats are sometimes required to seek approval from another agency or interest group before they can act (McCubbins, Noll and Weingast, 1987; Moe, 1989). Despite the ubiquity of such procedures, commentators are of two minds about their consequences, as exemplified in debates over the filibuster in the U.S. Senate. Critics of the filibuster complain about the minority's ability to engage in obstruction. However, defenders of the filibuster have argued that additional hurdles to policy enactment encourage constructive deliberation (Arenberg et al., 2012).

To understand the effects of veto players on policymaking it is important to consider both of these possibilities, i.e., to allow for both constructive policymaking and obstruction. In this paper we do so, building on the competitive policy development framework of Hirsch and Shotts (2015). The policy process is modeled as an open forum in which a decisionmaker considers policies crafted by policy-motivated actors, known as *policy developers*. Rather than promising policy-contingent transfers or furnishing general policy-relevant information,[1] these developers gain support for their policies by making costly, up-front policy-specific investments in their *quality*. Quality reflects characteristics of policies that are valued by all participants in the policy process, such as cost savings, promotion of economic growth, or efficient and effective administration. In the original model, competition between developers benefits a unitary decisionmaker because it prevents a developer from extracting all of the benefits of her quality investments in the form of ideological concessions. Herein we consider how the inclusion of veto players affects this process.

Generally speaking, veto players create additional hurdles to policy change, because they have

[1] See Grossman and Helpman (2001) for a review of theories of influence.

the power to block proposals that they find less desirable than the status quo. Their presence thus unambiguously harms a decisionmaker who would otherwise have full freedom of choice – *provided that* the set of policies from which he can choose is exogenous. However, when the set of potential policies is *endogenous* because policies must first be developed at some cost, the effect of veto players is no longer obvious. The additional hurdles that they pose could *discourage* policy development, because developers anticipate that it will be more difficult to achieve policy change. However, they could also *encourage* policy developers to invest more in quality in order to surmount those hurdles. Given these countervailing effects, it is also not ex ante obvious whether, and under what circumstances, the net effect of veto players is beneficial for the interests of a centrist decisionmaker.

We first examine how the presence of veto players affects *patterns of activity* in policy development, and show that they often lead to *asymmetric participation* between otherwise-symmetric policy developers. The reason is that a veto player may protect a non-centrist status quo that favors one developer; the favored developer is therefore *less-motivated* to develop a new policy, while the disfavored developer is *more-motivated* to do so. For a wide range of parameters this leads to an asymmetric equilibrium, in which the favored developer is sometimes or always inactive in policy development, while the disfavored developer always crafts a new policy for consideration. The model thus generates a natural and intuitive pattern frequently seen in real-world politics: the faction with the greatest interest in change actively invests to develop a credible policy alternative, while the faction that benefits from the status quo is less constructively engaged in policy development.

We next examine how the *extremity* of veto players affects patterns of participation. If veto players are moderate, then both developers craft new policies for consideration because each faces relatively modest hurdles to moving policy in her desired direction. However, as veto players become increasingly extreme, the pattern of activity becomes increasingly asymmetric; the reason is that greater hurdles to policy change disproportionately discourage policy development by the developer who is favored by the status quo. They do this both directly – by making it harder for her to get her preferred policies enacted – and indirectly – by protecting her from the possibility that the other side will successfully promulgate a policy that she finds highly unappealing. Finally, if veto players become very extreme, the pattern of activity once again becomes symmetric: both developers are completely discouraged from developing policies, and the result is gridlock.

We last examine when a centrist decisionmaker would benefit from *eliminating* veto players from the policy process. If veto players are highly moderate or highly extreme, the decisionmaker is indeed better off doing so; in the former case they simply dampen productive competition, while in the latter case they discourage policy development entirely. However, if veto players are *somewhat* extreme, then the decisionmaker benefits from *maintaining* them to protect non-centrist status quos; the disfavored developer is willing to make substantial quality investments to change such a status quo, and a somewhat extreme veto player protecting it forces her to do so. At the same time, the opposing developer favored by status quo is unmotivated to develop a competing policy. Our model thus counterintuitively predicts that veto players most strongly benefit the decisionmaker precisely when their presence inhibits *observable* competition. An important implication is that the absence of observable competition in policy development is not prima facie evidence of dysfunctional politics, or exogenous constraints on one side's participation—it could instead simply reflect competing groups' differential willingness and ability to invest in changing a lopsided status quo.

As applied to the filibuster in the U.S. Senate, our model has three main implications. First, it provides a novel explanation for shifting patterns of policymaking as the Senate has become increasingly polarized. Since the 1970s, the Senate has evolved from the "textbook Congress" in which members on both sides generated policy options (because veto players were relatively moderate), to highly asymmetric policymaking with the majority developing policies and the minority engaging in obstruction (as veto players became more extreme), and finally to the contemporary gridlocked Senate (as veto players became very extreme). Second, our model provides a novel rationale for centrist Senators' historical support of the filibuster as an institution; namely, that veto players who aren't too extreme can encourage development of reasonably-moderate and high-quality policies. Third, our model predicts that should Senate polarization persist or increase, centrists will become increasingly inclined to eliminate the filibuster as an institution and operate under majority rule.

The paper proceeds as follows. We first summarize related literature. We then introduce the model and show how the presence of veto players affects the set of feasible policies. We next explain the structure of equilibrium. We then present our main results on patterns of policy development activity and decisionmaker welfare, and apply these results to the U.S. Senate.

Related Literature

Our model first relates to a large literature studying the consequences of supermajority rules. While we do not model voting rules directly, the policy choice stage of our model can be interpreted as building on pivotal politics models of collective choice, where the median legislator acts as the decisionmaker, and supermajority rules create veto "pivots" on either side (Brady and Volden, 1997; Krehbiel, 1998). Among the many rationales for supermajority rules considered in the literature are policy stability (Barbera and Jackson, 2004; Caplin and Nalebuff, 1988), balanced budgets (Tabellini and Alesina, 1990), minority protections (Aghion and Bolton, 2003), insulation of the executive (Aghion, Alesina and Trebbi, 2004), intergenerational conflict (Messner and Polborn, 2004), information acquisition and aggregation (Persico, 2004), and maximizing campaign contributions (Diermeier and Myerson, 1999). More broadly, our model is related to theories in which constraints on a decisionmaker's discretion can improve her welfare by mitigating dynamic inconsistency problems; such constraints include delegating decisionmaking (Rogoff, 1985) and employing supermajority rules (Dal Bo, 2006). Our model diverges from this literature because we consider how constraints on a decisionmaker's discretion can improve the *set of alternatives from which she selects* by influencing the behavior of other strategic actors.

Our work also relates to previous research on veto players and blocking coalitions (Brady and Volden, 1997; Crombez, 1996; Dziuda and Loeper, 2018; Krehbiel, 1998; Tsebelis, 2002); the vast majority of this research adopts a purely-ideological model of policy choice. In contrast, policies in our model have an endogenous quality dimension; there thus exists the possibility for "buying" votes by developing high-quality policies.[2] An important feature of our model is that quality is *policy-specific* rather than applicable to policies anywhere in the ideological spectrum. Thus, our model contrasts with a large literature building on Crawford and Sobel (1982), in which policy-relevant information is not specific to any particular policy.[3] In so doing, we build on models of policymaking by Bueno De Mesquita and Stephenson (2007); Hirsch and Shotts (2012, 2018); Lax and Cameron (2007); Londregan (2000), and Ting (2011). A key feature of such models is that an expert is able to exert informal agenda power or "real authority" (Aghion and Tirole, 1997) by creating high-quality policies. Most closely related to our work is Hitt, Volden and Wiseman (2017), which briefly analyzes the case of a single developer who faces veto players. In addition to incorporating multiple potentially-active developers, our analysis differs in that we characterize effects on the moderation and quality of policies, as well whether centrists benefit from the presence of veto players.

Finally, because the cost of investing in quality is paid up-front, our model relates to previous work on all-pay contests (Baye, Kovenock and Vries, 1993; Che and Gale, 2003; Siegel, 2009). The developers in our model simultaneously make up-front payments to generate proposals with two dimensions (ideology and quality), and the decisionmaker chooses among them subject to the veto constraint. Our model has two primary differences from most previous contest models, both of which complicate the equilibrium analysis. The first difference is that the developers are policy-motivated rather than rent seeking, in that the "loser" cares about the exact policy crafted by the "winner."[4] Our model is thus better tailored to political environments where competing actors care

[2] Anesi and Bowen (2021) analyze veto players and vote buying with transfers in a model with a binary policy that is exogenously either good or bad.

[3] The Brownian motion approach developed by Callander (2008) is more similar to our model, but his model is purely spatial, whereas we model quality directly.

[4] Specifically, the model has a *second-order rank order spillover* (Baye, Kovenock and Vries (2012)).

about a collective policy decision. The second difference is that our model features players who can only *block* proposals, i.e., veto players. In the presence of veto players, investing in quality can be strategically beneficial in two distinct ways: higher quality can make a policy more appealing to the decisionmaker, but also help gain the support of the veto players. Both of these incentives play a crucial role in affecting equilibrium patterns of policy development.

The Model

The model takes place in three stages. First, two policy developers simultaneously craft new policies to add to the set of possible alternatives available for consideration. Second, a decisionmaker proposes a policy; either a new one crafted by a developer, or a preexisting one. Finally, a pair of veto players either approve the policy or block the proposal, in which case a status quo prevails.

Policy has two components: *ideology* $y \in \mathbb{R}$ and *quality* $q \in [0, \infty)$. Players' utility functions are:

$$U_i(b) = q - (x_i - y)^2$$

where x_i is i's ideological ideal point.

Policy development Each of two developers (L and R, with ideal points $x_L < 0$ and $x_R > 0$) may simultaneously invest costly resources to develop a new policy $b_i = (y_i, q_i)$ with ideology y_i and quality $q_i \geq 0$ at cost $\alpha_i q_i$. Costs are linear, and each developer's marginal cost of quality is assumed to exceed the total marginal benefit between them ($\alpha_i > 2$). This implies that a developer will only invest in quality if it increases the probability that her policy will be chosen.

Policy choice In Hirsch and Shotts (2015) and Hirsch (2023), policy is chosen by a single decisionmaker with an ideal ideology normalized to $x_D = 0$. In the present model we augment this decisionmaking process with two *veto players* $x_{VL} < 0$ and $x_{VR} > 0$ – if either veto player rejects the decisionmaker's proposal, a status quo policy b_0 prevails.

The set of policy alternatives consists of all 0-quality policies, any newly-developed policies crafted by the developers, and the status quo $b_0 = (y_0, q_0)$. This assumption reflects the idea that the decisionmaker and the veto players collectively have the power to *choose* policy, but not to *develop* it. Finally, for simplicity we assume that the status quo is low-quality ($q_0 = 0$) and within the "gridlock interval" for 0-quality policies ($x_{VL} < y_0 < x_{VR}$), and also that that the developers are more extreme than the veto players ($x_L \leq x_{VL}$ and $x_R \geq x_{VR}$). These assumptions ensure that the left developer wants to move policy leftward from the status quo, and the right developer rightward.

The Effect of Veto Players on Decisionmaking

In the absence of veto players, the decisionmaker can revise any status quo to a low quality policy that exactly reflects his ideal ideology; it is thus "as if" the status quo ideology is $y_0 = x_D = 0$. The decisionmaker is therefore willing to adopt any newly-crafted policy that he prefers over $(0, 0)$. This is depicted in the top panel of Figure 1; the set of acceptable policies is located above the green line that represents the decisionmaker's indifference curve through his ideal point with 0 quality.

The presence of veto players creates additional hurdles to policy change, which affects the decisionmaking process in two ways. First, it expands the range of potential status quos that the developers may face to include ones that are non-centrist: the status quo may be any low-quality policy with $y_0 \in [x_{VL}, x_{VR}]$. Because the status quo may not exactly reflect the decisionmaker's preferences, he will be more receptive to new proposals—this can be seen in the lower panel of Figure 1, where the developers' policies must only be above the decisionmaker's indifference curve through $y_0 \neq 0$ to gain his support. However, for policy change to occur, the new policy *also* must be acceptable to *both* veto players, who are collectively more opposed to policy change than the decisionmaker. This can be seen by observing that the veto players' indifference curves through the status quo (the dashed red lines) are steeper than the decisionmaker's indifference curve. To avoid a veto, the decisionmaker must choose a policy that is above the upper envelope of these two indifference curves. We henceforth refer to this shaded region as the *veto-proof set*.

The effect of veto players hinges on how this change in the set of acceptable policies affects the policy developers' strategic incentives to invest in quality. A developer may be *less* willing to invest

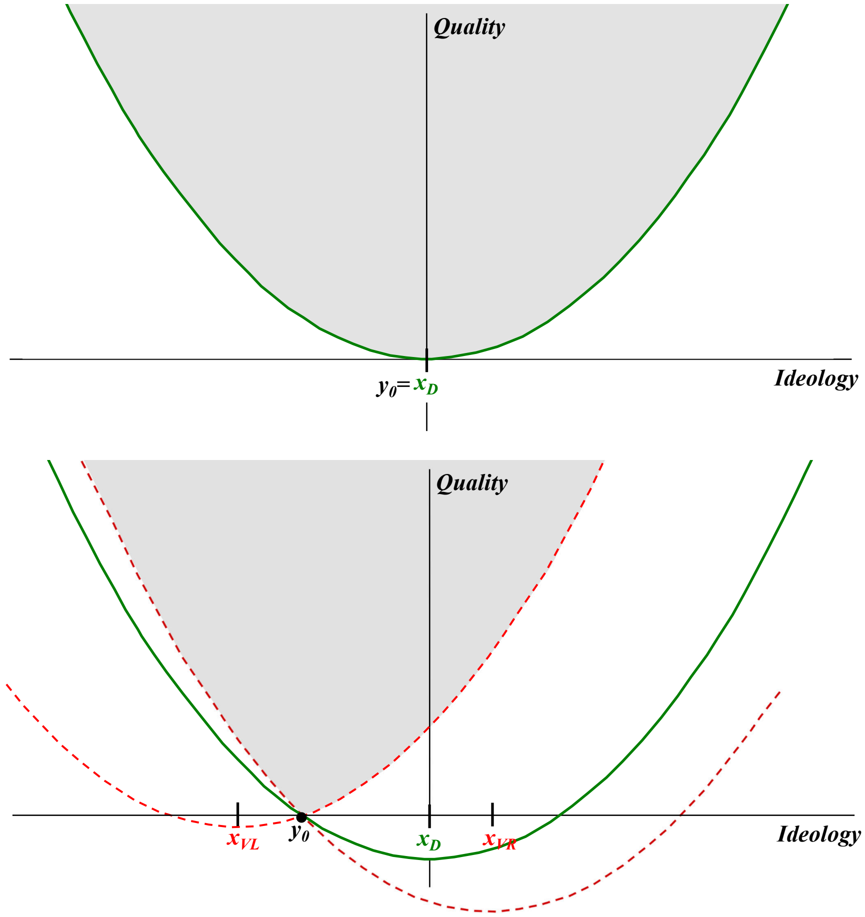

Figure 1: *The Effect of Veto Players on Decisionmaking*

if she is favorably disposed to the status quo, or unwilling to satisfy the veto players' demands. Alterantively, she may be more willing to invest in developing a high-quality policy, if she strongly dislikes the status quo and is willing to put in the extra work to satisfy the veto players' demands.

Notation To characterize how veto players affect the game, we introduce additional notation and terminology. We call the decisionmaker's utility from a policy its *score*, $s(y, q) = U_D(y, q) = q - y^2$. The concept of a score is useful because it fully characterizes how the decisionmaker will evaluate the available veto-proof policies. Recall that absent veto players, it is "as if" the score of the status quo is $s(0, 0) = 0$; the decisionmaker will then choose the policy with the highest score subject to

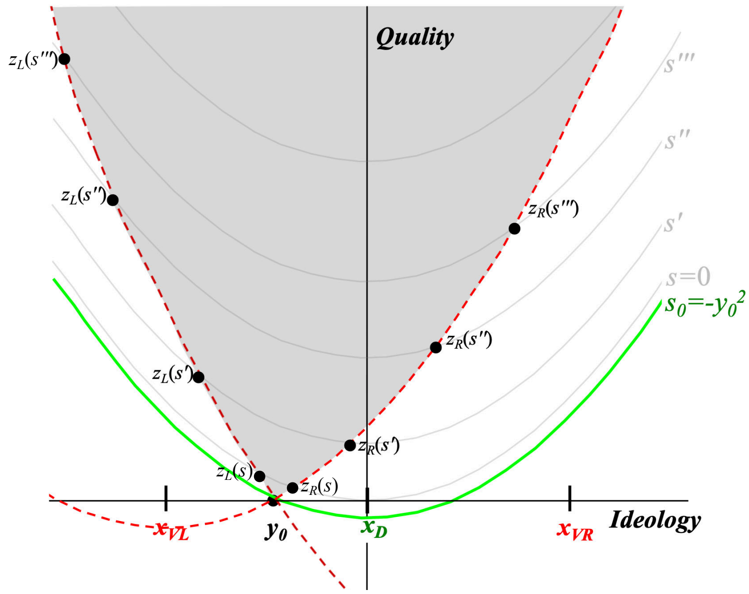

Figure 2: *The Veto-Proof Set*

the constraint that the score is ≥ 0. Veto players increase the range of scores the decisionmaker is willing to accept (to those $\geq U_D(y_0, 0) = -y_0^2$), but restrict the set of acceptable ideologies *given each score*. The following defines the set of veto-proof policies in terms of score s and ideology y.

Definition 1. *A policy (s, y) with score s and ideology y is veto-proof if and only if $y \in [z_L(s), z_R(s)]$, where $z_L(s) = y_0 - \frac{s-s_0}{2|x_{VR}|}$, $z_R(s) = y_0 + \frac{s-s_0}{2|x_{VL}|}$, and $s_0 = -y_0^2$ is the score of the status quo.*

Figure 2 depicts an example. The decisionmaker's indifference curves, i.e., the sets of policies with the same score, are depicted by gray lines. On any given "score curve" s, the range of veto-proof ideologies is $[z_L(s), z_R(s)]$; the right boundary is determined by the left veto player (who opposes rightward policy change), while the left boundary is determined by the right veto player (who opposes leftward policy change). After all policies have been developed, the decisionmaker optimally chooses the highest-score veto-proof policy available to him, and this becomes the final policy outcome.

Preliminary Analysis

Monopolist's Problem

To see how veto players influence policy development, it is instructive to first consider the case of a single developer who is a "monopolist."[5] Because a policy with score s and ideology y must have quality $q = s + y^2$, the up-front cost to developer i of crafting it is $\alpha_i \left(s + y^2\right)$, and her policy utility if it is adopted (whether or not she crafted it herself) is $V_i\left(s_i, y_i\right) = U_i\left(y_i, s_i + y_i^2\right) = -x_i^2 + s_i + 2x_i y_i$. A monopolist's objective is thus to craft a veto-proof policy (s_i, y_i) that maximizes $-\alpha_i \left(s_i + y_i^2\right) + V_i\left(s_i, y_i\right)$. The policy (s_i, y_i) satisfying this objective may be characterized as follows:

$$\underset{\{(s_i,y_i):s_i\geq s_0, y_i\in[z_L(s_i),z_R(s_i)]\}}{\arg\max} \left\{ \underbrace{-\left(\alpha_i - 1\right) s_i}_{\text{score effect}} + \underbrace{2x_i y_i - \alpha_i y_i^2}_{\text{ideology effect}} \right\}. \tag{1}$$

From Equation 1 it is easy to see that *if there were no veto players,* a monopolist would choose to craft a policy no better for the decisionmaker than $(0, 0)$, since it is "as if" this is the status quo and there is no constraint on ideology. This means setting $s_i = 0$ (minimizing the loss in the first term) and targeting the ideology that optimally trades off ideological concessions to the decisionmaker against the cost of compensating him with additional quality (maximizing the second term). The optimal ideology is then $y_i = \frac{x_i}{\alpha_i}$, which is a convex combination of the decisionmaker's and monopolist's ideal points weighted by the marginal cost of quality α_i.

The presence of veto players, however, *prevents the monopolist from doing this*, because they force her to develop a policy within the veto-proof set. This is precisely why veto players can benefit the decisionmaker–they force a developer to craft a higher-score policy if she wishes to move policy in her preferred ideological direction. What then will a monopolist do? She will develop a policy on the closer *boundary* of the veto-proof set ($z_L\left(s_L\right)$ for developer L or $z_R\left(s_R\right)$ for developer R), and select an ideology that trades off the marginal benefit of moving the outcome closer to her ideal

[5] See also Hirsch and Shotts (2015, 2018), Hitt, Volden and Wiseman (2017), and Hirsch (2023).

against the marginal cost of producing enough quality to gain the opposite-side veto player's support. Substituting the optimal ideology $y_i^*(s) = z_i(s_i)$ given a score s into Equation 1, straightforward optimization characterizes the ideological location of the optimal policy for a monopolist to develop. As in Hitt, Volden and Wiseman (2017), it is a convex combination of her own ideal point and the ideal point of the "binding" (opposite-side) veto player, weighted by the cost of producing quality:

$$\hat{y}_i = \begin{cases} \frac{1}{\alpha_L} x_L + \left(1 - \frac{1}{\alpha_L}\right) x_{VR} \text{ for } i = L \\ \frac{1}{\alpha_R} x_R + \left(1 - \frac{1}{\alpha_R}\right) x_{VL} \text{ for } i = R \end{cases}$$

If the status quo is already closer to the developer than $\hat{y}_i$, the marginal cost of moving policy in her direction is too high, so she develops no policy and the status quo is maintained. In sum, we have the following result.

Proposition 1. *When developer i is a monopolist, she crafts the policy (s_i^{M*}, y_i^{M*}), where*

$$y_i^{M*} = \begin{cases} \min\{y_0, \hat{y}_L\} \text{ for } i = L \\ \max\{y_0, \hat{y}_R\} \text{ for } i = R \end{cases}$$

and $z_i(s_i^{M*}) = y_i^{M*}$. A monopolist invests in policy development if and only if the status quo is farther away from her ideal point than her ideal monopoly policy: $s_i^{M*} > s_0 \iff |y_0 - x_i| > |\hat{y}_i - x_i|$.

Thus, *when* a new policy is developed, its optimal ideology $\hat{y}_i$ depends on the tradeoff at the margin between ideological gains and costs of quality. With linear costs, this ideology does not depend on the location of status quo. The optimal level of quality, however, *does* depend on the status quo, because a status quo closer to the opposite-side veto player's ideal point forces the developer to generate more quality to gain his support. This yields the following simple result.

Corollary 1. *At any status quo y_0 where policy development occurs ($s_0 < s_i^{M*}$), the monopoly score s_i^{M*} is strictly decreasing (increasing) in y_0 when $i = R\,(L)$.*

Intuitively, the farther is the status quo from the monopolist, the more change she wants, the more she must invest in quality to gain the binding veto player's support, and thus the more the decisionmaker benefits from her efforts.

Form of Equilibrium

Having established some intuition for the effet of veto players on strategic policy development, next describe key properties of subgame-perfect Nash Equilibria in the main model in which two developers compete. In our description we say that developer i is *active* when she develops a veto-proof policy with score $s_i > s_0$ (and therefore with strictly positive quality), and that she is *inactive* if she exerts no effort and "develops" the unique low-quality veto-proof policy, i.e., the status quo.

As it turns out, equilibrium of the competitive model may be in pure *or* mixed strategies. In the Appendix we fully characterize necessary and sufficient conditions for a strategy profile of each type to be a subgame-perfect Nash Equilibrium (see Lemma A.6 and Proposition B.1); herein we describe some key properties of each type of equilibrium.

Pure Strategy Equilibria

Whenever there exists an equilibrium in pure strategies, it takes the following form.

Lemma 1. *In a pure strategy equilibrium, the developer k with the lower monopoly score is inactive, while the other developer $-k$ crafts her monopoly policy $\left(s_{-k}^{M*}, y_{-k}^{M*}\right)$ from Proposition 1.*

In any pure strategy equilibrium at least one developer must be inactive. If both were active, one of them would be strictly better off either dropping out, or generating a slightly-higher quality policy and winning for sure. In addition, the inactive developer must have the lower monopoly score; otherwise, her opponent would strictly prefer crafting her monopoly policy to allowing her competitor to act as a monopolist, which results in an even worse policy outcome for her than the status quo.[6] Finally, the active developer must develop her monopoly policy, because absent competition her

[6] A developer's monopoly policy is strictly worse for her opponent than the status quo because it is crafted to just barely gain the support of the opposing veto player, who is weakly less extreme than the opposing developer.

incentives are the same as those of a monopolist. (If both developers' monopoly scores are s_0, then each prefers not to enter the contest, and in equilibrium both remain inactive).

While the preceding explains why pure strategy equilibria take a particular form, it does not explain why they exist at all—why doesn't the inactive developer simply craft a policy slightly better for the decisionmaker than her opponent's policy? Indeed, this is precisely what occurs in the model absent veto players, which lacks pure strategy equilibria even when the developers differ arbitrarily in their extremism and ability (Hirsch (2023)). The reason that the present model works differently is that veto players sometimes force the active monopolist to craft a policy that is sufficiently high-quality to (inadvertently) *insulate it* from potential competition.

Example 1: pure strategy equilibrium Figure 3 depicts a pure strategy equilibrium for a particular set of parameter values. In the example, R develops a new policy (represented by the blue dot in the figure) that is sufficiently high-quality to gain the support of the left veto player with ideal ideology x_{VL}; this requires substantial quality investments because the status quo is close to the left veto player. Moreover, these quality investments are sufficiently large that L prefers to sit out rather than develop her own competing policy; this is indicated by the purple dot at the status quo.

Mixed Strategy Equilibria

Mixed strategies in our model are quite complicated; developers could be either active or inactive, and when active could mix over a continuum of scores, as well combinations of ideology and quality to deliver each score. Despite this potential complexity, we show in the Appendix that it is without loss of generality to consider strategy profiles of the following form.

Remark 1. *We consider strategy profiles in which each developer*

1. *only crafts veto-proof policies* ($s_i \geq s_0$ *and* $y_i \in [z_L(s_i), z_R(s_i)]$)
2. *chooses the score* s_i *of her policy according to a cumulative distribution function* $F_i(s_i)$
3. *crafts a unique policy* $(s_i, y_i(s_i))$ *at each score* s_i.

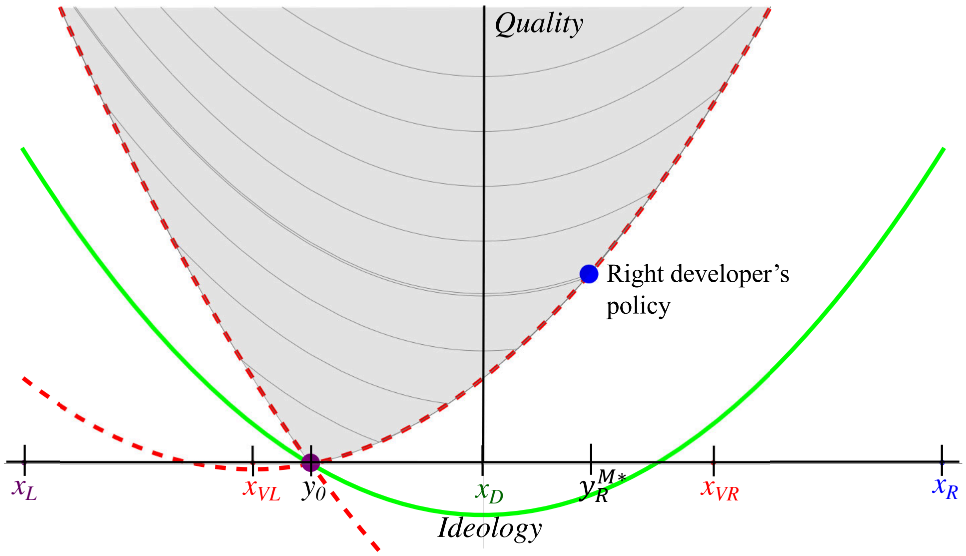

Figure 3: *A Pure Strategy Equilibrium*

To describe equilibrium it is useful to focus on the *cumulative distribution function* $F_i(s_i)$ of the *scores* of the policies that developer i crafts with positive probability. Recall that to get her policy enacted, a developer must craft a veto-proof policy that is better for the decisionmaker than any veto-proof policy that her opponent crafts; in a mixed-strategy equilibrium the developers randomize over scores as follows.

Proposition 2. *In any mixed strategy equilibrium that satisfies the conditions in Remark 1, there is a developer k and two scores $\underline{s}$ and $\overline{s}$ satisfying $s_0 \leq \underline{s} < \overline{s}$ such that*

- *developer k's score CDF F_k has support $s_0 \cup [\underline{s}, \overline{s}]$ and exactly one atom at s_0,*
- *developer $-k$'s score CDF F_{-k} has support $[\underline{s}, \overline{s}]$ and exactly one atom at $\underline{s}$.*

Mixed strategy equilibria have three properties. First, both developers mix smoothly over policies with scores in a *common interval* $[\underline{s}, \bar{s}]$. Second, one developer k has an atom at s_0, meaning that she is *sometimes inactive* in the sense of not developing any new policy. Third, the other developer $-k$ has an atom at $\underline{s}$; when $\underline{s} > s_0$ (which generically is the case in equilibrium) this means she is *always* *active*, but develops the exact policy $(\underline{s}, y_{-k}(\underline{s}))$ with strictly positive probability. Although the form of mixed strategy equilibria is reasonably intuitive, the details are cumbersome to derive. In the Appendix we analytically characterize equilibria for the centrist status quo case $y_0 = 0$ (Proposition C.1) and describe how we compute other mixed strategy equilibria numerically.

Example 2: mixed strategy equilibrium Figure 4 presents an example of a mixed strategy equilibrium; the left panel depicts score CDFs while the right panel depicts policies. In this example developer R is always active, whereas developer L is inactive with probability $F_L(s_0)$. This is intuitive because R is more dissatisfied with the status quo, which is $y_0 < 0$.

Looking at R's strategy, with probability $F_R(\underline{s})$ she develops a policy exactly at the blue dot in the right panel; otherwise she mixes smoothly over the policies on the blue curve with scores in $(\underline{s}, \bar{s}]$. Her policies are fully constrained by the left veto player, which is reflected by the fact that they are on the boundary of the veto-proof set. It may seem counterintuitive that R sometimes produces a policy at score $\underline{s}$ because L (when active) never develops a score below $\underline{s}$; R could therefore develop a lower-score policy and still win with the same probability, $F_L(s_0)$. However, R doesn't just care about the decisionmaker's support; she also needs to gain the assent of the left veto player. And just as a monopolist is willing to craft a policy at a score strictly greater than s_0 to gain a veto player's assent, so too is a developer whose opponent is sometimes inactive. In this example, R's optimal score-$\underline{s}$ policy trades off the up-front costs of developing a policy that will gain the left veto player's assent against the benefits of getting an ideological outcome closer to her ideal point when her opponent chooses to be inactive (which occurs with probability $F_L(s_0)$).

Turning now to developer L, with probability $F_L(s_0)$ she is inactive and develops no policy (the purple dot at the status quo on the right panel). With the remaining probability she mixes over policies on the purple curve with scores in $(\underline{s}, \bar{s}]$. She is willing to invest in developing these policies because they sometimes win due to the fact that R has an atom at $\underline{s}$. In this example, L's equilibrium policies are unconstrained by the veto players, i.e., they are not on the boundary of the veto-proof

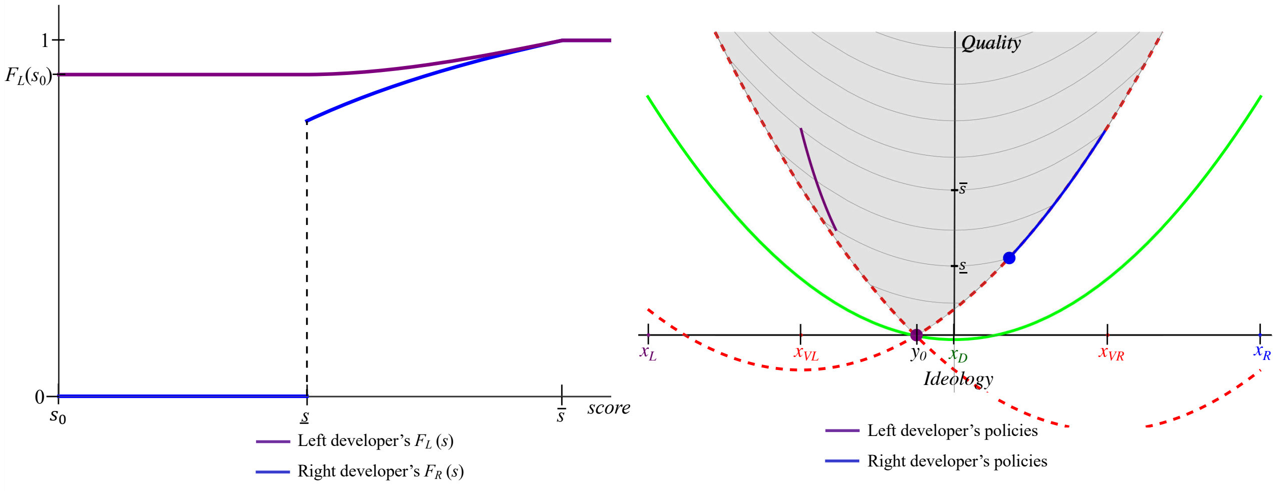

Figure 4: A Mixed Strategy Equilibrium

set.[7] Finally, the left panel shows that R's score CDF first order stochastically dominates L's score CDF, implying that the decisionmaker is strictly more likely to enact R's policy than L's policy.

Form of mixed strategy equilibria We now provide intuition about why mixed strategy equilibria must take the form in Proposition 2 – note that this subsection may be skipped with no loss of substantive insight. To do so, we consider and rule out other possible types of strategy profiles, focusing on the generic case $\underline{s} > s_0$. (If $\underline{s} = s_0$ both developers must have atoms at s_0, but the reasons are more subtle; see Appendix Proposition C.1).

First, suppose developer i sometimes crafts a policy with score $\tilde{s}_i$ that is strictly greater than the highest-score policy produced by her opponent, meaning that $\tilde{s}_i$ is strictly higher than necessary to ensure enactment of i's policy. For this to be optimal, developer i must *also* want to develop

[7] Appendix Proposition C.3 partially characterizes equilibria when, as in this example, $x_{VL} = -x_{VR}$ and $x_L = -x_R$. In particular, we show analytically that (i) the more-motivated developer's policy at $\underline{s}$ is on the boundary of the veto proof set, (ii) if a developer's policy at score $\hat{s}$ is off the boundary, it is also off the boundary at all $s > \hat{s}$, (iii) if the more-motivated developer's policy is off the boundary at s, so is the less-motivated developer's, and (iv) the more-motivated developer's score CDF first order stochastically dominates the less-motivated developer's.

this policy as a monopolist; otherwise some lower-score policy would be strictly better. But then the strategy profile must involve her *only* developing this policy (and none with other scores), further implying that $-i$ must be inactive, i.e., that the profile is actually in pure rather than mixed strategies. A similar argument rules out gaps within the common score interval $[\underline{s}, \overline{s}]$.

Another possibility is that developer i has an atom at some score $\hat{s}_i$ strictly inside the common score interval, $\hat{s}_i \in (\underline{s}, \overline{s})$. But then the policies that her opponent $-i$ is developing with scores slightly below this atom cannot be optimal, because $-i$ could profitably deviate to a score just above the atom and achieve a discrete increase in the probability that her policy is implemented. Moreover, it is not only possible, but necessary for each developer to have an atom at *either* s_0 or $\underline{s}$; otherwise her opponent would not develop policies with scores slightly above $\underline{s}$, because doing so would mean paying costs of policy development while almost always losing.

Finally, exactly one developer must have an atom at s_0, and the other at $\underline{s}$. To see why, first note that at most one developer can have an atom at $\underline{s}$. If both did, one could profitably deviate to a score just above the atom and achieve a discrete increase in the probability that her policy is enacted. Next, at most one developer can have an atom at s_0; otherwise, there would be at least one developer (say j) who has an atom at s_0 and faces an opponent *without* an atom at $\underline{s}$. Equilibrium requires that this developer be indifferent between crafting policies at s_0 and $\underline{s}$, because otherwise she would be unwilling to craft policies with scores slightly above $\underline{s}$. However, this is impossible, because any policy (s_j, y_j) with a score $s_j \in [s_0, \underline{s}]$ would have the same probability of winning, and over any such range there is a unique optimal score.

Main Results

We now state our main results. For simplicity we henceforth restrict attention to the case where developers are equally capable ($\alpha_L = \alpha_R = \alpha$) and developers and veto players are equidistant from the decisionmaker ($-x_L = x_R = x_E$, $-x_{VL} = x_{VR} = x_V$). Under these assumptions, any asymmetry in developers' incentives must arise from the location of the status quo.[8]

We refer to the developer farther from the status quo as *more-motivated*, and her opponent as *less-motivated*. The more-motivated developer is more likely to engage in policy development for two reasons. First, she has more to gain: because ideological loss functions are common and convex, she places a greater marginal value on shifts in her direction from the status quo. Second, she has an easier time persuading the opposing veto player to consent to policy changes; for example, if the status quo is $y_0 < 0$, it is easier to get the left veto player to agree to a rightward policy shift than it is to get the right veto player to agree to a leftward policy shift.

Patterns of Activity

Patterns of activity in our model depend on incentives to engage in policy development. What *incentivizes* a developer to be active? The prospect of shifting policy in her ideological direction; this prospect is more attractive when the alternative outcome (either the status quo or her opponent's policy) is far from her ideal point. On the other hand, what *deters* a developer from being active? The cost of developing a policy that can gain the support of both the veto players and the decisionmaker. This cost is higher when the opposing veto player is an extremist who demands substantial quality to compensate for small ideological movements, and is also higher when the opposing developer crafts

[8]Details for this special case are contained in Appendices C-D. Subsequently stated results are derived from a mixture of analytic and computational analysis; precise details of the analysis supporting each main text Proposition are located in Appendix D. Worth noting here is that the Appendix provides analytic necessary and sufficient conditions are provided for each *type* of equilibrium (pure or mixed and the exact form), and also analytically derives a mixed equilibrium for $y_0 = 0$ whenever the equilibrium is not pure. However, mixed equilibria with $y_0 \neq 0$ are computed numerically (albeit with several key properties of such equilibria shown analytically). While we are not able to analytically rule out coexistence of pure and mixed equilibria, or multiple mixed equilibria, we nevertheless find no parameter values exhibiting equilibrium multiplicity in our computational analysis.

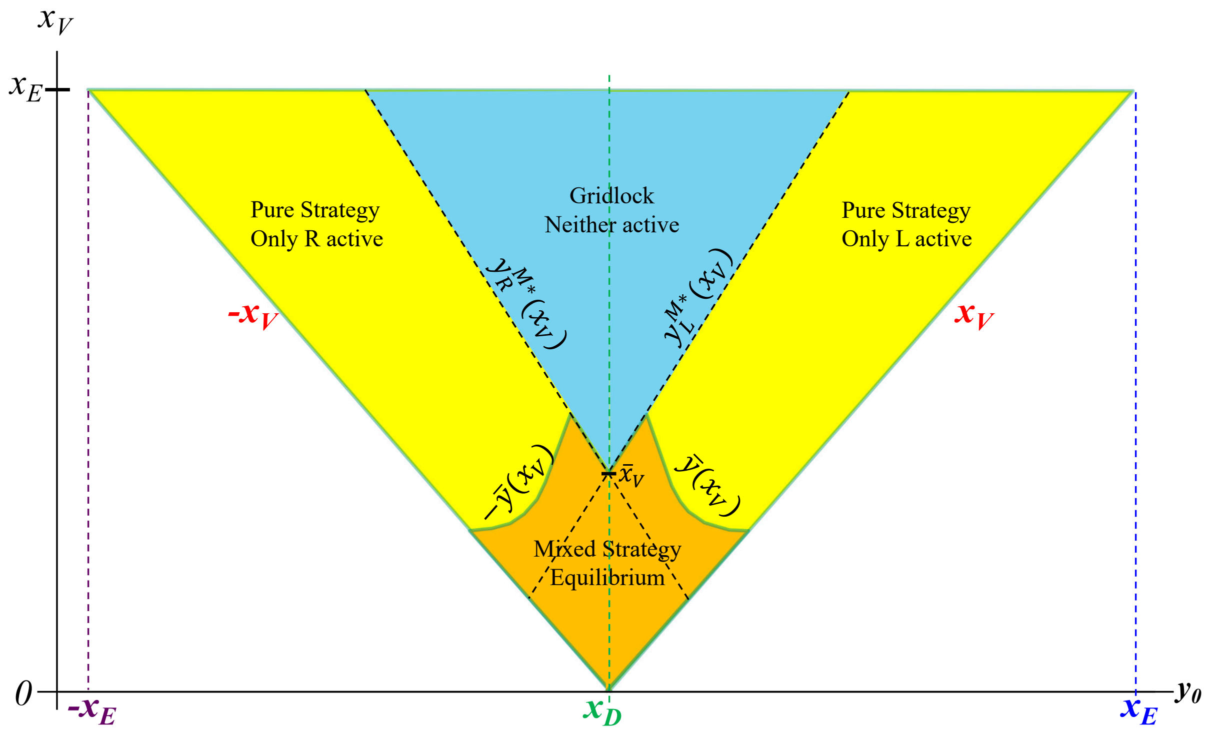

Figure 5: *Patterns of Activity as a Function of* y_0 *and* x_V

a high-quality policy that is very appealing to the decisionmaker.

The interplay between these motives generates three possible equilibrium patterns of activity: (i) neither developer is active, (ii) only the more-motivated developer is active, or (iii) the more-motivated developer is always active and the less-motivated developer is *sometimes* active (so that the equilibrium is in mixed strategies). Which of these patterns arises depends on the extremity of the veto players and the location of the status quo. Figure 5 provides an illustration for a fixed value of α, varying x_V (on the vertical axis, between 0 and x_E) and y_0 (on the horizontal axis, between $-x_V$ and x_V).

The first possibility (that neither developer is active) occurs in the blue region of Figure 5. In this region, the veto players are extreme and the status quo is moderate; each developer chooses not to develop a policy because it is too costly to get the opposing veto player's assent. The necessary condition for this case comes from our monopoly analysis. Recall from Proposition 1 that a monopolist refrains from developing a policy if the status quo is closer to her ideal point than her monopoly policy $\hat{y}_i(x_V)$ (denoting the dependence on x_V explicitly). There is thus a pure strategy equilibrium exhibiting *gridlock* in which neither developer is active if the status quo is both to the left of L's monopoly policy and to the right of R's monopoly policy, i.e., if it's sufficiently moderate, $y_0 \in [\hat{y}_R(x_V), \hat{y}_L(x_V)]$.[9] From the definition of the monopoly policy $\hat{y}_i(x_V)$, we can also see that the possibility of gridlock requires the veto players to be sufficiently extreme, i.e., $x_V \geq \bar{x}_V = \frac{x_E}{\alpha-1}$ (so that $\hat{y}_R(x_V) \leq \hat{y}_L(x_V)$).

The other two possibilities occur outside of the blue region of Figure 5, i.e., for parameter values such that at least one developer would be active as a monopolist. Not surprisingly, the set of active developers with competition always includes the more-motivated one. Whether the less-motivated developer is inactive (the yellow regions) or also active with strictly positive probability (the orange region) depends on what the developer would like to do when her more-motivated competitor acts as a monopolist; will she let this policy be enacted, or step in and develop her own alternative?

To answer this question, observe from Proposition 1 that the ideology of a monopolist's policy $\hat{y}_i(x_V)$ is *unaffected* by the location of the status quo, but its *quality* is greater the more distant is the status quo. By implication, if the more-motivated developer acts as a monopolist in the competitive model, it becomes both *more difficult* and *less intrinsically beneficial* for the less-motivated developer to craft a competing policy when the status quo is closer to her. Consequently, when the status quo is sufficiently close to the less-motivated developer, there will be a pure-strategy equilibrium in which the more motivated developer acts as a monopolist, and the less-motivated developer chooses to be inactive (the yellow regions in Figure 5); in the Appendix we characterize a cutpoint $\bar{y}(x_V)$ such that R is inactive if the status quo is to the right of $\bar{y}(x_V)$ and L is inactive if it is to the left of $-\bar{y}(x_V)$.

Conversely, if the status quo is more moderate than $\bar{y}(x_V)$ (the orange region in Figure 5), then

[9] From the definition of $\hat{y}_i$ in Proposition 1, if the cost of policy development is arbitrarily large ($\alpha \to \infty$) our model reduces to the classic spatial model with gridlock for status quos in $(-x_V, x_V)$.

equilibrium must sometimes involve active competition. The intuition is as follows. With moderate veto players and a moderate status quo, the more-motivated developer only needs to invest in a small amount of quality to get the opposing veto player to agree to a policy change. But if she did this, then her opponent would only need to invest in a small amount of quality to swing policy back in her preferred direction. Thus, in equilibrium both developers are active—the more-motivated developer always, and the less-motivated developer with strictly positive probability—and they compete to craft policies that are simultaneously appealing to the decisionmaker and acceptable to the veto players. In equilibrium, the more-motivated developer's policy is more appealing for the decisionmaker than her opponent's policy in a first-order stochastic dominance sense.[10]

Proposition 3. *Equilibria depend on the extremism of the veto players x_V and the status quo y_0:*

1. *If $x_V \geq \bar{x}_V = \frac{x_E}{\alpha - 1}$ and $y_0 \in [\hat{y}_R(x_V), \hat{y}_L(x_V)]$ neither developer is active.*
2. *Otherwise, at least one developer is active:*
   (a) *The more-motivated developer is active with probability 1.*
   (b) *If $y_0 \notin [-\bar{y}(x_V), \bar{y}(x_V)]$, the less-motivated developer is never active.*
   (c) *If $y_0 \in [-\bar{y}(x_V), \bar{y}(x_V)]$ there is a mixed strategy equilibrium in which the less-motivated developer is sometimes active.*
3. *The more-motivated developer's policies have first-order stochastically higher scores, and thus are strictly more likely to be enacted than the less-motivated developer's policies.*

At a broad level, the proposition shows that *asymmetric activity* is a fundamental feature of our model even when the developers are symmetrically extreme and equally capable, due to their

[10]This is trivially true in pure strategy equilibria with only one developer active, and also shown to be true analytically in any mixed equilibrium with $y_0 \neq 0$.

differential willingness and ability to shift policy from a non-centrist status quo.[11]

The effect of competition A natural question to ask is how the presence of each developer affects her opponent's decision over whether to engage in policy development. In one direction this question is trivial: the more-motivated developer is *always* willing to develop an alternative regardless of the presence or absence of a competing developer.[12] In contrast, the less-motivated developer's willingness to develop a new policy *is* affected by the presence of a more-motivated competitor. Surprisingly, however, a competitor's presence can either *increase* or *decrease* her policy development activity depending on the circumstances.

The differing effects of competition can be seen by considering different areas of the orange mixed-strategy region in Figure 5; recall that the dashed lines depict the ideologies of each developer's monopoly policies. When the status quo is within the mixed region but outside of the interval $[\min\{\hat{y}_L(x_V), \hat{y}_R(x_V)\}, \max\{\hat{y}_L(x_V), \hat{y}_R(x_V)\}]$, the less-motivated developer would *not* be active absent a competitor, but *is* active with one; the reason is that the status quo is insufficiently distasteful to motivate her activity, but the monopoly policy promulgated by her competitor is. This pattern is reminiscent of the Austen-Smith and Wright (1994) model of counteractive informational lobbying, in which there there are only two policy options and lobbyists have directly opposing preferences. However, in our model things are more complex; there is a range of potential policies, and developers make productive quality investments that are even valued by their opponents. Consequently, our model exhibits a second possibility; that a competitor's presence is *demotivating*, which occurs when the status quo is within both the mixed region *and* $(\hat{y}_L(x_V), \hat{y}_R(x_V))$. Under these circumstances, the less-developer *would* be willing to craft a new policy as a monopolist, but chooses not to do so when facing a high-quality policy crafted by a more-motivated ideological opponent.

[11] Only the special case $y_0 = 0$ exhibits symmetry in activity; see Appendix Proposition C.1.

[12] Of course, the policies she actually crafts depend on the presence of competition.

Effect of veto players' ideological extremism We next examine how the veto players' ideological extremism affects patterns of policy development. As can be seen toward the bottom of Figure 5, if veto players and the status quo are both moderate, then the more-motivated developer is always active, but her opponent only sometimes is. As x_V increases (moving vertically in the figure), the probability that the less-motivated developer is active decreases monotonically. For sufficiently high values of x_V, even the more-motivated developer may be deterred from developing an enactable policy. Formally, we have the following result.

Proposition 4. *The extremism of the veto players affects policy development activity as follows.*

1. *The probability that the less-motivated developer is active is strictly decreasing in x_V unless the equilibrium is in pure strategies, in which case it is constant at* 0.[13]

2. *The more-motivated developer is active if and only if the veto players are sufficiently moderate,* $x_V < \frac{\alpha|y_0|+x_E}{\alpha-1}$.

Thus, increasingly extreme veto players reduce the total amount of *participation* in policy development. At lower levels of extremism they make policy-development activity more asymmetric; the less-motivated developer increasingly disengages from developing policies, while the more motivated developer continues to participate. At higher levels of extremism they also deter the more motivated developer from engaging, resulting in gridlock and legislative stalemate.

Changes in Senate policymaking As a brief application, we argue that our model's predictions are broadly consistent with patterns of policymaking in the U.S. Senate since the 1970s. It is well-established that the Senate has become increasingly polarized during this period. More crucially

[13]Specifically, this property is exhibited by our computational solutions in the entire parameter space whenever the equilibrium is mixed. We also show analytically that whenever there is a pure equilibrium for a particular value of $\tilde{x}_V$ (so that the less-motivated developer is always inactive) there remains a pure equilibrium for strictly higher values of x_V – see Appendix D for details.

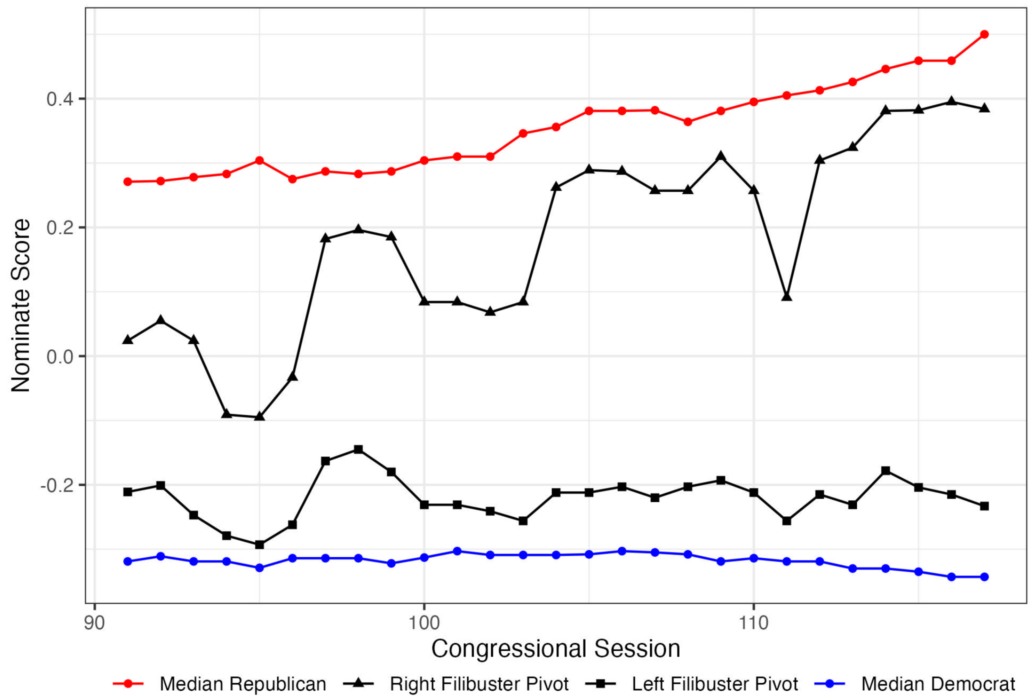

Figure 6: *Divergence of Senate Filibuster Pivots since 1970s*

from the perspective of our model, the Senate's veto players under the filibuster (the 40th and 61st most liberal senators) have diverged as well, as shown in Figure 6. Indeed, they have diverged more rapidly than the party medians; in the context of our model, this means that the *ratio* $\frac{x_V}{x_E}$ of veto player to developer extremism has increased over time. Moreover, status quos passed down from previous less-polarized Congresses are typically moderate compared to the increasingly-extreme veto players. Under these circumstances our model predicts first asymmetric policy development activity, followed by gridlock. Both of these patterns are well-documented in the empirical literature.

The first pattern—asymmetric activity—can be seen by contrasting the current highly-partisan policy development process with the traditional "textbook Congress," in which members of both parties actively worked in committees to develop proposals that could be enacted. Over time, Senate majority party leaders have played an increasingly central role in "negotiating the details of major bills" (Smith, 2011, p. 135) and "shaping the content of legislation" (Smith and Gamm, 2020, p.

216). For their part, members of the Senate minority have disengaged from creating policy proposals that might be enacted and have instead adopted a strategy of obstructionism, trying to block passage of the majority's legislation (Lee, 2016).

The second pattern—stalemate—is also well-established in the literature. It has become increasingly difficult for anyone, including majority party leaders, to get substantial new policies enacted. Nowadays, major policy changes most often occur via budget reconciliation (which doesn't require supermajorities) or during times of extraordinary crisis such as 9/11, the financial meltdown of 2007-8, and Covid-19. For most policy issues, including salient ones, legislative gridlock and stalemate have become common (Binder, 2015).

Thus, both the increasing asymmetry in policy development activity and the overall decline in successful policy development are consistent with our model. Many scholars see these twin developments as hallmarks of the decline of the Senate as an effective institution for crafting public policy; as noted by (Smith, 2014, p. 14), "an institution that once encouraged creativity, cross-party collaboration, individual expression, and the incubation of new policy ideas has become gridlocked."

We next turn our focus from the process of policy development activity to policy outcomes, by considering whether and when the decisionmaker would benefit from eliminating veto players.

Decisionmaker Welfare

In classic spatial models without costly policy development, the decisionmaker *always* benefits from eliminating veto players, since doing so allows him to revise any non-centrist status quo $y_0 \neq 0$ to reflect his own ideological preferences. With costly policy development, however, veto players don't always induce gridlock because policy change can still occur if the developers craft sufficiently-high quality policies to gain the veto players' assent. This opens up the possibility that the presence of veto players may benefit the decisionmaker because of how they affect policy development.

Welfare without veto players We first establish a baseline for decisionmaker utility in the absence of veto players. Crucially, the relevant baseline is *not* the decisionmaker's utility for a zero-quality policy at his ideal point, as it would be in a classic spatial model. Rather, it is his expected utility from *competitive policy development* in the absence of veto players; to calculate this utility we use the Hirsch and Shotts (2015) analysis of the model without veto players, in which the two developers are always active in equilibrium and mix over policies with strictly positive scores.

Corollary 2. *Absent veto players the decisionmaker's utility is*

$$EU_D^0 = 4x_E^2 \left(\int_0^1 2F \left(\int_0^F \frac{G}{\alpha(\alpha - G)} dG \right) dF \right) = 4x_E^2 \left(\alpha + \frac{1}{2} - \frac{2}{3\alpha} - (\alpha^2 - 1) \ln \left(\frac{\alpha}{\alpha - 1} \right) \right).$$

Absent veto players the decisionmaker's expected utility does not depend on the status quo, because it is "as if' the status quo is the decisionmaker's ideal point with 0 quality.[14] Also note that $EU_D^0 > 0$; this reflects the fact that a unitary decisionmaker strictly benefits from competitive policy development relative to receiving his own ideal point with 0 quality. Moreoever, the magnitude of this benefit depends on the marginal cost α of quality.

Welfare with veto players We next examine the decisionmaker's expected utility in the presence of veto players using Proposition 3, which we denote as $EU_D^{VP}(x_V, y_0)$. When there is gridlock this utility is is $-y_0^2$, which is unambiguously worse than his utility from competitive policy development absent veto players. When there is a pure strategy equilibrium with one active developer, $EU_D^{VP}(x_V, y_0)$ is the score of that developer's monopoly policy from Proposition 1. Finally, when there is a mixed strategy equilibrium we calculate $EU_D^{VP}(x_V, y_0)$ using numerical integration (see Appendix for details). Comparing $EU_D^{VP}(x_V, y_0)$ against EU_D^0 yields the following result.

Proposition 5. *The decisionmaker prefers to eliminate the veto players if the veto players **or** the status quo are sufficiently moderate. Otherwise he prefers to maintain them.*[15]

[14]This property holds even if the model is altered to restrict the decisionmaker's choices to *only* the developers' new policies and the status quo. The reason is that the more-motivated developer can profitably defeat the status quo by "developing" the decisionmaker's ideal point with 0 quality.

[15]This result is derived using a mixture of analytic and numerical analysis; see Appendix D for

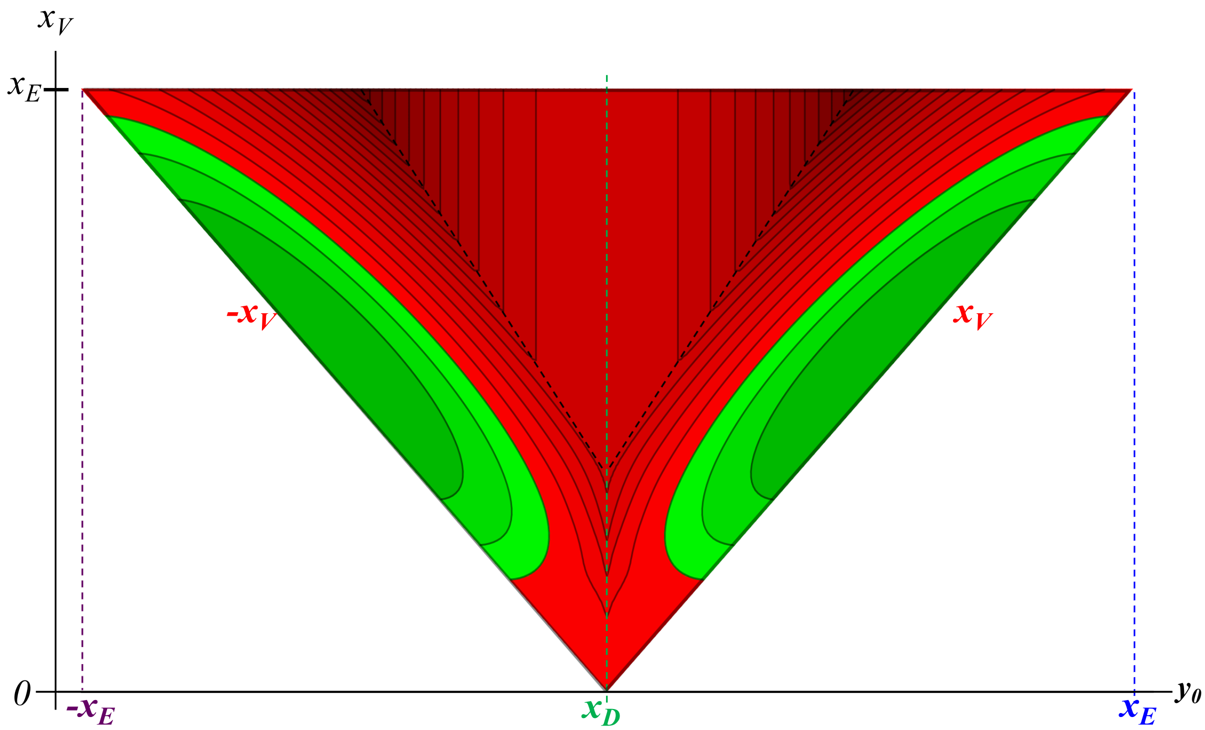

Figure 7: *Net Utility Gain from Eliminating Veto Players as Function of* y_0 *and* x_V. In red region, decisionmaker gains from eliminating veto players, whereas in green region he prefers to keep them. Darkness of shading indicates magnitude of gain or loss.

Figure 7 illustrates when the decisionmaker would be better off eliminating the veto players as a function of the veto players' extremism (on the vertical axis) and the location of the status quo (on the horizontal axis). In the red region the decisionmaker benefits from eliminating veto players; clearly, this must encompass the region where the veto players induce gridlock (the inner triangle). Conversely, in the green region he benefits from preserving veto players' role in the policy process.

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details. Specifically, we analytically derive necessary and sufficient conditions for a pure equilibrium with veto players as well as the decisionmaker's utility when these conditions hold. We further prove analytically that the decisionmaker is strictly worse off with veto players whenever $y_0 = 0$. Only when the equilibrium with veto players is mixed and also $y_0 \neq 0$ do we conduct the utility comparison using numerical integration of the computationally-derived mixed equilibria.

The figure has three important features. First, there *is* a green region—in contrast to a classic spatial model, the decisionmaker *can* sometimes benefit from the presence of veto players. Second, a necessary condition for the decisionmaker to benefit is that the status quo is *noncentrist*—this contrasts with a classic spatial model, in which the worst status quos for a centrist are those that are gridlocked far from his ideal point. Third, *observable* competition isn't necessary for the decisionmaker to benefit from the presence of veto players; indeed, within most of the green region in Figure 7 only the more-motivated developer is active, as can be seen by comparison with Figure 5.

Why can a centrist decisionmaker benefit from the presence of veto players when the status quo is noncentrist and only one developer is active? The crucial observation is that a developer is most willing to invest in policy change when she strongly dislikes the status quo, i.e., when it is far from her ideal point. It is thus under these circumstances that a somewhat-extreme opposing veto player can benefit the decisionmaker by credibly demanding higher quality to consent to policy change. Why then does this coincide with reduced participation in policy development? Because by forcing the more-motivated developer to craft a higher-quality policy (Corollary 1), a more extreme veto player also (inadvertently) induces her to *deter* her less-motivated competitor from crafting an alternative.[16] Overall, the surprising empirical implication is that the absence of *observable* competition—and apparent monopoly over policy development by one side—is not necessarily indicative of dysfunctional politics. Rather, this can occur when there is an extreme status quo on a policy issue, so that only one side is highly motivated to change it. Under these conditions, the veto player favorable to the status quo already extracts substantial quality investments from the more motivated developer, so

[16] The intuition is similar for parameter values at which both developers are active with strictly positive probability and the decisionmaker benefits from veto players (i.e., the lowest part of the green regions in Figure 7, which overlaps with the orange mixed strategy region in Figure 5). In this case, the more-motivated developer's policies are sufficiently high quality to sometimes, but not always, deter the less-motivated developer from participation.

potential competing developers rationally calculate that they are better off remaining inactive.

Having discussed when and why the decisionmaker can benefit from the presence of veto players, we now discuss what can go wrong, i.e., what happens in the red region of Figure 7 where the presence of veto players harms the decisionmaker. Veto players can have three distinct negative effects: (i) dampening productive competition, (ii) inducing gridlock, and (iii) allowing for new policies that are non-centrist and relatively low quality.

The first effect occurs when both the veto players and the status quo are very moderate, as in the bottom center of Figure 7. In this region, policy change is easy to achieve, but the developers aren't highly motivated to invest in quality because the status quo is also moderate. Thus, although the equilibrium involves both developers sometimes being active (see Figure 5), the presence of veto players simply dampens the intensity of productive competition between them. Specifically, from each developer's perspective, veto players limit both the upside of engaging in development (by constraining policy change in her own direction), and the downside of disengaging from development (by constraining policy change in her opponent's direction).

The second effect occurs when the veto players are more extreme but the status quo is still moderate. In this case, the veto players demand a lot of quality to consent to policy change, but a moderate status quo limits the developers' motivation to provide this quality. The result is gridlock, with both developers declining to craft a new policy (see the triangular region in the top center of Figure 7, which corresponds to the blue triangle in Figure 5). In fact, veto-player induced gridlock in our model is *worse for the decisionmaker* than in the classic spatial model because it does not just stop the decisionmaker from getting his ideal; it also prevents productive competition.[17]

The third effect occurs when the veto players are more extreme and the status quo is *neither* sufficiently moderate to induce gridlock, nor sufficiently extreme to motivate the more-distant developer. This effect dominates in the portions of the red region in Figure 7 for which only the more-distant

[17]The decisionmaker's loss from veto players is $EU_D^0 + y_0^2$ instead of y_0^2 as in a classic spatial model.

developer is active (i.e., the overlap with the yellow regions of Figure 5). In these regions the active developer crafts a non-centrist policy of sufficient quality to gain the veto players' support over the status quo, but of insufficient quality to surpass the benefit from unconstrained competition.

One final property worth noting is when the costs of policy development are sufficiently high ($\alpha > \tilde{\alpha} \approx 3.68$, as in Figure 7, where $\alpha = 3.75$), then it is not only *very moderate* veto players who are unambiguously harmful – it is *very extreme* ones as well.[18] Under these circumstances, there is no feasible status quo that is extreme enough to induce the more-motivated developer to craft a policy better for the decisionmaker than what he would receive under unrestricted competition. We shortly revisit this observation in our discussion of potential future developments in the U.S. Senate.

Filibusters We conclude by using our model to reexamine a critical question in legislative studies: why does the U.S. Senate allow a submajority of 41 members to block legislation that a majority prefers to the status quo? The U.S. Constitution dictates that the Senate is a self-organizing body, and both constitutional scholarship and Senate history support the proposition that a simple majority may eliminate or modify the filibuster (Gold and Gupta, 2004). However, as documented by Binder and Smith (2001), there has there never been a Senate majority in support of eliminating the legislative filibuster by reducing the cloture requirement to 51 votes. Most recently, in early 2022 the Senate voted 52-48 against a one-time exception to the filibuster that would have made it possible to pass a voting rights bill. At the time, 21 Democratics supported eliminating the filibuster, 27 supported changes such as requiring a "talking filibuster," and two of the most moderate Democrats (Senators Manchin and Sinema) opposed any changes (Rieger and Adrian, 2022).

From the perspective of classic spatial models of policymaking, centrist Senators' support for the filibuster presents a puzzle. In such models, supermajority rules harm centrists by preventing them from altering policies to reflect their own ideal point. One explanation previously offered is

[18]The derivation is straightforward, because the equilibrium for such parameter values is always in pure strategies, with either gridlock or a single active developer. See Appendix for details.

that centrists use supermajority requirements to *counterbalance* the power of non-centrists *agenda-setters* (Krehbiel and Krehbiel, 2023; Peress, 2009). However, it is unclear whether formal agenda setting power is actually present in the U.S. Senate. The absence of germaneness requirements gives individual members considerable power to force proposals onto the agenda, and party leaders expend extraordinary effort to accommodate the scheduling demands of individual members (Oleszek et al., 2015). Moreover, to the extent that formal agenda setting power *is* present, that power can only exist with the consent of a Senate majority (Krehbiel, 1992). Thus, any theoretical explanation of the filibuster that relies on formal agenda power must also address why centrists would add an additional procedure to address its shortcomings, rather than simply revoke that agenda power.

In contrast, our model shows that *even in the absence of formal agenda power*, centrist Senators can benefit from maintaining supermajority requirements that create de facto veto players. The reason is that policy developers' need to satisfy those veto players can force them to craft more moderate and higher-quality policies. As shown in Proposition 5 and Figure 7, centrists are most likely to benefit from the filibuster when the implied veto players (the 41st and 60th most liberal Senators) are somewhat non-centrist and the status quo is also non-centrist. A non-centrist status quo could occur in policy areas that are rapidly changing, such as financial regulation or health care; given limits on Senators' time and attention, the issues most likely to receive legislative attention are arguably precisely these issues.

Finally, our model does *not* imply that centrists *always* benefit from the filibuster. Rather, as shown in Proposition 5 and Figure 7, there are several circumstances under which veto players harm centrists. In the context of current debates about the filibuster, one of these is particularly relevant: when veto players are very extreme and policy development is very costly. Does this describe the contemporary Senate? As shown in Figure 6, the veto players induced by the filibuster, i.e., the 40th and 61st most liberal Senators, have indeed become increasingly polarized over the past few decades. Simultaneously, Congress has disinvested in its own capacity for policymaking—despite the fact that policy issues have become vastly more complicated—by substantially reducing the number of staffers, allowing personnel funding to remain constant or decrease in real terms, and reducing funding for agencies like the CBO, CRS, and GAO (Reynolds, 2020). Given these changes, scholars and commentators have become concerned that it is increasingly difficult for members' offices to craft high-quality policies. Thus, our model suggests that although centrists may have benefitted from the filibuster in the past, calls for reform may become increasingly persuasive if these trends persist.

Conclusion

In this paper we have explored a model of costly policy development in political environments where actors have divergent objectives but also a shared interest in enacting high-quality policies. In this setting, policy developers can obtain informal agenda power by crafting policies that are well-designed but that also promote their own objectives. We have assessed how the inclusion of veto playersin decisionmaking affects the policies enacted as well as the welfare of centrist decisionmakers.

Absent veto players, competing developers will always craft policies that benefit a centrist decisionmaker irrespective of the status quo policy. The effect of including veto players in decisionmaking depends on the status quo. If veto players are quite moderate, the dominant effect will be to dampen productive competition, thereby making the decisionmaker worse off. However, if they are sufficiently noncentrist, then the developer dissatisfied with the status quo will be willing to work hard to craft a high-quality alternative, and an opposing veto player will force her to do so, thereby benefitting the decisionmaker. By implication, veto players will most benefit a centrist decisionmaker precisely when standard spatial models predict that they are most harmful, i.e., when the status quo is non-centrist. In addition, under such circumstances the developer satisfied with the status quo will often or always refrain from crafting a competing policy, which reflects the fact that veto players have already forced her competitor to craft a reasonably-moderate and high quality policy. Thus, veto players will be most beneficial to the decisionmaker precisely when their presence also inhibits observable competition.

Our model also yields testable predictions on the number of well-developed policy proposals that will be created for a given issue: multiple serious proposals are likely to be developed when the status quo is centrist or when veto players are absent. It further yields predictions about the quality of policies that are adopted. Quality is difficult to measure empirically because it comes from a variety of characteristics. However, if measurement issues can be overcome, one could test the model's prediction that centrist policies that are successfully enacted tend to be of lower quality relative to noncentrist ones, because the latter must be more carefully crafted to gain broad approval.

Finally, our model has surprising implications for institutional design of policymaking capacity in Congress. A natural intuition is that the best way to allocate policymaking capacity in polarized times is to invest in shared resources that can be used by all members for policy development. Our model suggests this intuition may be off-target, and that reformers might instead do better by giving resources to non-centrist policy developers. A natural fear is that such policy developers will inevitably use this capacity to further their own extreme objectives, as suggested by the pejoriative characterization in Drutman and LaPira (2020) of the current regime in Congress as "adversarial clientilism." However, if policy developers are constrained—either by potential competition or opposing veto players—then they will need to focus their energies on generating policies that are relatively high-quality and moderate in the hopes of getting them enacted.

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Supporting Information for

Veto Players and Policy Development

Contents

A General Equilibrium Analysis

Recall that the decisionmaker's utility for a policy (y, q) is its *score* $s(y, q)$, and that for the analysis we reparameterize policies (y, q) to be expressed in terms of score and ideology (s, y). The following definitions are then easily verified.

Definition A.1.

1. *The implied quality of a policy* (s, y) *is* $q = s + y^2$*, and the score of the status quo is* $s_0 = -y_0^2$.

2. *Player* i*'s utility for policy* (s, y) *is* $V_i(s, y) = U_i\left(y, s + y^2\right) = -x_i^2 + s + 2x_i y$

3. *Developer* i*'s cost to craft policy* (s, y) *is* $\alpha_i\left(s + y^2\right)$

4. *All veto players weakly prefer policy* (s, y) *to the status quo if and only if* $s \geq s_0$ *and* $y \in Y_V(s) = [z_L(s), z_R(s)]$*, where* $z_i(s) = y_0 - \frac{s - s_0}{2x_{V-i}}$.

In the policymaking stage the decisionmaker is an agenda-setter vis-a-vis the veto players. As is customary in agenda-setting models we henceforth restrict attention to strategy profiles in which both veto players break indifference in favor of the decisionmaker's proposal.

In this appendix, we first prove Proposition 1 for the monopoly model. We then give a detailed analysis of the structure of equilibria in the competitive model. Next, we link the paper's results for the competitive model to this analysis. The final section of the appendix describes the data used in Figure 6.

A.1 Proof of Proposition 1 (Equilibrium of Monopoly Variant)

First recall that (i) the status quo (s_0, y_0) is the unique veto-proof policy already available to the decisionmaker (who can only access policies with strictly positive quality if they are developed by a developer), and (ii) any policy weakly preferred by *both* veto players to the status quo will also be weakly preferred by the decisionmaker to the status quo (because his ideal point is between them). Consequently, it is optimal for the decisionmaker to simply propose whatever the monopoly developer crafts – any other feasible proposal will be vetoed and result in the same outcome (the status quo), while proposing the developer's policy will only result in an outcome different from the status quo when it is veto proof and therefore also weakly preferred by the decisionmaker to the status quo. For simplicity we henceforth restrict attention to such profiles.

With the preceding restriction the game is "as if" the developer is an agenda setter directly proposing her policy to the veto players, who will break indifference in favor of her proposal. This further implies that we may restrict the developer's choice space to veto-proof policies without loss of generality, since "developing" the status quo is a free veto-proof option available to her, and developing any policy that fails veto-proofness will result in the same outcome (the status quo) at weakly higher cost. With these observations in hand it is easily verified (as stated in the main text) that an optimal policy (s_i^{M*}, y_i^{M*}) for the developer must satisfy

$$\underset{\{(s_i,y_i):s_i\geq s_0,y_i\in[z_L(s_i),z_R(s_i)]\}}{\arg\max}\left\{\underbrace{-\left(\alpha_i-1\right)s_i}_{\text{score effect}}+\underbrace{2x_iy_i-\alpha_iy_i^2}_{\text{ideology effect}}\right\}. \tag{A.1}$$

Now suppose without loss of generality that the monopoly developer is $i = R$. Our assumptions from the main text that $y_0 \in [x_{VL}, x_{VR}]$ and $x_{VR} \leq x_R$ imply that $y_0 \leq x_R$, so the monopoly developer wishes to move any potential status quo rightward. We now proceed in three steps.

Step 1. Developer R never develops a policy (s_R, y_R) with $y_R < y_0$. Doing so entails paying strictly positive costs to move policy away from her ideal point, and it is straightforward to see from Equation A.1 that the developer would be better off proposing the status quo, at zero cost. Note also that because $x_{VL} < 0 < x_{VR}$, any policy developed with $y_R \geq y_0$ will be veto proof (and therefore enacted if developed) if and only if the left veto player weakly prefers it to the status quo.

Step 2. At any $y_R \geq y_0$ the optimal policy for R to develop must satisfy $y_R = z_R(s_R)$; because $\alpha_R > 1$, for any policy (s_R, y_R) not on the boundary of the veto proof set, R is strictly better off developing a lower quality policy at the same ideology that is on the boundary of the veto proof set, i.e., $(\tilde{s}_R, y_R)$, where $y_R = z_R(\tilde{s}_R)$.

Step 3. We find the optimal policy (s_R, y_R) for R to develop with $y_R \geq y_0$ and $y_R = z_R(s_R)$. Applying Step 2 and inverting the relationship between score and ideology in Definition A.1, the optimal score s_R for ideology y_R is $s_R = 2x_{VL}(y_0 - y_R) - s_0$. Substituting into Equation A.1, R maximizes:

$$-(\alpha_R - 1)(2x_{VL}(y_0 - y_R) - s_0) + 2x_R y_R - \alpha_R y_R^2,$$

which is strictly concave in y_R. Differentiating respect to y_R and setting equal to zero yields

$$\hat{y}_R = \frac{1}{\alpha_R} x_R + \left(1 - \frac{1}{\alpha_R}\right) x_{VL}.$$

For $y_0 \leq \hat{y}_R$ it is optimal for the developer to develop a policy at $\hat{y}_R$, whereas for $\hat{y}_R < y_0$ the developer's utility is strictly higher from sitting out than it is for developing any (s_R, y_R) with $y_R > y_0$ and $y_R = z_R(s_R)$.

A.2 Preliminary Analysis of Competitive Model

A developer pure strategy (s_i, y_i) is a two-dimensional element of the set of scores and ideologies that imply weakly positive-quality policies: $\mathbb{B} \equiv \left\{(s, y) \in R^2 \mid \left(s - y_0^2\right) + y^2 \geq 0\right\}$. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$. For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution. In this section we first derive five key conditions for equilibrium.

As in the model with a monopoly developer, we first argue that it suffices to restrict attention to strategy profiles in which both developers only develop veto-proof policies.

Lemma A.1. *Consider an equilibrium strategy profile in which the developers sometimes develop policies that fail veto-proofness; then the modified strategy profile in which each developer develops the status quo whenever the original profile called for her to develop a policy that failed veto-proofness is also an equilibrium that yields the same distribution over outcomes and payoffs.*

Proof: First recall that because $x_{VL} < 0 < x_{VR}$, the status quo (s_0, y_0) is both the unique score-minimizing policy among those that are veto proof, and the unique veto-proof policy that is 0-quality. Now consider *any* profile of policies $\{(s_i, y_i), (s_{-i}, y_{-i})\}$ such that i's policy fails veto-proofness; we argue that the alternative profile $\{(s_i = s_0, y_i = y_0), (s_{-i}, y_{-i})\}$ in which i "develops" the status quo yields the same probability distribution over outcomes. If (s_{-i}, y_{-i}) is both veto-proof and distinct from the status quo then $s_{-i} > 0$, in either profile the decisionmaker is strictly better off proposing (s_{-i}, y_{-i}) than any other feasible policy, and it will be accepted for sure. Otherwise, in either profile any feasible proposal either fails veto-proofness or is equal to the status quo, so any feasible proposal by the decisionmaker will result in the status quo for sure.

The preceding observation then immediately yields the desired result through a series of observations. First, in any equilibrium strategy profile, each developer must *never* develop a *strictly* positive-quality policy that fails veto proofness; a developer who did so would be strictly better off deviating to a strategy in which they instead "develop" the status quo, because outcomes would be unaffected and they would strictly save on the costs of policy development. Second, whenever developing a 0-quality policy that fails veto-proofness is a best response for i, so too is developing the status quo; thus, altering i's equilibrium strategy to have him develop the status quo whenever he previously developed a 0-quality policy that failed veto-proofness remains a best response for i. Finally, altering developer i's equilibrium strategy to have him develop the status quo whenever he previously developed a 0-quality policy that failed veto proofness does not change $-i$'s utility from develop any particular competing policy (s_{-i}, y_{-i}), and therefore the set of strategies that are a best response for her. **QED**

Having restricted the strategy space to the set of veto-proof policies Y_V, let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i; when both developers' policies are in the veto proof set, a policy with the strictly highest score will be the outcome for sure. We now derive necessary and sufficient equilibrium conditions in a series of lemmas. Let $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility for developing a policy $(s_i, y_i) \in Y_V$ if a "score tie" would be broken in her favor. Clearly this is i's expected utility from developing any policy with $s_i \geq s_0$ where $-i$ has no atom, and regardless of whether $-i$ has an atom at s_i, i can always achieve utility arbitrarily close to $\bar{\Pi}_i (s_i, y_i; \sigma_{-i})$ by developing an $\varepsilon-$higher score policy. Now

$$\bar{\Pi}_i (s_i, y_i; \sigma_{-i}) = -\alpha_i \left(s_i + y^2\right) + F_{-i} (s_i) \cdot V_i (s_i, y_i) + \int\limits_{s_{-i} > s_i} V_i (s_{-i}, y_{-i})\, d\sigma_{-i}. \tag{A.2}$$

The first term is the up-front cost of generating the quality. With probability $F_{-i} (s_i)$, i's opponent develops a policy with a lower score, i's policy in this case will then be proposed and passed for sure, and this yields utility $V_i (s_i, y_i)$. With the remaining probability $-i$'s policy will be proposed and passed for sure, yielding utility $V_i (s_{-i}, y_{-i})$.

Note that only the first two terms of Equation A.2 are affected by y_i. Taking the first derivative w.r.t. y_i yields $-2\alpha_i y_i + 2F_{-i} (s_i) x_i$, which is strictly decreasing in y_i. Given s_i, there is thus a unique *strictly* optimal value of y_i in the veto-proof interval $[z_L (s_i), z_R (s_i)]$, yielding the following.

Lemma A.2. *At any score $s \geq s_0$ where $F_{-i} (\cdot)$ has no atom or i would win in a tie for sure, the policy $(s, y_i^* (s))$ is i's strictly best score-s veto-proof policy, where $y_i^*(s) = \hat{y}_i(s; F_{-i}(s))$ and*

$$\hat{y}_i (s; F_{-i}) = \min \left\{ \max \left\{ z_L (s), \frac{x_i}{\alpha_i} F_{-i} \right\}, z_R (s) \right\}.$$

Lemma A.2 states that for almost every score $s > s_0$, developer i's best combination of ideology and quality to generate a veto-proof policy with that score is unique. Specifically, the optimal ideology is the closest veto-proof ideology to the *unconstrained optimal ideology* $\frac{x_i}{\alpha_i} F_{-i}(s)$; notably, the unconstrained optimum only depends on the score s *indirectly* through its impact on the probability of "winning" $F_{-i}(s)$.[1] The expression in the lemma may be written more intuitively as

$$y_i^*(s) = \begin{cases} z_L(s) & \text{if } \frac{x_i}{\alpha_i} F_{-i}(s) < z_L(s) \\ \frac{x_i}{\alpha_i} F_{-i}(s) & \text{if } z_L(s) \leq \frac{x_i}{\alpha_i} F_{-i}(s) \leq z_R(s) \\ z_R(s) & \text{if } z_R(s) < \frac{x_i}{\alpha_i} F_{-i}(s) \end{cases}.$$

Finally, we say that a strategy profile satisfies *ideological optimality* if each developer's policy (s_i, y_i) targets the strictly best veto proof ideology (i.e. $y_i = y_i^*(s_i)$) given its score s_i with probability 1.

The next lemma establishes that in equilibrium there is 0 probability of a tie at scores strictly larger than s_0, a property we term *no ties*. The absence of score ties is an intuitive consequence of opposing ideological interests and the fact that generating quality is all pay. However, the distribution of outcomes conditional on a tie is potentially complicated, because it can depend on each developer's distribution of proposals as well the decisionmaker's rule for choosing between each possible pair of proposals. We show that if ties occur at strictly positive scores, at least one developer will find it in her interest to either invest up-front in a bit more quality and make a proposal that she prefers over the expected ideological outcome from a tie, or develop no new policy.

Lemma A.3. *In equilibrium there is 0-probability of a tie at scores $s > s_0$.*

Proof: We prove by contradiction. Suppose not, so that in some equilibrium there is a strictly positive probability of a tie at a score $s > s_0$; we show that at least one developer must have a strictly profitable deviation. We begin by introducing notation. First, let $p_i^s > 0$ denote the probability developer i crafts a policy with score exactly equal to s; further note that this may involve mixing over distinct policies with the same score. Second, let $\bar{y}_i^s$ denote the *expected ideology of i's policy* conditional on crafting a score-s policy; further observe that the up-front cost to developer i of crafting the exact policy $(s, \bar{y}_i^s)$ is weakly lower than the *expected cost* of mixing according to her

[1] This simplifying property is critical to deriving the simple analytic characterization of equilibria in the asymmetric game without veto players – see Hirsch (2023).

strategy conditional on crafting a score s policy (since from part 3 of Definition A.1 quality costs are convex in ideology holding score fixed). Third, let

$$y_D^s = \max\left\{\min\left\{0, z_R\left(s\right)\right\}, z_L\left(s\right)\right\} = \begin{cases} z_L\left(s\right) & \text{if } z_L\left(s\right) > 0 \\ 0 & \text{if } z_L\left(s\right) \leq 0 \leq z_R\left(s\right) \\ z_R\left(s\right) & \text{if } z_R\left(s\right) < 0 \end{cases}.$$

This is the ideological location closest to 0 (the decisionmaker's ideal ideology) that can be attached to a score-s policy and remain veto proof; from part 3 of Definition A.1 the policy (s, y_D^s) is the *cheapest* score-s veto-proof policy that either developer can craft. Finally, let $\bar{y}^s$ denote the *expected ideology of the final policy outcome* conditional on a tie at score s (this will implicitly depend on the exact policies each developer crafts with score s, as well as the decisionmaker's tie-breaking rules).

Now to see that at least one developer must have a strictly profitable deviation, first observe (from the definition of Nash equilibrium) that each developer can achieve her equilibrium utility by mixing according to her strategy *conditional* on developing only score-s policies; it therefore suffices to show that at least one developer can do strictly better than this by deviating. Next recall that each developer's policy utility $V_i(s, y)$ is linear in y – thus, getting outcome (s, y) for sure or a mix of score-s outcomes with expected ideology y yields identical policy utility. There are three subcases.

Suppose first that $\bar{y}^s \neq y_D^s$, so that the expected ideology of the final outcome conditional on a tie at score s is *distinct* from the cheapest veto-proof ideology to target at score s. Because the developers wish to move ideology in strictly opposite directions holding score fixed (by part 2 of Definition A.1), exactly one developer k *strictly* prefers policy (s, y_D^s) to policy $(s, \bar{y}^s)$, implying that her policy utility from getting the former for sure is strictly higher than her expected policy utility from a tie at score s. If this developer k plays according to her strategy conditional on crafting a score-s policy, then she gets policy utility $V_k(s, \bar{y}_k^s)$ with probability $F_{-k}(s) - p_{-k}^s$ (when her opponent crafts a policy with *strictly* lower score than s) and policy utility $V_k(s, \bar{y}^s)$ with probability p_{-k}^s (when her opponent crafts a policy with score exactly equal to s). If she were instead to deviate to crafting policy (s, y_D^s) with probability $\frac{p_{-k}}{F_{-k}(s)}$ and policy $(s, \bar{y}_k^s)$ with probability $1 - \frac{p_{-k}}{F_{-k}(s)}$ *and always win in a tie at score* s, she would again get policy utility $V_k(s, \bar{y}_k^s)$ with probability $F_{-k}(s) - p_{-k}^s$, but policy utility $V_k(s, y_D^s) > V_k(s, \bar{y}^s)$ with probability p_{-k}^s; this would be a strictly profitable deviation since the latter strategy yields strictly higher policy utility at a weakly lower up-front cost. Finally, while developer k can't achieve exactly this utility with this deviation (since she's not assured to win in a tie at score s) she can achieve utility arbitrarily close to it by using otherwise-identical policies with scores just above s, and therefore has a strictly profitable deviation.

Suppose next that $y_D^s = \bar{y}^s$, so that the expected ideology of the final outcome in a tie at score s is equal to the cheapest veto-proof ideology to target at score s. If at least one developer k crafts a policy other than (s, y_D^s) with strictly positive probability, then the deviation described in the preceding paragraph would again be strictly profitable – not because the policy outcome would be strictly better with probability p_{-k}^s (it would be equivalent), but because she would strictly save on the up-front cost of policy development when she crafts $(s, \bar{y}_k^s)$.

Suppose finally that $y_D^s = \bar{y}^s$ and that both developers craft the exact policy (s, y_D^s) with probability 1 conditional on crafting a score-s policy. Then each developer's utility from crafting (s, y_D^s) is "as if" she always wins in a tie (or equivalently as if her opponent has no atom at score s), and so her equilibrium utility is exactly $\bar{\Pi}_i\left(s, y_D^s; \sigma_{-i}\right)$. If at least one developer k's strictly-best veto proof ideology $y_k^*(s)$ at score s from Lemma A.2 differs from y_D^s, then it follows immediately that she has a strictly profitable deviation to developing a policy at ideology $y_k^*(s)$ with a score just above s. Alternatively, if both developers' strictly-best veto proof ideologies at score s are exactly y_D^s, then (using the definition of $y_i^*(s)$ in Lemma A.2 and that $F_i(s) > 0\ \forall i$) it must be that y_D^s is on the boundary of the veto-proof set ($y_D^s = z_j(s)$ for some $j \in \{L, R\}$). If the developer $-j$ on the *opposite* side of the decisionmaker from the relevant boundary were to deviate by dropping out of the contest (i.e., crafting (s_0, y_0)), then her net utility gain would be $\alpha_{-j}\left(s + (z_j(s))^2\right) + \int_{s_0}^{s}\left(V_{-j}(s_j, y_j) - V_{-j}(s, z_j(s))\right) d\sigma_j$. But since each developer is weakly more extreme than her same-sided veto player ($|x_i| \geq |x_{Vi}|$), it is easily verified that $(s, z_j(s))$ is the *weakly worst veto proof policy with score* $s_{-j} \in [s_0, s]$ for developer $-j$; hence the net utility gain is *strictly* positive and this is a profitable deviation for $-j$. **QED**

Lemmas A.2 – A.3 jointly imply that in equilibrium, developer i can compute her expected utility *as if* her opponent only crafts veto-proof policies of the form $\left(s_{-i}, y^*_{-i}(s_{-i})\right)$. Her expected utility from crafting *any* veto-proof policy (s_i, y_i) with score $s_i \geq s_0$ where $-i$ has no atom (or a tie would be broken in i's favor) is therefore

$$\bar{\Pi}^*_i(s_i, y_i; \mathbf{F}) = -\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y^*_{-i}(s_{-i})\right) dF_{-i}, \tag{A.3}$$

and her utility from crafting the *best* veto-proof policy with score s_i (where $-i$ has no atom or a tie would be broken in her favor) is therefore $\bar{\Pi}^*_i(s_i, y^*_i(s_i); \mathbf{F}) =$

$$\bar{\Pi}^*_i(s_i; \mathbf{F}) = -\alpha_i\left(s_i + [y^*_i(s_i)]^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y^*_i(s_i)) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y^*_{-i}(s_{-i})\right) dF_{-i}. \tag{A.4}$$

We now establish several useful properties of this function.

Lemma A.4. $\bar{\Pi}^*_i(s; \mathbf{F})$ *is right-continuous; in addition* $\lim_{s \to \hat{s}^-}\left\{\bar{\Pi}^*_i(s; \mathbf{F})\right\} \leq \bar{\Pi}^*_i(\hat{s}; \mathbf{F}) \, \forall \hat{s} > s_0$ *when the strategy profile satisfies ideological optimality and no ties.*

Proof: It is straightforward to verify from the definition that $\bar{\Pi}^*_i(s; \mathbf{F})$ inherits the right-continuity of $F_{-i}(s)$, and that its only potential points of discontinuity over $\hat{s} > s_0$ are at scores where $-i$ has an atom $p^{\hat{s}}_{-i} > 0$. Next, we establish that $\lim_{s \to \hat{s}^-}\left\{\bar{\Pi}^*_i(s; \mathbf{F})\right\} \leq \bar{\Pi}^*_i(\hat{s}; \mathbf{F})$ for $\hat{s} > s_0$; intuitively, if i were to deviate from her strategy to any score $\hat{s} > s_0$, it would be weakly better to just win than to just lose at that score. Clearly the property is trivial if $\bar{\Pi}^*_i(s; \mathbf{F})$ is continuous at $\hat{s}$, so suppose $-i$ has an atom at $\hat{s}$; then (by no ties) i does not, and at the atom $-i$ develops $\left(\hat{s}, y^*_{-i}(\hat{s})\right)$. Let $y^{\hat{s}-}_i = \lim_{s \to \hat{s}^-}\{y^*_i(s)\}$ denote i's optimal ideology if she were to just lose at score $\hat{s}$. Then it is easily verified that $\bar{\Pi}^*_i\left(\hat{s}, y^{\hat{s}-}_i; \mathbf{F}\right) - \lim_{s \to \hat{s}^-}\left\{\bar{\Pi}^*_i(s; \mathbf{F})\right\} = p^{\hat{s}}_{-i}\left(V_i\left(\hat{s}, y^{\hat{s}-}_i\right) - V_i\left(\hat{s}, y^*_{-i}(\hat{s})\right)\right)$. Finally $V_i\left(\hat{s}, y^{\hat{s}-}_i\right) - V_i\left(\hat{s}, y^*_{-i}(\hat{s})\right) \geq 0$ since $V_i\left(\hat{s}, y^{\hat{s}-}_i\right) \geq V_i\left(\hat{s}, y^{\hat{s}}_D\right) \geq V_i\left(\hat{s}, y^*_{-i}(\hat{s})\right)$, recalling that $y^{\hat{s}}_D$ is defined in the proof of Lemma A.3 as the ideology closest to the decisionmaker's ideal that may be attached a score-$\hat{s}$ policy and remain veto-proof. The first inequality comes from Lemma A.2 applied to i; the second inequality comes from Lemma A.2 applied to $-i$. Finally $\bar{\Pi}_i^*(\hat{s};\mathbf{F}) \geq \bar{\Pi}_i^*\left(\hat{s}, y_i^{\hat{s}-};\mathbf{F}\right)$ from the definition of $\bar{\Pi}_i^*(\hat{s};\mathbf{F})$. **QED**

Having established these properties, we next show that in equilibrium any score in the support of developer i's score CDF $F_i(s)$ must maximize $\bar{\Pi}^*(s_i;\mathbf{F})$, a property we term *score optimality.*

Lemma A.5. *For all i and $\hat{s}$ in the support of $F_i(\cdot)$, $\bar{\Pi}_i^*(\hat{s};\mathbf{F}) = \max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$.*

Proof: First observe that equilibrium requires $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$ to be well-defined – otherwise i would not have a best response. Next, for any score $\hat{s} \geq s_0$ it is trivially the case that $\bar{\Pi}_i^*(\hat{s};\mathbf{F}) \leq \max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$; hence it suffices to show that $\hat{s}$ in the support of F_i implies $\bar{\Pi}_i^*(\hat{s};\mathbf{F}) \geq \max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$.

Suppose first that $\hat{s} \geq s_0$ is the support of $F_i(\cdot)$ but a neighborhood below $\hat{s}$ is not ($\Pr(s_i \in (\hat{s}-\epsilon,\hat{s}))$ $= 0$ for sufficiently small ϵ). Then either i has an atom at $\hat{s}$ or she has support in any neighborhood above $\hat{s}$ (i.e. $\Pr(s_i \in (\hat{s},\hat{s}+\epsilon)) > 0\ \forall\epsilon$). If i has an atom at $\hat{s}$ then her utility from choosing $\hat{s}$ must be exactly $\bar{\Pi}_i^*(\hat{s};\mathbf{F})$ – either because $\hat{s} > s_0$ and $-i$ has no atom there by Lemma A.3, or because $\hat{s} = s_0$ and there is a unique veto proof ideology y_0 (so the decisionmaker's tie breaking rule doesn't matter). Regardless of which is the case, we must have $\lim\limits_{s\to\hat{s}^+}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\} = \bar{\Pi}_i^*(\hat{s};\mathbf{F}) \geq \max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$; otherwise i would have a strictly profitable deviation taking probability weight from $\hat{s}$ or a neighborhood above and reallocating to scores yielding utility arbitrarily close to $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$.

Suppose next that $\hat{s} \geq s_0$ is in the support of $F_i(\cdot)$ and a neighborhood below is as well (i.e. $\Pr(s_i \in (\hat{s}-\epsilon,\hat{s})) > 0\ \forall\epsilon > 0$). Then $\hat{s} > s_0$ (by the restriction to veto-proof policies) and $\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\} \geq \max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$, since otherwise i would have a strictly profitable deviation taking probability weight from a neighborhood below $\hat{s}$ and reallocating to scores that yield utility arbitrarily close to $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$. But by Lemma A.4 we have $\bar{\Pi}_i^*(\hat{s};\mathbf{F}) \geq \lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$ so again $\bar{\Pi}_i^* (\hat{s}; \mathbf{F}) \geq \max_{s \geq s_0} \left\{\bar{\Pi}_i^* (s; \mathbf{F})\right\}$. **QED**

Finally, we conclude the general analysis by showing that the preceding necessary conditions are also sufficient for equilibrium.

Lemma A.6. *When each developer only crafts veto-proof policies, the properties of ideological optimality, no ties, and score optimality are jointly necessary and sufficient for equilibrium.*

Proof: Necessity is already shown. Now it is straightforward that a strategy profile satisfying no ties, ideological optimality, and score optimality yields utility equal to $\max_{s \geq s_0} \left\{\bar{\Pi}_i^* (s; \mathbf{F})\right\} = U_i^*$ by construction. Also recall that i's utility for developing any (s_i, y_i) where $-i$ has no atom is $\bar{\Pi}_i^* (s_i, y_i; \mathbf{F}) \leq \bar{\Pi}_i^* (s_i; \mathbf{F}) \leq U_i^*$, so consider a policy $(\hat{s}, \hat{y}_i)$ at a score $\hat{s}$ where $-i$ has an atom; then i's actual utility $\Pi_i^* (\hat{s}, \hat{y}_i; \mathbf{F})$ from developing $(\hat{s}, \hat{y}_i)$ is $\leq \max \left\{ \bar{\Pi}_i^* (\hat{y}_i, \hat{s}; \mathbf{F}), \lim\limits_{s \to \hat{s}^-} \left\{\bar{\Pi}_i^* (\hat{y}_i, \hat{s}; \mathbf{F})\right\} \right\}$ since either $V_i (\hat{s}, \hat{y}_i) \geq V_i (\hat{s}, \hat{y}_{-i} (\hat{s}))$ (so i prefers to always win at the atom) or $V_i (\hat{s}, \hat{y}_i) < V_i (\hat{s}, \hat{y}_{-i} (\hat{s}))$ (so i prefers to always lose at the atom). But both quantities are $\leq \bar{\Pi}_i^* (\hat{s}; \mathbf{F}) \leq U_i^*$ since we have that $\lim\limits_{s \to \hat{s}^-} \left\{\bar{\Pi}_i^* (\hat{y}_i, \hat{s}; \mathbf{F})\right\} \leq \lim\limits_{s \to \hat{s}^-} \left\{\bar{\Pi}_i^* (\hat{s}; \mathbf{F})\right\} \leq \bar{\Pi}_i^* (\hat{s}; \mathbf{F})$ by Lemmas A.2 and A.4. QED

B Characterizing Score-Optimal CDFs

In this section we use the key properties shown in Lemmas A.1 – A.5 (no ties, ideological optimality, and score optimality) to more precisely characterize the form of equilibria and derive conditions that allow us to numerically compute CDFs which satisfy score-optimality.

It is helpful to first rule out the possibility that *in equilibrium* a developer will work to craft a positive-quality veto-proof policy (which therefore has score $s_i > s_0$) with an ideology $y_i^*(s_i)$ that is *weakly further* away from her ideal ideology than the status quo ideology y_0. This is intuitive, since both developers are working to pull policy in their desired ideological direction from the status quo. However, it is not obvious, since the full policy $(s_i, y_i^*(s))$ may be better for her than both the status quo (s_0, y_0) as well as her optimal opponent's score-s_i policy $(s_{-i}, y_{-i}^*(s))$. A further useful implication of Lemma B.1 is that in any equilibrium a developer will only craft policies *interior* to the veto proof set or on the *closest* boundary.

Lemma B.1. *If $s_i > s_0$ is $\in supp\{F_i(\cdot)\}$ then $F_{-i}(s) > \frac{y_0}{x_i/\alpha_i}$ and $|x_i - y_i^*(s)| < |x_i - y_0|$.*

Proof: We show that $|x_i - y_i^*(s_i)| \geq |x_i - y_0|$ implies $\bar{\Pi}_i^*(s_i; \mathbf{F}) - \bar{\Pi}_i^*(s_0; \mathbf{F}) < 0$, which yields our desired property by contrapositive and score optimality. Suppose $|x_i - y_i^*(s_i)| \geq |x_i - y_0|$; it is easily verified from the definition of $y_i^*(s_i)$ that we must have $sign(x_i) = sign(y_0)$ (i is on the same side of the decisionmaker as y_0) and $F_{-i}(s) \leq \frac{y_0}{x_i/\alpha_i}$. Now i's utility difference from developing *any* veto proof policy (s_i, y_i) with score $s_i > s_0$ vs. developing no policy at all is:

$$\bar{\Pi}_i^*(s_i, y_i; \mathbf{F}) - \bar{\Pi}_i^*(s_0; \mathbf{F}) = -\alpha_i\left(s_i + y_i^2\right) + \int_{s_0}^{s_i}\left(V_i(s_i, y_i) - V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right)\right) dF_{-i}$$

Recall that (since each developer is weakly more extreme than the same-sided veto player) policy $(s_i, z_{-i}(s_i))$ is the *weakly worst* veto proof policy among all those with scores $s \in [s_0, s_i]$; hence from part 2 of Definition A.1 the above utility difference is less than or equal to

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot 2x_i\left(y_i - z_{-i}(s_i)\right) \tag{A.5}$$

We last argue that Equation A.5 is strictly negative when $F_{-i}(s) \leq \frac{y_0}{x_i/\alpha_i}$. Using that y_i is veto proof yields that the preceding is less than or equal to $-\alpha_i\left(s_i + y_i^2\right) + \left(\frac{y_0}{x_i/\alpha_i}\right) \cdot 2x_i\left(y_i - z_{-i}(s_i)\right)$; from Lemma A.2 the veto proof y_i maximizing the preceding is $y_i^* = \min\left\{\max\left\{z_{-i}(s), \hat{y}_i\left(s; \frac{y_0}{x_i/\alpha_i}\right), z_i(s)\right\}\right\} = y_0$; substituting and simplifying yields $-\alpha_i\left(1 - \frac{y_0}{x_{Vi}}\right)\left(s_i + y_0^2\right)$, which is < 0 since the status quo y_0 is always weakly more moderate than the same-sided veto player x_{Vi}. QED

Having constrained where ideologically-optimal veto proof policies might lie equilibrium, we now consider the form of the equilibrium score CDFs, which depends critically on the shape of each developer's objective function $\bar{\Pi}_i^*(s_i; \mathbf{F})$ at points of continuity. To understand this shape we study the derivative $\frac{\partial \bar{\Pi}_i^*(s_i; \mathbf{F})}{\partial s_i}$ from Equation A.4;[2] differentiating and simplying yields that $\frac{\partial \bar{\Pi}_i^*(s_i; \mathbf{F})}{\partial s_i}$ must

[2] At points of left-discontinuity the expression that follows is actually the right-derivative.

equal the following at points of continuity:

$$\begin{aligned}\frac{\partial \bar{\Pi}_i^*\left(s_i;\mathbf{F}\right)}{\partial s_i} &= -\left(\alpha_i - F_{-i}\left(s_i\right)\right) + \alpha_i \frac{\partial\left(y_i^*\left(s_i\right)\right)}{\partial s_i}\cdot 2\left(F_{-i}\left(s_i\right)\frac{x_i}{\alpha_i} - y_i^*\left(s_i\right)\right) \\ &\quad + f_{-i}\left(s_i\right)\cdot\left(V_i\left(s_i, y_i^*\left(s_i\right)\right) - V_i\left(s_i, y_{-i}^*\left(s_i\right)\right)\right) \qquad \text{(A.6)}\end{aligned}$$

The first and third terms are identical to the model without veto players (Hirsch and Shotts (2015)) – they are the net quality cost to a developer of raising her score $\alpha_i - F_{-i}\left(s_i\right)$ and the net ideological benefit of doing so (which is the "probability" $f_{-i}\left(s_i\right)$ her opponent $-i$ crafts a score s_i policy times the net policy benefit $V_i\left(s_i, y_i^*\left(s\right)\right) - V_i\left(s_i, y_{-i}^*\left(s\right)\right)$ of moving the ideological outcome from her opponent's policy to her own). Absent veto players, the fact that increasing score is intrinsically costly implies that a developer will never craft a higher-score policy unless it also has a higher chance of defeating the opponent's policy (i.e. $\hat{s}_i > s_0$ and $\hat{s}_i \in \text{supp}\{F_i\left(\cdot\right)\} \rightarrow F_{-i}\left(s_i\right) < F_{-i}\left(\hat{s}_i\right)\ \forall s_i < \hat{s}_i$).

The second term, however, is new to the model with veto players; it captures a developer's benefit from increasing her score when her optimal policy $y_i^*\left(s\right)$ is *constrained* by the veto players. Moreover (by Lemma B.1) whenever developer i is constrained, her optimal ideology is on the closest boundary $z_i\left(s_i\right)$ of the veto proof set. Thus, whenever this new term is not equal to 0 it is strictly positive and equal to the value with $y_i^*\left(s_i\right) = z_i\left(s_i\right)$ substituted in, which is exactly $D_i\left(s_i, F_{-i}\left(s_i\right)\right)$ where

$$D_i\left(s, F\right) = \frac{\alpha_i}{\left|x_{V_{-i}}\right|}\cdot sign\left(x_i\right)\cdot\left(F\frac{x_i}{\alpha_i} - z_i\left(s\right)\right). \qquad \text{(A.7)}$$

(Note that $D_i\left(s, F\right)$ is continuous in s and F, strictly decreasing (increasing) in the former (latter), and $y_i^*(s) = \frac{x_i}{\alpha_i}F_{-i}(s) \rightarrow D_i\left(s, F_{-i}(s)\right) \leq 0$). In words, whenever this new term is strictly positive, it will be twice the difference between i's *unconstrained* ideological optimum $F_{-i}\left(s_i\right)\frac{x_i}{\alpha_i}$ and *constrained* optimum $z_i(s_i)$, times the marginal cost α_i of quality, times the rate $\frac{1}{2\left|x_{V_{-i}}\right|}$ at which the binding boundary of the veto-proof set increases in score.

Intuitively, this term reflects the fact that veto players may provide an additional incentive beyond competing for the decisionmaker's support to craft higher-score policies. When it is strictly positive there is effectively an endogenous "discount" $D_i\left(s_i, F_{-i}\left(s_i\right)\right)$ on the net marginal cost $\alpha_i - F_{-i}\left(s_i\right)$

of increasing score because a higher score allows a developer to target a veto proof ideology $y_i^*(s_i) = z_i(s)$ strictly closer to her unconstrained optimum $F_{-i}(s_i)\frac{x_i}{\alpha_i}$. Thus, $\frac{\partial \bar{\Pi}_i^*(s_i;\mathbf{F})}{\partial s_i}$ may be rewritten as

$$\begin{aligned}\frac{\partial \bar{\Pi}_i^*(s_i;\mathbf{F})}{\partial s_i} &= -(\alpha_i - F_{-i}(s_i)) + \max\{D_i(s_i, F_{-i}(s_i)), 0\} \\ &\quad + f_{-i}(s_i)\cdot\left(V_i(s_i, y_i^*(s_i)) - V_i\left(s_i, y_{-i}^*(s_i)\right)\right) \qquad \text{(A.8)}\end{aligned}$$

Finally, recalling that $y_i^*(s_i) = \hat{y}_i(s_i; F_{-i}(s_i))$ (Lemma A.2), $\bar{\Pi}_i^*(s_i;\mathbf{F})$ from Equation A.4 may also be rewritten as:

$$\begin{aligned}\bar{\Pi}_i^*(s_i;\mathbf{F}) &= F_{-i}(s_i)\cdot V_i(s_0, y_0) + \int_{s_0}^{s_i}\left(-(\alpha_i - F_{-i}(s_i)) + \max\{D_i(\tilde{s}, F_{-i}(s_i)), 0\}\right)d\tilde{s} \\ &\quad + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right)dF_{-i} \qquad \text{(A.9)}\end{aligned}$$

by observing $-\alpha_i\left(s_i + [\hat{y}_i(s_i; F_{-i}(s_i))]^2\right) + F_{-i}(s_i)\cdot V_i(s_i, \hat{y}_i(s_i; F_{-i}(s_i)))$

$$\begin{aligned}&= -\alpha_i\left(s_0 + [\hat{y}_i(s_0; F_{-i}(s_i))]^2\right) + F_{-i}(s_i)\cdot V_i(s_0, \hat{y}_i(s_0; F_{-i}(s_i))) \\ &\quad + \int_{s_0}^{s_i}\frac{\partial}{\partial \tilde{s}}\left(-\alpha_i\left(\tilde{s} + [\hat{y}_i(\tilde{s}; F_{-i}(s_i))]^2\right) + F_{-i}(s_i)\cdot V_i(\tilde{s}, \hat{y}_i(\tilde{s}; F_{-i}(s_i)))\right)d\tilde{s} \\ &= F_{-i}(s_i)\cdot V_i(s_0, y_0) + \int_{s_0}^{s_i}\left(-(\alpha_i - F_{-i}(s_i)) + \max\{D_i(\tilde{s}, F_{-i}(s_i)), 0\}\right)d\tilde{s}\end{aligned}$$

The final equality uses the fact that $\hat{y}_i(s_0; F_{-i}(s_i)) = y_0$ and $-\alpha_i\left(s_0 + [\hat{y}_i(s_0; F_{-i}(s_i))]^2\right) = 0$.

Using these observations we now state our first key lemma on the form of equilibrium score CDFs. In contrast to the model without veto players, in the present model it *is* possible for a developer to target a particular score $\hat{s}_i > s_0$ even if there is a veto-proof lower score $s_i \in [s_0, \hat{s}_i)$ that would gain the decisionmaker's support with equal probability (i.e., $F_{-i}(s_i) = F_{-i}(\hat{s}_i)$). However, *if* this is the case, then it must *only* be because the developer is constrained at this score by the opposing veto player (i.e., $y_i^*(\hat{s}_i) = z_i(\hat{s}_i)$); in addition, $\hat{s}_i$ must be the *lowest* score in the support of i's score CDF, which we henceforth denote $\underline{s}_i = min_s\{\text{supp}\{F_i(\cdot)\}\}$.

Lemma B.2. *If $\hat{s}_i \in supp\{F_i(\cdot)\}$ and there exists some $s_i \in [s_0, \hat{s}_i)$ such that $F_{-i}(s_i) = F_{-i}(\hat{s}_i)$, then $F_{-i}(\hat{s}_i) > 0$, $\hat{s}_i = \underline{s}_i$ and $y_i^*(\hat{s}_i) = z_i(\hat{s}_i)$.*

Proof: Suppose $\hat{s}_i \in$supp$\{F_i(\cdot)\}$ and $\exists s \in [s_0, \hat{s}_i)$ such that $F_{-i}(s) = F_{-i}(\hat{s}_i)$; now let $\underline{\hat{s}}_i \geq s_0$ be the *lowest* score that gains the decisionmaker's support with the same probability $F_{-i}(\hat{s}_i)$, i.e., $\min\{s_i : F_{-i}(s_i) = F_{-i}(\hat{s}_i)\}$, which is well-defined by the right-continuity of CDFs.

We first argue that the existence of lower support points that can gain the decisionmaker's support with the same probability implies that $\hat{s}_i$ must be developer i's *only support point* over the interval $[\underline{\hat{s}}_i, \hat{s}_i]$. Observe from Equation A.8 that since $F_{-i}(s_i)$ is constant and equal to $F_{-i}(\hat{s}_i)$ over this interval, $\bar{\Pi}_i^*(s_i; \mathbf{F})$ is continuous and $\forall s_i \in (\underline{\hat{s}}_i, \hat{s}_i)$ we have $\frac{\partial \bar{\Pi}_i^*(s_i;\mathbf{F})}{\partial s_i}$ also continuous and equal to:

$$\frac{\partial \bar{\Pi}_i^*(s_i; \mathbf{F})}{\partial s_i} = -(\alpha_i - F_{-i}(\hat{s}_i)) + \max\{D_i(s_i, F_{-i}(\hat{s}_i)), 0\} \tag{A.10}$$

We must therefore have that $y_i^*(\hat{s}_i) \neq F_{-i}(\hat{s}_i)\frac{x_i}{\alpha_i}$ (since otherwise the derivative would be strictly negative immediately below $\hat{s}_i$ and it could therefore not be in the support by score optimality), further implying $y_i^*(s_i) = z_i(s_i)$ $\forall s_i \in [\underline{\hat{s}}_i, \hat{s}_i]$ (i's optimal policy is on the boundary for any score in $[\underline{\hat{s}}_i, \hat{s}_i]$). Now it is easily verified that Equation A.10 is linearly and strictly decreasing in s_i, implying that $\bar{\Pi}_i^*(s_i; \mathbf{F})$ is strictly concave over $[\underline{\hat{s}}_i, \hat{s}_i]$ and has a unique strict maximizer; therefore if $\hat{s}_i$ is in i's support it can be the only such maximizer, and must satisfy

$$-(\alpha_i - F_{-i}(\hat{s}_i)) + D_i(s_i, F_{-i}(\hat{s}_i)) > 0 \quad \forall s_i \in [s_0, \hat{s}_i)$$

Finally, it is easily verified that we must have $F_{-i}(\hat{s}_i) > 0$, since if $F_{-i}(\hat{s}_i) = 0$ then the above evaluated at $s_i = s_0$ is equal to $-\alpha_i\left(1 - \frac{y_0}{x_{V_{-i}}}\right) < 0$.

We now show that $\hat{s}_i$ must be i's *lowest* support point. Suppose not. Since supports are closed, developer i must have a *next lowest* support point $s_i' \in [s_0, \underline{\hat{s}}_i)$, and at this support point it must be the case that $F_{-i}(s_i') < F_{-i}(\hat{s}_i)$, further implying that her opponent $-i$ must have a strictly positive probability of crafting a score in the interval $(s_i', \underline{\hat{s}}_i]$. We next argue that (a) developer $-i$ must have an atom at exactly $\underline{\hat{s}}_i \in (s_i', \hat{s}_i)$ where she crafts a policy $y_{-i}^*(\underline{\hat{s}}_i) = z_{-i}(\underline{\hat{s}}_i)$ on her respective boundary of the veto proof set, and (b) $-i$'s score CDF $F_{-i}(s_i)$ is constant around s_i' (implying that i works on the boundary of the veto proof set at the lower support point s_i' as well). To see this, recall that since developer i has no support over $(s_i', \hat{s}_i)$, her own score CDF $F_i(s_{-i})$ is constant over the interval $[s_i', \underline{\hat{s}}_i]$ (recalling that $\underline{\hat{s}}_i < \hat{s}_i$ and CDFs are right-continuous). By the argument in the preceding paragraph, but applied to $-i$ rather than i, this implies that $-i$ has a unique support point over this closed interval at which she crafts a policy on her boundary of the veto proof set. This then yields the desired properties when combined with the definition of $\underline{\hat{s}}_i$ and $F_{-i}(s_i') < F_{-i}(\underline{\hat{s}}_i) = F_{-i}(\hat{s}_i)$.

Finally, using the preceding we show that $\bar{\Pi}_i^*(\hat{s}_i;\mathbf{F}) - \bar{\Pi}_i^*(s_i';\mathbf{F})$ is strictly positive, contradicting score optimality at s_i' and thus implying that s_i' cannot be in i's support. Observe that we may rewrite $\bar{\Pi}_i^*(\hat{s}_i;\mathbf{F}) = -\alpha_i\left(\hat{s}_i + [z_i(\hat{s}_i)]^2\right) + F_{-i}(\hat{s}_i)\cdot V_i(\hat{s}_i, z_i(\hat{s}_i)) + \int_{\hat{s}_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}$ as

$$\begin{aligned}
& -\alpha_i\left(s_i' + \left[z_i\left(s_i'\right)\right]^2\right) + F_{-i}(\hat{s}_i)\cdot V_i\left(s_i', z_i\left(s_i'\right)\right) \\
& + \int_{s_i'}^{\hat{s}_i} \frac{\partial}{\partial s_i}\left(-\alpha_i\left(s_i + [z_i(s_i)]^2\right) + F_{-i}(\hat{s}_i)\cdot V_i(s_i, z_i(\hat{s}_i))\right) ds_i + \int_{\hat{s}_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i} \\
= \; & -\alpha_i\left(s_i' + \left[z_i\left(s_i'\right)\right]^2\right) + F_{-i}\left(s_i'\right)\cdot V_i\left(s_i', z_i\left(s_i'\right)\right) + p_{-i}^{\underline{\hat{s}}_i}\cdot V_i\left(s_i', z_i\left(s_i'\right)\right) \\
& + \int_{s_i'}^{\hat{s}_i} \left(-\left(\alpha_i - F_{-i}(\hat{s}_i)\right) + D_i\left(s_i, F_{-i}(\hat{s}_i)\right)\right) ds_i + \int_{\hat{s}_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}
\end{aligned}$$

which then yields (again recalling $y_i^*(s_i') = z_i(s_i')$) that $\bar{\Pi}_i^*(\hat{s}_i;\mathbf{F}) - \bar{\Pi}_i^*(s_i';\mathbf{F}) =$

$$\int_{s_i'}^{\hat{s}_i} \left(-\left(\alpha_i - F_{-i}(\hat{s}_i)\right) + D_i\left(s_i, F_{-i}(\hat{s}_i)\right)\right) ds_i + p_{-i}^{\underline{\hat{s}}_i}\cdot\left(V_i\left(s_i', z_i\left(s_i'\right)\right) - V_i\left(\underline{\hat{s}}_i, z_{-i}\left(\underline{\hat{s}}_i\right)\right)\right)$$

The second term is positive since $(\underline{\hat{s}}_i, z_{-i}(\underline{\hat{s}}_i))$ is the weakly worst veto proof policy for i with score $\in [s_0, \underline{\hat{s}}_i]$ (recalling that each developer is more extreme than the same sided veto player). The first term has already been shown to be strictly positive. **QED**

With the preceding critical lemma in hand, we are now in a position to more precisely characterize necessary and sufficient conditions for equilibrium to use for numerical computation. There are effectively four types of potential equilibria in the model, with each type characterized by the combination of two critical properties: (a) whether or not one of the developers is *always active* (i.e., crafts a policy with score $s > s_0$ with probability 1), and (b) whether the equilibrium is in *pure* or *mixed* strategies. Which type of equilibrium a particular candidate set of score CDFs $(F_L(\cdot), F_R(\cdot))$ must fall into, and thus the necessary and sufficient conditions needed to satisfy score optimality, may be determined by considering two quantities; (a) the maximum *lowest score* in the support of the two CDFs – which we denote $\underline{s} = \max_i\{\underline{s}_i\}$ – and the maximum *highest score* in the support of the two CDFs – which we denote $\bar{s} = \max_i\{\bar{s}_i\}$, where $\bar{s}_i = \max_s\{\text{supp}\{F_i(\cdot)\}\}$. (Note that $F_i(\bar{s}) = 1\ \forall i$ by the definition of $\bar{s}$). Necessary and sufficient conditions for a pair of score CDFs to satisfy score optimality and therefore support an equilibrium are then as follows.

Proposition B.1. *Let* $D_i(s, F) = \frac{\alpha_i}{\left|x_{V_{-i}}\right|} \cdot \mathit{sign}(x_i) \cdot \left(F\frac{x_i}{\alpha_i} - z_i(s)\right)$. *Then a profile of score CDFs* $\mathbf{F}$ *satisfies score optimality i.f.f. the following hold.*

1. *If* $s_0 = \underline{s} = \overline{s}$ *(so that* $F_i(s_0) = 1\ \forall i$*), then* $\alpha_i - 1 \geq D_i(s_0; 1)\ \forall i$.

2. *If* $s_0 < \underline{s}$*, then there exists a* $k \in \{L, R\}$ *such that*

   - *developer* k *is* ***sometimes-inactive*** *(i.e.* $s_0 = \underline{s}_k < \underline{s}$*), never crafts a policy with score* $\in (s_0, \underline{s}]$ *(so that* $0 < F_k(s_0) = F_k(\underline{s})$*), and has a probability of inactivity* $F_k(\underline{s})$ *satisfying*

   $$\alpha_{-k} - F_k(\underline{s}) = D_{-k}(\underline{s}; F_k(\underline{s}))$$

   - *developer* $-k$ *is* ***always-active*** *(i.e.* $s_0 < \underline{s}_{-k} = \underline{s}$ *so that* $F_{-k}(s) = 0\ \forall s \in [s_0, \underline{s})$*), and has a probability* $F_{-k}(\underline{s})$ *of crafting a score-*$\underline{s}$ *policy satisfying* $\bar{\Pi}_k^*(s_0; \mathbf{F}) \geq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$*, which may written in the following two equivalent forms:*

   $$\alpha_k\left(\underline{s} + [y_k^*(\underline{s})]^2\right) \geq F_{-k}(\underline{s}) \cdot 2x_k \cdot (y_k^*(\underline{s}) - z_{-k}(\underline{s}))$$

   $$\int_{s_0}^{\underline{s}} ((\alpha_k - F_{-k}(\underline{s})) - \max\{D_k(s; F_{-k}(\underline{s})), 0\})\, ds \geq F_{-k}(\underline{s}) \cdot \left(\frac{x_k}{x_{V_k}} - 1\right)(\underline{s} - s_0)$$

3. *If* $\underline{s} < \bar{s}$*, then* $\forall i \in \{L, R\}$ *and* $s \in [\underline{s}, \overline{s}]$ *the score CDF* $F_{-i}(s)$ *is continuous and satisfies:*

$$(\alpha_i - F_{-i}(s)) - \max\{D_i(s; F_{-i}(s)), 0\} = f_{-i}(s) \cdot 2|x_i|(y_R^*(s) - y_L^*(s))$$

4. *If **both** $s_0 < \underline{s}$ and $\underline{s} < \bar{s}$, then developer $-k$'s probability $F_{-k}(\underline{s})$ of crafting a score-$\underline{s}$ policy also satisfies $\bar{\Pi}_k^*(s_0; \mathbf{F}) \leq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$*

Proof: ***(Necessity)*** We first show the necessity of property (1). If $s_0 = \underline{s} = \overline{s}$ then $F_i(s_i) = 1\ \forall i \in \{L, R\}$ and $s_i \geq s_0$ and $\bar{\Pi}_i^*(s_i; \mathbf{F})$ is continuously differentiable; thus score optimality requires that $\left.\frac{\partial \bar{\Pi}_i^*(s_i;\mathbf{F})}{\partial s_i}\right|_{s_i = s_0} \leq 0\ \forall i$, which is exactly the stated condition.

We now show the necessity of property (2). Suppose $\underline{s} > s_0$; then there must be *exactly one* developer with $\underline{s}_i = \underline{s}$ (because if both had $\underline{s}_i = \underline{s}$ then by Lemma B.2 each must have an atom at $\underline{s}$, which would violate the no ties property in Lemma A.3). Henceforth denote this developer $-k$. Note by the definition of $\underline{s}$ we have $\underline{s}_k \in [s_0, \underline{s})$. Because $F_{-k}(s_k) = 0\ \forall s_k \in (s_0, \underline{s})$, no such score can be in the support of $F_{-k}(\cdot)$ by the first property in Lemma B.2, further implying that $\underline{s}_k = s_0$.

Next, note that k cannot have an atom at $\underline{s}$ (because then $-k$ could not have an atom at $\underline{s}$ by the no ties property in Lemma A.3, which would in turn imply a contradiction via the first part of Lemma B.2). Thus developer k must have an atom at the lowest score s_0 exactly equal to $F_k(\underline{s})$, with $F_k(s_k)$ constant over $s_k \in [s_0, \underline{s}]$. Moreover, again applying Lemma B.2, the fact that $\underline{s} \in$supp$\{F_{-k}\}$ and $F_k(s_k)$ is constant over $s_k \in [s_0, \underline{s}]$ implies that $F_{-k}(\underline{s}) > 0$.

Having established the form of the developers' strategies, we now turn to the characterization of $F_{-k}(\underline{s})$. The fact that $\bar{\Pi}_k^*(s_0; \mathbf{F}) \geq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$, as noted in the second bullet point of property (2), follows from the score optimality property in Lemma A.5. For the two equivalent conditions on $F_{-k}(\underline{s})$, first recall that, as shown above, k doesn't have an atom at $\underline{s}$, which implies, via Lemma B.2, that $y_{-k}^*(\underline{s}) = z_{-k}(\underline{s})$. Using this substitution, the first equivalence in the second bullet point of property (2) comes from the standard form of $\bar{\Pi}_i^*(s_i; \mathbf{F})$ in Equation A.4 and the second equivalence comes from the integral form of $\bar{\Pi}_i^*(s_i; \mathbf{F})$ in Equation A.9.

We last turn to to the characterization of $F_k(\underline{s})$. Since CDFs are right continuous, $\bar{\Pi}_{-k}^*(s_{-k}; \mathbf{F})$

is constant over $[s_0, \underline{s})$ and continuous over $[s_0, \underline{s} + \epsilon)$ for sufficiently small ϵ, so the expression

$$-\left(\alpha_{-k} - F_k\left(\underline{s}\right)\right) + D_{-k}\left(\underline{s}, F_k\left(\underline{s}\right)\right) + f_k\left(\underline{s}\right) \cdot \left(V_{-k}\left(\underline{s}, y^*_{-k}\left(\underline{s}\right)\right) - V_{-k}\left(\underline{s}, y^*_k\left(\underline{s}\right)\right)\right)$$

is the right derivative of $\bar{\Pi}^*_{-k}\left(\underline{s};\mathbf{F}\right)$, the first two terms are the left derivative of $\bar{\Pi}^*_{-k}\left(\underline{s};\mathbf{F}\right)$, and the third term line is weakly positive. Thus the first two terms must exactly equal 0; if they were strictly negative (positive) a score s_{-k} a little bit below (above) $\underline{s}$ would yield a strictly higher value of $\bar{\Pi}^*_{-k}\left(\underline{s};\mathbf{F}\right)$, violating score optimality.

We next show the necessity of property (3). Consider $\underline{s} < \overline{s}$. We first argue that for any $s_i > \underline{s} \geq s_0$ in the support of $F_i\left(\cdot\right)$ we must have $F_{-i}\left(s\right) < F_{-i}\left(s_i\right)$ $\forall s \in [s_0, s_i)$; if not then by Lemma B.2 we have $s_i = \underline{s}_i$, contradicting the definition of $\underline{s}$. Next we argue that any support points strictly above $\underline{s}$ must be common; if not then there $\exists$ $\hat{s}_i \in$supp$\{F_i\left(\cdot\right)\}$ such that $\hat{s}_i > \underline{s}$ and $\hat{s}_i \notin$supp$\{F_{-i}\left(\cdot\right)\}$ and therefore $F_{-i}\left(\hat{s}_i - \epsilon\right) = F_{-i}\left(\hat{s}_i\right)$ for sufficiently small ϵ, which by Lemma B.2 implies that $\underline{s}_i = \hat{s}_i > \underline{s}$, a contradiction.

Next we argue that the set of (common) support points strictly above $\underline{s}$ must be convex. If not, there would exist $\hat{s} > \underline{s} \geq s_0$ in the common support such that neither developer has support in a neighborhood immediately below, so $F_i\left(s\right) < F_i\left(\hat{s}\right)$ $\forall s \in [s_0, \hat{s})$ would require both developers to have atoms at $\hat{s}$, contradicting the no ties property in Lemma A.3. Since supports are closed, the closed interval $[\underline{s}, \bar{s}]$ must therefore be in the support of both developer's CDFs. Thus by the score optimality property in Lemma A.5, $\bar{\Pi}^*_i\left(s;\mathbf{F}\right) = U^*_i$ $\forall s \in [\underline{s}, \bar{s}]$, further implying that the score CDFs are absolutely continuous over $(\underline{s}, \bar{s})$, and therefore that $\frac{\partial}{\partial s}\left(\bar{\Pi}^*_i\left(s;\mathbf{F}\right)\right) = 0$ for almost all $s \in [\underline{s}, \bar{s}]$. This straightforwardly yields the stated differential equation.

We last show the necessity of property (4). If $\underline{s}_0 < \underline{s} < \overline{s}$ then by implication of property (3) the score $\underline{s}$ is also in the support of $F_k\left(\cdot\right)$; score optimality thus requires $\bar{\Pi}^*_k\left(s_0;\mathbf{F}\right) \leq \bar{\Pi}^*_k\left(\underline{s};\mathbf{F}\right)$.

This concludes the argument that properties $(1) - (4)$ are necessary for score optimality.

(Sufficiency) Observe that for all possibilities we have $\bar{\Pi}_i^* (\underline{s}; \mathbf{F}) = \bar{\Pi}_i^* (\bar{s}; \mathbf{F})\ \forall i$ (since $\underline{s} = \bar{s}$ or $\underline{s} < \bar{s}$ and both are in the support of both developers' score CDFs); thus to show score optimality for i we need only show that scores $s_i > \bar{s}$ and scores $s_i \in [s_0, \underline{s}]$ outside of i's support cannot deliver a strictly higher value of $\bar{\Pi}_i^* (\cdot; \mathbf{F})$.

To argue that $\bar{\Pi}_i^* (s_i; \mathbf{F}) \leq \bar{\Pi}_i^* (\bar{s}; \mathbf{F})\ \forall i$ and $s_i > \bar{s}$, observe that from Equation A.10 we have $\frac{\partial}{\partial s_i} \left(\bar{\Pi}_i^* (s; \mathbf{F})\right) = -(\alpha_i - 1) + \max\{D_i (s_i, 1), 0\}$ for $s_i > \bar{s}$ and is weakly decreasing in s_i; thus it suffices to show that at $\bar{s}$ we have $-(\alpha_i - 1) + \max\{D_i (\bar{s}, 1), 0\} \leq 0$. If property (1) holds then this is immediate. If property (3) holds then this follows immediately from the differential equation since $y_R^* (\bar{s}) > y_L^* (\bar{s})$.

If neither property (1) nor property (3) hold then $s_0 < \underline{s} = \bar{s}$ so $F_i (\underline{s}) = F_i (\bar{s}) = 1$. Then property (2) implies that there is an always-active developer $-k$, and the desired condition holds for $-k$ from the first bullet point of property (2) since $F_k (\underline{s}) = 1$. For developer k, the integral formulation of the second bullet point in property (2) combined with $s_0 < \underline{s} = \bar{s}$ and $F_i (\underline{s}) = F_i (\bar{s}) = 1$ yields that

$$\int_{s_0}^{\underline{s}} ((\alpha_k - 1) - \max\{D_k (s; 1), 0\})\, ds \geq \left(\frac{x_k}{x_{V_k}} - 1\right)(\bar{s} - s_0).$$

Thus $-(\alpha_k - 1) + \max\{D_k (\bar{s}; 1), 0\} > 0$, combined with the fact that $D_k (s; F)$ is strictly decreasing in s, would imply that the left hand side is strictly negative, contradicting the inequality since $\left(\frac{x_k}{x_{V_k}} - 1\right)(\bar{s} - s_0) \geq 0$.

Finally, the property that $\bar{\Pi}_i^* (s_i; \mathbf{F}) \leq \bar{\Pi}_i^* (\underline{s}; \mathbf{F})\ \forall i$ and $s_i \leq \underline{s}$ that are also *not* in supp$\{F_i (\cdot)\}$ is true by construction of the necessary conditions. To revisit, observe that the potential existence of such a score requires that $s_0 < \underline{s}$ (property 2), a sometimes inactive developer k with $\underline{s}_k = 0$, and an always active developer $-k$ with $\underline{s}_{-k} = \underline{s}$. Now the condition in the first bullet point of property (2) implies $\bar{\Pi}_{-k}^* (\underline{s}; \mathbf{F}) > \bar{\Pi}_{-k}^* (s_{-k}; \mathbf{F})\ \forall s_{-k} \in [s_0, \underline{s})$. In addition $F_{-k} (s_k) = 0\ \forall s_k \in [s_0, \underline{s})$ implies $\bar{\Pi}_k^* (s_0; \mathbf{F}) > \bar{\Pi}_k^* (s_k; \mathbf{F})$, and the condition in the second bullet point of property (2) is that $\bar{\Pi}_k^* (s_0; \mathbf{F}) \geq \bar{\Pi}_k^* (\underline{s}; \mathbf{F})$, completing the argument. QED

Before proceeding, we also briefly note that because $F_{-i} (s)$ is assumed to be a mixture of discrete and absolutely continuous distributions it must be absolutely continuous over $[\underline{s}, \bar{s}]$; in addition, $D_i(s; F_{-i}(s))$, and $y_i^*(s) = \hat{y}_i(s; F_{-i}(s))$ are continuous in $F_{-i}(s)$; thus, part (3) of Proposition B.1 implies that $f_i(s)$ is continuous over $[\underline{s}, \bar{s}]$.

C Symmetric Special Case

We now impose the additional assumptions of our symmetric special case ($|x_{Vl}| = |x_{Vr}| = x_V \leq |x_L| = |x_R| = x_E$ and $\alpha_L = \alpha_R = \alpha_R$) and analytically prove several properties of equilibrium for the special case. Under symmetry we have $z_i(s) = 2y_0 - z_{-i}(s)$ at any score s (a developer's respective boundary of the veto proof set is the reflection point about the status quo of his opponent's respective boundary), a developer i is strictly constrained by the veto players if and only if $\frac{x_E}{\alpha} F_{-i}(s) > sign(x_i) \cdot z_i(s) = sign(x_i) \cdot y_0 + \frac{s-s_0}{2x_V}$, so that

$$D_i(s, F) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - \left(sign(x_i) \cdot y_0 + \frac{s-s_0}{2x_V}\right)\right)$$

and finally $D_i(s,F) - D_{-i}(s,F) = -sign(x_i) \cdot \frac{\alpha}{x_V} 2y_0$. From this it is straightforward that $y_0 = 0 \to D_L(s,F) = D_R(s,F)$ and $y_0 < (>) \, 0 \to D_R(s,F) > (<) \, D_L(s,F)$ – in words, the developer who is strictly more distant from the status quo (i.e., who is more motivated) has a strictly higher "discount function" for any values of (s, F). These properties crucially determine which developer must be more active in any equilibrium where there is some participation in policy development.

C.1 Equilibrium with $y_0 = 0$

We first characterize equilibrium when the status quo is already located at the decisionmaker's ideal ($y_0 = 0$) so that the developers are equally motivated.

Proposition C.1. *Suppose $y_0 = 0$ so that $s_0 = 0$, $D_L(s,F) = D_R(s,F) = D(s,F) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - \frac{s}{2x_V}\right)$, and $-z_L(s) = z_R(s) = z(s) = \frac{s-s_0}{2x_V}$).*

- *If $\alpha - 1 \geq D_R(s_0, 1) \iff \alpha \geq \frac{x_E}{x_V} + 1$, then in equilibrium both developers are inactive with probability $F(0) = 1$ (so $0 = \underline{s} = \bar{s}$)*

- *If* $\alpha \in \left(2, 1+\frac{x_E}{x_V}\right)$ *then in equilibrium both developers are inactive with probability* $F(0) = \frac{\alpha}{1+\frac{x_E}{x_V}} < 1$ *so* $0 = \underline{s} < \bar{s}$*. Now let* $s(F)$ *denote the inverse of* $F(s)$ *(so* $s(F(0)) = 0$*), let* $\breve{F} = \min\left\{F(0)\cdot\left(\frac{1+3\frac{x_E}{x_V}}{1+2\frac{x_E}{x_V}}\right), 1\right\}$*, and let* $\breve{s}$ *solve* $\frac{x_E}{\alpha}\breve{F} = z(\breve{s}) \iff \breve{s} = \frac{2x_E x_V}{\alpha}\breve{F}$*.*

  – *for* $F \in \left[F(0), \breve{F}\right]$ *we have* $|y_i^*(s(F))| = z(s)$ *and* $s(F)$ *equal to*

$$\hat{s}(F) = \frac{2x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\cdot(F - F(0))$$

  – *for* $F \in (\breve{F}, 1]$ *we have* $|y_i^*(s(F))| = \frac{x_E}{\alpha}F$ *and* $s(F)$ *equal to*

$$\tilde{s}(F) = \breve{s} + \int_{\breve{F}}^{F} 4x_E^2 \frac{G}{\alpha(\alpha - G)}dG = \breve{s} + 4x_E^2\left(\ln\left(\frac{\alpha - \breve{F}}{\alpha - F}\right) - \frac{F - \breve{F}}{\alpha}\right)$$

Finally, the decisionmaker's equilibrium utility is $\int_{F(0)}^{\breve{F}} 2F\cdot\hat{s}(F)\,dF + \int_{\breve{F}}^{1} 2F\cdot\tilde{s}(F)\,dF =$

$$\frac{4x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\left(\frac{[F(0)]^3 - \breve{F}^2\left(3F(0) - 2\breve{F}\right)}{6}\right)$$
$$+\breve{s}\cdot\left(1-\breve{F}^2\right) + 4x_E^2\left(\left(1-\breve{F}\right)\left(\alpha + \frac{1+\breve{F}}{2} - \frac{2-\breve{F}\left(1-\breve{F}\right)}{3\alpha}\right) - \left(\alpha^2 - 1\right)\log\left(\frac{\alpha - \breve{F}}{\alpha - 1}\right)\right)$$

Proof: We show by construction that there exists a solution to score optimality satisfying $\underline{s} = s_0 = 0$, and that it is the unique solution with this property.

For the first bullet point of the proposition, note that if $D(s_0, 1) = \frac{x_E}{x_V} \leq \alpha - 1 \iff \frac{\alpha}{1+\frac{x_E}{x_V}} \geq 1$ then clearly inactivity ($s_0 = \underline{s} = \bar{s}$) is the unique equilibrium of the form $s_0 = \underline{s}$.

For the second bullet point, note that if $D(s_0, 1) = \frac{x_E}{x_V} > \alpha - 1$ then clearly inactivity ($0 = s_0 = \underline{s} = \bar{s}$) is *not* an equilibrium. We solve for a solution of the form $0 = s_0 = \underline{s} < \bar{s}$ which is unique by construction. Since $y_i^*(s_0) = y_0 = 0\ \forall i$ we must have $\alpha - F_{-i}(0) - D(0; F_{-i}(0)) = 0 \iff F_{-i}(0) = \frac{\alpha}{1+\frac{x_E}{x_V}}\ \forall i$. In a neighborhood above 0 we have $y_i^*(s) = z_i(s)$ and $D(s_i, F_i(s)) \geq 0$; substituting into the differential equations from part 3 of Proposition B.1, simplifying, and rearranging yields that:

$$\alpha - \left(1 + \frac{x_E}{x_V}\right)F_{-i}(s) + \frac{\alpha}{2x_V^2}s = f_{-i}(s)\cdot 2\frac{x_E}{x_V}\cdot s \quad \forall i$$

Since the differential equation and boundary condition at $s_0 = 0$ is the same $\forall i$ the solution in this neighborhood is a common score CDF $\hat{F}(s)$ satisfying

$$\alpha - \left(1 + \frac{x_E}{x_V}\right) \hat{F}(s) + \frac{\alpha}{2x_V^2} s = \hat{f}(s) \cdot 2\frac{x_E}{x_V} \cdot s$$

and the boundary condition $\alpha - \left(1 + \frac{x_E}{x_V}\right) \hat{F}(0) = 0$. It is easily verified that the system has the following simple linear solution:

$$\hat{F}(s) = \left(\frac{\alpha}{2x_V^2}\right) \left(\frac{1}{3\frac{x_E}{x_V} + 1}\right) s + F(0), \tag{A.11}$$

whose inverse is the function $\hat{s}(F)$ in the proposition.

Now by linearity of $\hat{F}(\cdot)$ and $z(s)$ there is unique value $\breve{s}$ such that $\frac{x_E}{\alpha}\hat{F}(\breve{s}) = \frac{\breve{s}}{2x_V}$, which is exactly $\breve{s} = 2x_V^2 \left(\frac{1+3\frac{x_E}{x_V}}{1+\frac{x_E}{x_V}}\right)\left(\frac{\frac{x_E}{x_V}}{1+2\frac{x_E}{x_V}}\right)$ so that $\hat{F}(\breve{s}) = \alpha \left(\frac{\left(1+3\frac{x_E}{x_V}\right)}{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}\right)$, and another unique value $\hat{s}$ such that $\hat{F}(\hat{s}) = 1$, which is exactly $\hat{s} = 2x_V^2 \left(\frac{1+3\frac{x_E}{x_V}}{1+\frac{x_E}{x_V}}\right)\left(\frac{1+\frac{x_E}{x_V}}{\alpha} - 1\right)$. Then we may have the following two possibilities for the unique solution to the system.

First, we may have $\breve{s} \geq \hat{s}$, which occurs exactly when $\alpha \geq \frac{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}{\left(1+3\frac{x_E}{x_V}\right)}$. In this case $\frac{x_E}{\alpha}\hat{F}(s) > \frac{s}{2x_V}$ $\forall s \in [0, \hat{s})$ and so $0 = \underline{s} < \bar{s} = \hat{s}$ with $F_i(s) = \hat{F}(s)$ $\forall i$ is the unique solution of the form $s_0 = \underline{s} < \bar{s}$ so the inverse is $\hat{s}(F)$ for $F \in [F(0), 1]$.

Second, we may have $\breve{s} < \hat{s}$. In this case $\hat{F}(\breve{s}) = \breve{F} = \alpha \left(\frac{\left(1+3\frac{x_E}{x_V}\right)}{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}\right) < 1$ and the differential equation in a neighborhood above $\breve{s}$ is

$$\alpha - F_{-i}(s) = f_{-i}(s) \cdot \frac{x_E^2}{\alpha}\left(F_{-i}(s) + F_i(s)\right) \qquad \forall i$$

with boundary condition $F_{-i}(\breve{s}) = F_i(\breve{s}) = \breve{F}$. It is easily verified that the solution is a common CDF $F_{-i}(s) = F_i(s) = \tilde{F}(s)$ satisfying $\tilde{F}(\breve{s}) = \breve{F}$ and $\tilde{f}(s) = \frac{\alpha}{4x_E^2}\frac{\alpha - \tilde{F}(s)}{\tilde{F}(s)}$ $\forall s \in [\breve{s}, \bar{s}]$. This solution is straightforwardly strictly concave; since $z(s)$ is linear we then have $\frac{x_E}{\alpha}\tilde{F}(s) < z(s)$ for $s \in (\breve{s}, \bar{s}]$ as required. To derive an analytic expression observe that the inverse $\tilde{s}(F)$ of the function $\tilde{F}(s)$ satisfies $\tilde{s}'(F) = \frac{4x_E^2}{\alpha}\left(\frac{F}{\alpha - F}\right)$ and $\tilde{s}\left(\breve{F}\right) = \breve{s}$, implying directly that $\tilde{s}(F) = \breve{s} + \int_{\breve{F}}^{F} 4x_E^2 \frac{G}{\alpha(\alpha - G)} dG$.

Lastly, in any symmetric mixed strategy equilibrium with $\underline{s} = s_0 = 0 \leq \bar{s}$ the decisionmaker's payoff is the *maximum* of the two scores offered, and the CDF of the maximum is $[F(s)]^2$. The decisionmaker's utility is thus

$$\begin{aligned} \int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s}\left([F(s)]^2\right) ds &= \int_0^{\bar{s}} s \cdot 2F(s) f(s)\, ds = \int_0^{\bar{s}} 2F(s) \cdot s(F(s)) \cdot f(s)\, ds \\ &= \int_{F(0)}^{1} 2F \cdot s(F)\, dF \end{aligned}$$

where the last equality follows from a change of variables and $F(\bar{s}) = 1$. Thus in the model with veto players the decisionmaker's utility is

$$\begin{aligned} &\int_{F(0)}^{\breve{F}} 2F \cdot \hat{s}(F)\, dF + \int_{\breve{F}}^{1} 2F \cdot \tilde{s}(F)\, dF \\ =\ &\int_{F(0)}^{\breve{F}} 2F \cdot \left(\frac{2x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\cdot(F - F(0))\right) dF + \int_{\breve{F}}^{1} 2F \cdot \left(\breve{s} + \int_{\breve{F}}^{F} 4x_E^2 \frac{G}{\alpha(\alpha - G)} dG\right) dF \\ =\ &\frac{4x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\int_{F(0)}^{\breve{F}} F \cdot (F - F(0))\, dF + \left(1 - \breve{F}^2\right)\breve{s} + 4x_E^2 \int_{\breve{F}}^{1} 2F\left(\int_{\breve{F}}^{F} \frac{G}{\alpha(\alpha - G)} dG\right) dF \end{aligned}$$

which evaluates to the expression in the proposition. QED

We conclude this subsection by showing that when $y_0 = 0$, the presence of veto players always *strictly harms* the decisionmaker.

Proposition C.2. *If $y_0 = 0$ then the decisionmaker's equilibrium payoff in the model without veto players first-order stochastically dominates her payoff in the symmetric equilibrium that we characterize in in Proposition C.1 for the model with veto players.*

Proof: Let $F_V(s)$ denote the developers' common score CDF in the symmetric equilibrium with veto players and $y_0 = 0$ characterized in Proposition C.1, and let $F_C(s)$ denote the developers' common score CDF in the unique symmetric equilibrium of the model without veto players characterized in Hirsch and Shotts (2015). In both models the decisionmaker chooses the policy with the maximum score, so the CDF of her utility is the *square* of the developers' common score CDF. Thus, to show first order stochastic dominance of the decisionmaker's utility in the model without veto players it suffices to show first order stochastic dominance of the equilibrium score CDF.

Throughout this proof we will let $r = \frac{x_E}{x_V}$ denote the ratio of the developers' extremism to the veto players' extremism, and note that $r \geq 1$ by assumption. From Hirsch and Shotts (2015) $F_C(s)$ is a continuous strictly increasing function over $[0, \bar{s}_C]$ satisfying $F_C(0) = 0$ and $F_C(\bar{s}_C) = 1$ where $\bar{s}_C = 4x_E^2 \left(\log\left(\frac{\alpha}{\alpha-1}\right) - \frac{1}{\alpha}\right) > 0$; the CDF has a (well-defined) inverse $s_C(F)$ over $F \in [0,1]$ that is equal to the function $\tilde{s}_C(F) = 4x_E^2 \int_0^F \frac{G}{\alpha(\alpha-G)} dG$.

Now using Proposition C.1, clearly first-order stochastic dominance holds when $\alpha \geq 1 + r$, i.e., the equilibrium of the model with veto players is in pure strategies with full inactivity. So consider next when the equilibrium with veto players has activity and is mixed, i.e. $\alpha \in (2, 1+r)$ so $F_V(0) = \frac{\alpha}{1+r} \in (0,1)$. We wish to show that $F_C(s) < F_V(s)$ $\forall s \in [0, \bar{s}_V]$ (where $F_V(\bar{s}_V) = 1$), further implying that $F_C(s) < F_V(s) = 1$ $\forall s \in (\bar{s}_V, \bar{s}_C)$ and $F_C(s) = F_V(s) = 1$ $\forall s \geq \bar{s}_C$. To do so we may work with the inverse score CDFs and show that $s_C(F) > s_V(F)$ $\forall F \in [F_V(0), 1]$.

Now, recall from Proposition C.1 that $s_V(F) = \hat{s}(F) = \frac{2x_V^2}{\alpha}(3r+1)(F - F_V(0))$ for $F \in \left[F(0), \breve{F}\right]$, where $\breve{F} = \min\left\{\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right), 1\right\} > F_V(0)$. Further recall that $\hat{s}(F)$ is linear and note that $\tilde{s}_C(F)$ is strictly convex. And, it is easily verified that $\hat{s}'\left(\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right)\right) = \tilde{s}'_C\left(\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right)\right) = \frac{2x_V^2}{\alpha}(3r+1)$. It thus suffices to show that $\tilde{s}_C\left(\breve{F}\right) > \hat{s}\left(\breve{F}\right)$ since (using Proposition C.1) either (a) $\breve{F} = 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) \geq 1$, $s_V(F) = \hat{s}(F)$ and $\tilde{s}'_C(F) < \hat{s}'(F)$ $\forall F \in [F(0), 1]$, or (b) $\breve{F} < 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$, $s_V(F) = \hat{s}(F)$ and $\tilde{s}'_C(F) < \hat{s}'(F)$ $\forall F \leq \left[\breve{F}(0), \breve{F}\right]$, and $s_V(F) = \tilde{s}(F)$ and $\tilde{s}'(F) = \tilde{s}'_C(F) = 4x_E \frac{G}{\alpha(\alpha-G)}$ $\forall F \in (\breve{F}, 1]$. So we need only show that

$$4x_E^2 \int_0^{\breve{F}} \frac{G}{\alpha(\alpha-G)} dG > \frac{2x_V^2}{\alpha}(3r+1)\left(\breve{F} - \left(\frac{\alpha}{1+r}\right)\right)$$

which simplifies to

$$2r^2 \int_0^{\breve{F}} \frac{G}{\alpha(\alpha-G)} dG > \frac{1}{\alpha}(3r+1)\left(\breve{F} - \left(\frac{\alpha}{1+r}\right)\right) \tag{A.12}$$

Now either $\breve{F} = \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$ and the r.h.s. reduces to $\frac{1}{\alpha}\breve{F}r$, or $\breve{F} = 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) \geq 1 \iff$

$\frac{\alpha}{1+r} \geq \frac{1+2r}{1+3r}$ in which case the r.h.s is weakly smaller than $\frac{1}{\alpha}(3r+1)\left(1-\frac{1+2r}{1+3r}\right) = \frac{1}{\alpha}r = \frac{1}{\alpha}\breve{F}r$. So in either case, the desired inequality holds when

$$2r^2 \int_0^{\breve{F}} \frac{G}{\alpha(\alpha-G)} dG > \frac{1}{\alpha}\breve{F}r$$

which reduces to $2r\int_0^{\breve{F}} \frac{G}{\alpha-G} dG > \breve{F}$. Now since $\alpha - G \geq \alpha - 1$ the preceding holds if the yet stronger inequality $\frac{2r}{\alpha-1}\int_0^{\breve{F}} G \cdot dG > \breve{F}$ holds, which reduces to $\frac{r}{\alpha-1}\breve{F} > 1$. If $\breve{F} = 1$ this holds, because $F_V(0) < 1$ implies $\alpha < 1+r$. If $\breve{F} = \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$ then because $\frac{1+3r}{1+2r} > 1$ it suffices to show that $\frac{r}{\alpha-1}\frac{\alpha}{1+r} > 1$ which holds because $\alpha < 1+r$. QED

C.2 Equilibrium with $y_0 \neq 0$

We next partially characterize equilibrium when the status quo is located away from the decisionmaker's ideal ($y_0 \neq 0$) so the developer on the opposite side of the status quo is more motivated; wlog we consider when the right developer is more motivated ($y_0 < 0$).

Proposition C.3. *Suppose $y_0 < 0$ so that $s_0 = -y_0^2 < 0$, and*

$$\begin{aligned} D_L(s,F) &= \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} + z_L(s)\right) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - z_R(s) + 2y_0\right) \\ &< \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - z_R(s)\right) = D_R(s,F) \ \ \forall (s,F) \end{aligned}$$

Then in any equilibrium with activity ($s_0 < \bar{s}$), participation is asymmetric (so $s_0 < \underline{s} \leq \bar{s}$)

- *if a developer is unconstrained at a particular score ($F_{-i}(\hat{s})\frac{x_E}{\alpha} \leq sign(x_i) \cdot z_i(\hat{s})$) they are unconstrained at all higher scores ($F_{-i}(s)\frac{x_E}{\alpha} < sign(x_i) \cdot z_i(s)\ \forall s > \hat{s}$)*
- *developer L is sometimes inactive ($L = k$); developer R is always active ($R = -k$) and therefore strictly constrained by the veto players at the lowest score $\underline{s}$*
- *whenever developer R is unconstrained ($\frac{x_E}{\alpha}F_L(s) \leq z_R(s)$) so is developer L ($\frac{x_E}{\alpha}F_R(s) > z_L(s)$)*
- *developer R's score CDF is first-order stochastically dominant, i.e. $F_R(s) \leq F_L(s)$, and $\exists \breve{s} \in (\underline{s}, \bar{s}]$ such that $F_R(s) < F_L(s)$ for $s < \breve{s}$ and $F_R(s) = F_L(s)$ for $s \geq \breve{s}$.*

Proof: We first consider pure strategy equilibria with activity ($s_0 < \underline{s} = \bar{s}$). From Proposition B.1 and imposing symmetry for all parameters except y_0, any such pure strategy equilibrium with activity must be asymmetric and satisfy $\alpha - 1 = D_{-k}(\underline{s}; 1)$ and

$$\int_{s_0}^{\underline{s}} \left((\alpha - 1) - \max\left\{D_k(s; 1), 0\right\}\right) ds \geq \left(\frac{x_E}{x_V} - 1\right)(\underline{s} - s_0), \tag{A.13}$$

observing that the right hand side of Equation A.13 is weakly positive. Now suppose that $-k = L$ and $k = R$; then $D_L(\underline{s}; 1) = \alpha - 1$ (from the first condition). But since $D_L(\underline{s}; 1) < D_R(s; 1)$ and $D_i(s; 1)$ is strictly decreasing in s, the left hand side of Equation A.13 would be strictly negative, a contradiction. Thus any pure strategy equilibrium with activity must have $k = L$ and $-k = R$, and it is easily verified that such a pure strategy equilibrium (which is unique) satisfies the remaining properties in Proposition C.3.

We next consider mixed strategy equilibria ($s_0 \leq \underline{s} < \bar{s}$).To begin, we show that at any $s \in (\underline{s}, \bar{s}]$ where $D_i(s; F_i(s)) < 0 \iff \frac{x_E}{\alpha} F_{-i}(s) < sign(x_i) \cdot z_i(s)$ we must have $F_{-i}(s)$ strictly concave. To see this, observe from the differential equations in part (3) of Proposition B.1 that

$$f_{-i}(s) = \frac{\alpha - F_{-i}(s)}{2x_E \cdot \left(y_R^*(s) - y_L^*(s)\right)}$$

which is straightfowardly strictly decreasing in s.

Next, we show that at any $\breve{s} \in (\underline{s}, \bar{s}]$ where $D_i(\breve{s}; F_{-i}(\breve{s})) = 0 \iff \frac{x_E}{\alpha} F_{-i}(\breve{s}) = z_i(\breve{s})$ and also $D_i(s; F_{-i}(s)) > 0 \iff \frac{x_E}{\alpha} F_{-i}(s) > sign(x_i) \cdot z_i(s)$ in a neighborhood immediately below, $F_{-i}(s)$ is also strictly concave at $\breve{s}$ and in a neighborhood below, and $\frac{x_E}{\alpha} f_{-i}(\breve{s}) < \frac{\partial(sign(x_i) \cdot z_i(s))}{\partial s} = \frac{1}{2x_V}$. To see this first observe that we must have $\frac{x_E}{\alpha} f_{-i}(\breve{s}) \leq \frac{\partial(sign(x_i) \cdot z_i(s))}{\partial s} = \frac{1}{2x_V}$; otherwise $\frac{x_E}{\alpha} F_{-i}(\breve{s}) = z_i(\breve{s})$ would imply $\frac{x_E}{\alpha} F_{-i}(s) < sign(x_i) \cdot z_i(s)$ in a neighborhood below $\breve{s}$, contradicting our premise. Next, our premises imply that in a neighborhood below $\breve{s}$ we have

$$(\alpha - F_{-i}(s)) - D_i(s; F_{-i}(s)) = f_{-i}(s) \cdot 2x_E \cdot \left(y_R^*(s) - y_L^*(s)\right) \iff$$

$$f_{-i}(s) = \frac{(\alpha - F_{-i}(s)) + \frac{\alpha}{x_V}\left(F_{-i}(s)\frac{x_E}{\alpha} - sign(x_i) \cdot z_i(s)\right)}{2x_E \cdot \left(y_R^*(s) - y_L^*(s)\right)}$$

Since $y_R^*(s) - y_L^*(s)$ is strictly increasing in s, to show $f_{-i}(s)$ strictly decreasing (i.e., $F_{-i}(s)$ strictly concave) in a neighborhood below $\breve{s}$ it suffices to show the numerator is strictly decreasing. The derivative of the numerator is $-f_{-i}(s) + \frac{\alpha}{x_V}\left(f_{-i}(s)\frac{x_E}{\alpha} - \frac{1}{2x_V}\right)$, which is indeed < 0 at $\breve{s}$ and in a neighborhood below $\breve{s}$ since $f_{-i}(\breve{s})\frac{x_E}{\alpha} - \frac{1}{2x_V} \leq 0$. Lastly, since $F_{-i}(s)$ is strictly concave at $\breve{s}$ and in a neighborhood below $\breve{s}$ and $sign(x_i) \cdot z_i(s)$ is linear, we cannot have $f_{-i}(\breve{s})\frac{x_E}{\alpha} = \frac{1}{2x_V}$ since if so we would have $\frac{x_E}{\alpha}F_L(s) < sign(x_i) \cdot z_i(s)$ in a neighborhood below $\breve{s}$, again contradicting our premise.

Finally, note that the preceding arguments jointly imply that whenever $\frac{x_E}{\alpha}F_{-i}(s)$ "reaches" $sign(x_i) \cdot z_i(s)$ from above, it crosses at exactly a single point $\breve{s}$ and stays strictly below thereafter (since $z_i(s)$ is linear, $F_{-i}(s)$ is strictly concave in a neighborhood around $\breve{s}$, and once $\frac{x_E}{\alpha}F_{-i}(s)$ is strictly below $sign(x_i) \cdot z_i(s)$ the CDF $F_{-i}(s)$ remains strictly concave).

Now, having established basic properties about the score CDFs in neighborhoods around "crossings" between a developer's unbounded optimum and their boundary of the veto proof set, we make some statements about $F_L(s) - F_R(s)$ in neighborhoods below scores where it crosses 0 (i.e., scores $\breve{s} \in (\underline{s}, \overline{s}]$ where $F_L(\breve{s}) - F_R(\breve{s}) = 0$). Specifically, if $F_L(\breve{s}) - F_R(\breve{s}) = 0$ and $\frac{\partial}{\partial s}(F_L(s) - F_R(s)) = f_L(s) - f_R(s) < 0$ in a neighborhood below $\breve{s}$ then $F_L(\breve{s}) - F_R(\breve{s})$ is strictly decreasing in a neighborhood below $\breve{s}$, implying $F_L - F_R(s) > 0$ in a neighborhood below $\breve{s}$. To sign $F_L(s) - F_R(s)$ in a neighborhood below crossings of $F_L(s) - F_R(s)$ with 0 it thus suffices to sign $f_L(s) - f_R(s)$. Now from part (3) of Proposition B.1 we have $\forall i \in \{L, R\}$ and $s \in (\underline{s}, \overline{s}]$ the score CDFs are continuous and satisfy:

$$\begin{aligned} (f_R(s) - f_L(s)) \cdot 2x_E(y_R^*(s) - y_L^*(s)) &= F_L(s) - F_R(s) \\ &+ \max\{D_R(s; F_L(s)), 0\} - \max\{D_L(s; F_R(s)), 0\} \end{aligned} \tag{A.14}$$

with $y_R^*(s) - y_L^*(s) > 0$, which we use to sign $f_R(s) - f_L(s)$ around crossings $F_L(\breve{s}) - F_R(\breve{s}) = 0$.

(Case 1) Suppose first that $D_R(\breve{s}; F_L(\breve{s})) > 0$ (the right developer is strictly constrained at score $\breve{s}$). Then we immediately have that $f_R(\breve{s}) > f_L(\breve{s})$ (using that $D_R(s;F) > D_L(s;F)\ \forall (s,F)$), which immediately implies that $F_L(s) - F_R(s) > 0$ and strictly decreasing in a neighborhood below $\breve{s}$ (using that the score CDFs $\mathbf{F}$ are absolutely continuous over $(\underline{s}, \overline{s})$).

(Case 2) Suppose next that $D_R(\breve{s}; F_L(\breve{s})) \leq 0$ (the right developer is weakly unconstrained *at* score $\breve{s}$). Then $D_R(s;F) - D_L(s;F) = \frac{\alpha}{x_V} 2|y_0| > 0$ and $F_L(\breve{s}) - F_R(\breve{s}) = 0$ jointly imply that $D_L(\breve{s}; F_R(\breve{s})) < 0$ (the left developer is strictly unconstrained at score at $\breve{s}$ and a neighborhood below). We then consider two subcases.

(Subcase 2.i) Suppose we also have $D_R(s; F_L(s)) \leq 0$ (the right developer is also weakly unconstrained) in a neighborhood below $\breve{s}$. Then over this region the differential equations are

$$\alpha - F_{-i}(s) = f_{-i}(s) \cdot 2x_E\left(y_R^*(s) - y_L^*(s)\right)\ \forall i.$$

Then $\frac{f_i(s)}{\alpha - F_i(s)} = \frac{f_{-i}(s)}{\alpha - F_{-i}(s)} \to \int_s^{\breve{s}} \frac{f_i(t)}{\alpha - F_i(t)} dt = \int_s^{\breve{s}} \frac{f_{-i}(t)}{\alpha - F_{-i}(t)} dt \to \log\left(\frac{\alpha - F_i(s)}{\alpha - F_i(\breve{s})}\right) = \log\left(\frac{\alpha - F_{-i}(s)}{\alpha - F_{-i}(\breve{s})}\right) \to$ $F_i(s) - F_{-i}(s) = 0$ when combined with the boundary condition $F_i(\breve{s}) = F_{-i}(\breve{s})$.

(Subcase 2.ii) Suppose we instead have $D_R(s; F_L(s)) > 0$ (the right developer is strictly constrained) in a neighborhood below $\breve{s}$, further implying $D_R(\breve{s}; F_L(\breve{s})) = 0 \iff \frac{x_E}{\alpha} F_L(\breve{s}) = z_R(\breve{s})$ when combined with $D_R(\breve{s}; F_L(\breve{s})) \leq 0$ by continuity. By our initial arguments we immediately have that $F_L(s)$ is strictly concave in a neighborhood below $\breve{s}$ and $\frac{x_E}{\alpha} f_L(\breve{s}) < z_R'(\breve{s}) = \frac{1}{2x_V}$. We then argue that this implies $F_L(s) - F_R(s) > 0$ and strictly decreasing in a neighborhood below $\breve{s}$. Under our premises, in a neighborhood below $\breve{s}$ Equation ($A.14$) reduces to

$$\begin{aligned}
\left(f_R(s) - f_L(s)\right) \cdot 2x_E\left(y_R^*(s) - y_L^*(s)\right) &= \left(F_L(s) - F_R(s)\right) + D_R(s; F_L(s)) \\
&= \left(F_L(s) - F_R(s)\right) + \frac{\alpha}{x_V}\left(F_L(s)\frac{x_E}{\alpha} - z_R(s)\right)
\end{aligned}$$

Now using that $F_L(\breve{s}) = F_R(\breve{s})$ and $F_L(\breve{s})\frac{x_E}{\alpha} = z_R(\breve{s})$ we may rewrite the right hand side for s in a neighborhood below $\breve{s}$ as:

$$\begin{aligned}&\int_{s}^{\breve{s}}\left(f_R\left(t\right)-f_L\left(t\right)\right)dt+\frac{\alpha}{x_V}\int_{s}^{\breve{s}}\left(z_R'\left(t\right)-f_L\left(t\right)\frac{x_E}{\alpha}\right)dt\\=&\int_{s}^{\breve{s}}\left(\left(f_R\left(t\right)-f_L\left(t\right)\right)+\frac{\alpha}{x_V}\left(\frac{1}{2x_V}-f_L\left(t\right)\frac{x_E}{\alpha}\right)\right)dt\end{aligned}$$

Since $\lim_{s\to\breve{s}}\left(\left(f_R\left(s\right)-f_L\left(s\right)\right)+\frac{\alpha}{x_V}\left(\frac{1}{2x_V}-f_L\left(s\right)\frac{x_E}{\alpha}\right)\right)=\frac{\alpha}{x_V}\left(\frac{1}{2x_V}-f_L\left(\breve{s}\right)\frac{x_E}{\alpha}\right)>0$, the right hand side is strictly positive for s in a neighborhood below $\breve{s}$, implying that the left hand side is also strictly positive in a neighborhood below $\breve{s}$, implying $f_R\left(s\right)-f_L\left(s\right)>0$ in a neighborhood below $\breve{s}$, which yields the desired property.

We last apply the preceding to prove the main results. We first show weak first-order stochastic dominance ($F_R\left(s\right)\leq F_L\left(s\right)\ \forall s\in\left[\underline{s},\bar{s}\right]$). Suppose not, so $\exists s\in\left[\underline{s},s\right)$ such that $F_L\left(s\right)-F_R\left(s\right)<0$. Since $F_L\left(\bar{s}\right)-F_R\left(\bar{s}\right)=0$ there must exist some $\breve{s}\in\left(s,\bar{s}\right]$ such that $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$ and $F_L\left(s\right)-F_R\left(s\right)<0$ in a neighborhood below; but the preceding arguments already collectively imply that $F_L\left(s\right)-F_R\left(s\right)\geq 0$ in a neighborhood below any $\breve{s}$ where $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$.

We next argue $s_0<\underline{s}<\bar{s}$ (any mixed equilibrium is asymmetric). Suppose $s_0=\underline{s}$; since $y_R^*\left(s_0\right)=y_L^*\left(s_0\right)=0$ by part (3) of Proposition B.1 we must have $\alpha-F_{-i}\left(s\right)=D_i\left(s_0;F_{-i}\left(s_0\right)\right)$ $\forall i$; but since $D_R\left(s;F\right)>D_L\left(s;F\right)\ \forall\left(s,F\right)$ and $D_i\left(s,F\right)$ is strictly increasing in F, satisfying both equalities would require that $F_L\left(s_0\right)<F_R\left(s_0\right)$, contradicting first-order stochastic dominance.

We next argue that in any asymmetric equilibrium we must have $k=L$ and $-k=R$. Part (2) of Proposition B.1 requires that $\alpha-F_k\left(\underline{s}\right)=D_{-k}\left(\underline{s};F_k\left(\underline{s}\right)\right)$ and

$$\int_{s_0}^{\underline{s}}\left(\left(\alpha-F_{-k}\left(\underline{s}\right)\right)-\max\left\{D_k\left(s;F_{-k}\left(\underline{s}\right)\right),0\right\}\right)ds\geq F_{-k}\left(\underline{s}\right)\cdot\left(\frac{x_E}{x_V}-1\right)\left(\underline{s}-s_0\right)$$

where the right hand side is weakly positive. Suppose that instead $k=R$ and $-k=L$; then $\alpha-F_R\left(\underline{s}\right)=D_L\left(\underline{s};F_R\left(\underline{s}\right)\right)$ implying $\alpha-F_R\left(\underline{s}\right)<D_R\left(\underline{s};F_R\left(\underline{s}\right)\right)$. Then $F_L\left(\underline{s}\right)\geq F_R\left(\underline{s}\right)$ would imply that $\int_{s_0}^{\underline{s}}\left(\left(\alpha-F_L\left(\underline{s}\right)\right)-\max\left\{D_R\left(s;F_L\left(\underline{s}\right)\right),0\right\}\right)ds<0$, which would violate the inequality, so we must instead have $F_L\left(\underline{s}\right)<F_R\left(\underline{s}\right)$, but this would violate first-order stochastic dominance.

We next show $F_R(s) < F_L(s)\ \forall s \in [s_0, \underline{s}]$, which is equivalent to showing $F_R(\underline{s}) < F_L(\underline{s})$; by first-order stochastic dominance it suffices to rule out $F_R(\underline{s}) = F_L(\underline{s})$. Suppose so. By the first bullet point of part (2) of Proposition B.1 we have $D_R(\underline{s}; F_L(\underline{s})) = \alpha - F_L(\underline{s}) > 0$; letting $f_i^+(s) = \lim_{s \to \underline{s}^+}(f_i(s))$, Equation A.14 would then imply that

$$\begin{aligned}\left(f_R^+(\underline{s}) - f_L^+(\underline{s})\right) \cdot 2x_E\left(y_R^*(\underline{s}) - y_L^*(\underline{s})\right) &= D_R(s; F_L(\underline{s})) - \max\{D_L(s; F_R(\underline{s})), 0\} \\ &= D_R(s; F_L(\underline{s})) - \max\{D_L(s; F_L(\underline{s})), 0\} > 0\end{aligned}$$

implying $f_R^+(\underline{s}) > f_L^+(\underline{s})$; but then $F_R(s) - F_L(s) > 0$ in a neighborhood above $\underline{s}$, violating first-order stochastic dominance.

We last show there $\exists \breve{s} \in (\underline{s}, \bar{s}]$ such that $F_R(s) < F_L(s)$ for $s < \breve{s}$ and $F_R(s) = F_L(s)$ for $s \geq \breve{s}$. To see this, recall from the beginnning of the proof that (i) $F_L(s)$ is strictly concave at any $s \in (\underline{s}, \bar{s}]$ where $D_R(s; F_L(s)) < 0$, and (ii) at any $\breve{s} \in (\underline{s}, \bar{s}]$ where $D_R(\breve{s}; F_L(\breve{s})) = 0 \iff \frac{x_E}{\alpha} F_L(\breve{s}) = z_R(\breve{s})$ and also $D_R(s; F_L(s)) > 0 \iff \frac{x_E}{\alpha} F_L(s) > z_R(s)$ in a neighborhood immediately below, we must have $\frac{x_E}{\alpha} f_L(\breve{s}) < z_R'(\breve{s}) = \frac{1}{2x_V}$. When combined with the property that $D_R(\underline{s}; F_L(\underline{s})) = \alpha - F_L(\underline{s})$ these jointly imply that $D_R(s; F_L(s))$ crosses 0 at most once at a $\breve{s}$ (since $D_R(\breve{s}; F_L(\breve{s})) = 0$ and $\frac{x_E}{\alpha} f_L(\breve{s}) < z_R'(\breve{s})$ imply $D_L(s; F_L(s)) < 0$ in a neighborhood above $\breve{s}$ and therefore $F_L(s)$ is strictly concave and remains strictly concave thereafter by the first argument in this section of the proof on mixed strategy equilibria). Thus we can only have (i) $D_R(s; F_L(s)) > 0\ \forall s \in [\underline{s}, \bar{s})$ implying $F_R(s) < F_L(s)\ \forall s \in [\underline{s}, \bar{s})$, or (ii) there $\exists \breve{s} \in (\underline{s}, \bar{s})$ such that $D_R(s; F_L(s)) > (<)(=) 0 \iff s < (>)(=) \breve{s}$. Since $F_R(s) \leq F_L(s)\ \forall s$ and $D_R(s; F) \geq D_L(s; F)$ we must therefore also have $D_R(s; F_L(s)) \leq 0 \to D_L(s; F_R(s)) < 0$ (i.e., whenever (R) is weakly unconstrained (L) is strictly unconstrained), so over $[\breve{s}, \bar{s}]$ the differential equations are simply $\alpha - F_{-i}(s) = f_{-i}(s) \cdot 2x_E(y_R^*(s) - y_L^*(s))\ \forall i$ which as shown previously in this proof (see *Subcase 2.i*) requires $F_i(s) - F_{-i}(s) = 0$ when combined with the boundary condition that $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$. QED

Computational Procedure

The preceding analytical results justify the following computational procedure to calculate asymmetric equilibria ($y_0 \neq 0$) numerically. We describe the process for the case $y_0 < 0$ (the case $y_0 > 0$ is identical, switching L and R). First, we check whether the parameters are such that no activity ($\bar{s} = s_0$) is a pure strategy equilibrium – it is easy to verify from the score optimality conditions that when this is an equilibrium, it is necessarily the unique one. Next, when inactivity fails to be an equilibrium, we check whether the unique required strategy profile for a pure strategy equilibrium with activity ($s_0 < \underline{s} = \overline{s}$) with $k = L$ is indeed an equilibrium. If it is not, we then search for an asymmetric mixed equilibrium with $s_0 < \underline{s} < \overline{s}$ with $k = L$, which we identify for all parameters (indeed, it is possible to prove equilibrium existence using Simon and Zame (1990)).

Our analytical results handle some, but not all, potential issues of equilibrium multiplicity. Specifically, when inactivity is an equilibrium it is the unique one. When inactivity fails to be an equilibrium, *any* equilibrium exhibits asymmetric participation ($s_0 < \underline{s}$) with the more-motivated developer always active. And whenever an asymmetric pure strategy equilibrium exists, it is the unique pure strategy equilibrium. However, our analytical results do not rule out coexistence of a pure and mixed asymmetric equilibrium both with the more motivated developer always active, nor coexistence of two distinct mixed asymmetric equilibria both with the more motivated developer always active. We nevertheless conjecture (and our computational analysis supports) that equilibrium with symmetric developers and veto players is unique.

D Competitive Model Results

Having characterized the competitive model, we now show how results in the main paper follow from this characterization.

Lemma 1

Proposition B.1 implies that in a pure strategy equilibrium at most one developer is active. The case where neither developer is active is covered in part (1) of the proposition, noting that in this case each developer's optimal policy to develop is the status quo (s_0, y_0). When exactly one developer is active, she must develop her monopoly policy. To see that the active developer must be the developer with the higher monopoly score, suppose $s_{-k}^{M*} > s_k^{M*} > s_0$ and only k is active, developing $\left(s_k^{M*}, y_k^{M*}\right)$. But $-k$ strictly prefers to develop $\left(s_{-k}^{M*}, y_{-k}^{M*}\right)$ and have it enacted rather than having the status quo (s_0, y_0) and because $|x_{-k}| \geq |x_{V-k}|$, $-k$ strictly prefers (s_0, y_0) over $\left(s_k^{M*}, y_k^{M*}\right)$. Thus $-k$ strictly prefers to enter, i.e., there cannot exist a pure-strategy equilibrium in which the less-motivated developer is active.

Remark 1

Part 1. Follows from Lemma A.1.

Part 3. Follows from Lemmas A.2 and A.3

Part 2. Given part 3, the only choice for the developer is the CDF she uses when choosing score.

Proposition 2

Follows from Proposition B.1.

Proposition 3

Part 1. A necessary and sufficient condition for L to develop a policy if R sits out is $y_0 > \hat{y}_L(x_V) = -\frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)x_V$ and a necessary and sufficient condition for R to develop a policy if L sits out is $y_0 < \hat{y}_R(x_V) = \frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)(-x_V)$. Combining these conditions yields a necessary and sufficient condition for existence of a pure strategy equilibrium in which both sit out:

$$\hat{y}_R(x_V) \leq y_0 \leq \hat{y}_L(x_V)$$

For this condition to hold requires

$$\begin{aligned}\hat{y}_R(x_V) &\le \hat{y}_L(x_V)\\ \frac{1}{\alpha}x_E+\left(1-\frac{1}{\alpha}\right)(-x_V) &\le -\frac{1}{\alpha}x_E+\left(1-\frac{1}{\alpha}\right)x_V\\ x_V &\ge \frac{x_E}{\alpha-1}=\bar{x}_V.\end{aligned}$$

Part 2. As shown above, we know that in equilibrium at least one developer must be active when $y_0 \notin [\hat{y}_R(x_V), \hat{y}_L(x_V)]$. From Proposition C.3, the more-motivated developer is always active. The question is whether the less-motivated developer must be active as well, in a mixed strategy equilibrium as characterized in Proposition C.3.

We characterize a cutpoint $\bar{y}(x_V) \ge 0$ such that there is a pure strategy equilibrium in which the less-motivated developer is inactive iff $y_0 \notin [-\bar{y}(x_V), \bar{y}(x_V)]$. WLOG let $y_0 > 0$ so L is the more-motivated developer and always active. We determine whether R's best response is to enter or to sit out when L develops her monopoly policy from Lemma 1. Recall that L's monopoly policy is $\left(s_L^{M*}, y_L^{M*}\right)$ where

$$y_L^{M*}=-\frac{1}{\alpha}x_E+\left(1-\frac{1}{\alpha}\right)x_V=z_L\left(s_L^{M*}\right).$$

Also, note that from Definition A.1 $z_R(s)-z_L(s)=2(y_0-z_L(s))$ and thus $z_R(s)=2y_0-z_L(s)$.

We first note that it is never optimal for R to develop a policy with a score higher than s_L^{M*}. To see this, note that because $y_0>0$, $|z_R(s)|>|z_L(s)|$ so from Equation A.7 $D_R(s,1)<D_L(s,1)$.

Thus by Lemma A.2, if R's best response is to enter and beat L's monopoly policy, it will be at score s_L^{M*} and ideology $\min\left\{\max\left\{z_L\left(s_L^{M*}\right), \frac{x_E}{\alpha}\right\}, z_R\left(s_L^{M*}\right)\right\}$.

Obviously if $z_L\left(s_L^{M*}\right)=\min\left\{\max\left\{z_L\left(s_L^{M*}\right), \frac{x_E}{\alpha}\right\}, z_R\left(s_L^{M*}\right)\right\}$, R won't enter because doing so would mean paying costs to develop the same policy L develops. So the best ideology for R to enter at with a score-s_L^{M*} policy is $\min\left\{\frac{x_E}{\alpha}, z_R\left(s_L^{M*}\right)\right\}$.

We first note that $z_R\left(s_L^{M*}\right) \leq \frac{x_E}{\alpha}$ iff $y_0 \leq \frac{\alpha-1}{2\alpha}x_V$:

$$\begin{aligned}
z_R\left(s_L^{M*}\right) &\leq \frac{x_E}{\alpha} \\
2y_0 - z_L\left(s_L^{M*}\right) &\leq \frac{x_E}{\alpha} \\
2y_0 + \frac{1}{\alpha}x_E - \left(1-\frac{1}{\alpha}\right)x_V &\leq \frac{x_E}{\alpha} \\
y_0 &\leq \frac{\alpha-1}{2\alpha}x_V
\end{aligned}$$

We first consider the case $y_0 \leq \frac{\alpha-1}{2\alpha}x_V$, for which R's optimal score-s_L^{M*} is on the boundary and R's net benefit from entering at, $\left(s_L^{M*}, z_R\left(s_L^{M*}\right)\right)$ is

$$\begin{aligned}
& 2x_E\left(z_R\left(s_L^{M*}\right) - z_L\left(s_L^{M*}\right)\right) - \alpha\left(s_L^{M*} + \left[z_R\left(s_L^{M*}\right)\right]^2\right) \\
= \; & 4x_E\left(y_0 - z_L\left(s_L^{M*}\right)\right) - \alpha\left(s_L^{M*} + \left(2y_0 - z_L\left(s_L^{M*}\right)\right)^2\right) \\
= \; & 4x_E\left(y_0 - y_L^{M*}\right) - \alpha\left(s_L\left(y_L^{M*}; y_0\right) + \left(2y_0 - y_L^{M*}\right)^2\right) \\
= \; & 4x_E\left(y_0 - y_L^{M*}\right) - \alpha\left(-y_0^2 + 2x_V\left(y_0 - y_L^{M*}\right) + \left(2y_0 - y_L^{M*}\right)^2\right) \\
= \; & 4x_E\left(y_0 - y_L^{M*}\right) - \alpha\left(2x_V\left(y_0 - y_L^{M*}\right) + 2y_0\left(y_0 - y_L^{M*}\right) + \left(y_0 - y_L^{M*}\right)^2\right) \\
= \; & \left(y_0 - y_L^{M*}\right) \cdot \left(4x_E - \alpha\left(2x_V + 3y_0 - y_L^{M*}\right)\right)
\end{aligned}$$

Note that this is a concave quadratic function of y_0 with zeroes at $y_0 = y_L^{M*}$ and at $\check{y}\left(x_V\right)$ that solves

$$\begin{aligned}
0 &= 4x_E - \alpha\left(2x_V + 3y_0 - y_L^{M*}\right) \\
3\alpha y_0 &= 4x_E - 2\alpha x_V + \alpha y_L^{M*} \\
3\alpha y_0 &= 4x_E - 2\alpha x_V - \alpha\frac{1}{\alpha}x_E + \alpha\left(1-\frac{1}{\alpha}\right)x_V \\
\check{y}\left(x_V\right) &= \frac{x_E}{\alpha} - \frac{x_V\left(\alpha+1\right)}{3\alpha} \qquad \text{(A.15)}
\end{aligned}$$

Recall that for L to enter as a monopolist requires that $y_0 > y_L^{M*}$, and thus for R to have a profitable deviation to enter and win with policy $\left(s_L^{M*}, z_R\left(s_L^{M*}\right)\right)$ also requires that $y_0 < \check{y}\left(x_V\right)$.

We now consider the case $y_0 > \frac{\alpha-1}{2\alpha}x_V$, for which R's optimal score-s_L^{M*} is off the boundary and R's net benefit from entering at, $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ is:

$$\begin{aligned}
\tilde{G}\left(y_0; x_V, x_E\right) &= 2x_E\left(\frac{x_E}{\alpha} - z_L\left(s_L^{M*}\right)\right) - \alpha\left(s_L^{M*} + \left[\frac{x_E}{\alpha}\right]^2\right) \\
&= 2x_E\left(\frac{x_E}{\alpha} - y_L^{M*}\right) - \alpha\left(\left(-y_0^2 + 2x_V\left(y_0 - y_L^{M*}\right)\right) + \left[\frac{x_E}{\alpha}\right]^2\right) \\
&= 2x_E\left(\frac{x_E}{\alpha} + \frac{1}{\alpha}x_E - \left(1 - \frac{1}{\alpha}\right)x_V\right) \\
&\quad -\alpha\left(-y_0^2 + 2x_V y_0 - 2x_V\left(-\frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)x_V\right) + \left[\frac{x_E}{\alpha}\right]^2\right) \\
&= \alpha y_0^2 - 2\alpha x_V y_0 + 2x_E\left(\frac{2x_E}{\alpha} - \left(1 - \frac{1}{\alpha}\right)x_V\right) \\
&\quad -2x_V\alpha\frac{1}{\alpha}x_E + 2x_V\alpha\left(1 - \frac{1}{\alpha}\right)x_V - \alpha\left[\frac{x_E}{\alpha}\right]^2 \\
&= \alpha y_0^2 - 2\alpha x_V y_0 + 3\frac{x_E^2}{\alpha} + \left(-2x_E + 2x_V\alpha\right)\left(1 - \frac{1}{\alpha}\right)x_V - 2x_V x_E \\
&= \alpha y_0^2 - 2\alpha x_V y_0 + 3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1 - \frac{1}{\alpha}\right) \qquad \text{(A.16)}
\end{aligned}$$

From the quadratic formula, this has zeroes at

$$y_0 = x_V \pm \frac{1}{\alpha}\sqrt{\alpha^2 x_V^2 - \alpha\left(3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1 - \frac{1}{\alpha}\right)\right)}.$$

Because R's net benefit is a quadratic convex function of y_0, if the determinant is negative then the benefit of entering at $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ is strictly positive $\forall y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, x_V\right]$. Otherwise, R strictly gains from entering iff $y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, \tilde{y}\left(x_V\right)\right)$ where:

$$\tilde{y}\left(x_V\right) = x_V - \frac{1}{\alpha}\sqrt{\alpha^2 x_V^2 - \alpha\left(3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1 - \frac{1}{\alpha}\right)\right)}$$

Thus, letting

$$\bar{y}\left(x_V\right) = \begin{cases} \check{y}\left(x_V\right) \text{ if } y_0 \in \left(0, \frac{\alpha-1}{2\alpha}x_V\right] \\ \tilde{y}\left(x_V\right) \text{ if } y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, x_V\right] \end{cases} \qquad \text{(A.17)}$$

we have shown that for $y_0 > 0$ there is an equilibrium with only L active iff $y_0 \in \left(0, \bar{y}\left(x_V\right)\right]$. A symmetric argument applies to R when $y_0 < 0$.

Although not covered explicitly in part 2 of Proposition 3 in the main text, we briefly note what happens for $y_0 = 0$, in which case the developers are equally-motivated. Proposition C.1 characterizes a symmetric equilibrium for this case We also note that if $y_0 = 0$ there cannot be an equilibrium with exactly one developer active because developer $-k$'s monopoly policy $\left(0, y_{-k}^{M*}(0)\right)$ is developer k's strictly-worst zero-score policy, so if $-k$ develops $\left(0, y_{-k}^{M*}(0)\right)$ then k is strictly better off entering than sitting out.

Part 3. Follows directly from Proposition C.3.

Proposition 4

Part 1. First note that within a pure strategy equilibrium, the less-motivated developer's probability of being active is constant at 0.

We next show that if there is a pure strategy equilibrium at $\tilde{x}_V$ then there is a pure strategy equilibrium $\forall x_V > \tilde{x}_V$. WLOG, we show this for $y_0 > 0$.

First note that because a mixed strategy equilibrium requires that the more-motivated developer L be active, there is a pure strategy equilibrium with neither developer active iff

$$\begin{aligned} y_L^{M*} &> y_0 \\ -\frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)x_V &> y_0 \\ x_V &> \frac{\alpha y_0 + x_E}{\alpha - 1}. \end{aligned}$$

Otherwise, any pure strategy equilibrium must have L active. From Proposition 3, a pure strategy equilibrium with L active must have $y_0 > \bar{y}(x_V)$. where $\bar{y}(x_V) = \begin{cases} \check{y}(x_V) & \text{if } y_0 \in \left(0, \frac{\alpha-1}{2\alpha}x_V\right] \\ \tilde{y}(x_V) & \text{if } y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, x_V\right] \end{cases}$ from Equation A.17.

For $y_0 \leq \frac{\alpha-1}{2\alpha} x_V$, from Equation A.15 a pure strategy equilibrium requires:

$$
\begin{aligned}
y_0 &> \breve{y}(x_V) = \frac{x_E}{\alpha} - \frac{x_V(\alpha+1)}{3\alpha} \\
y_0 &> \frac{x_E}{\alpha} - \frac{x_V(\alpha+1)}{3\alpha} \\
x_V &> \frac{3\alpha}{\alpha+1}\left(\frac{x_E}{\alpha} - y_0\right).
\end{aligned}
$$

For $y_0 \geq \frac{\alpha-1}{2\alpha} x_V$, a pure strategy equilibrium requires that R's net benefit from entering at $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ if L develops her monopoly policy must be negative, i.e., from Equation A.16 it must be the case that $\tilde{G}(y_0; x_V, x_E) \leq 0$. We show that if $\tilde{G}(y_0; \tilde{x}_V, x_E) \leq 0$ for some $\tilde{x}_V \in (0, x_E)$ then $\tilde{G}(y_0; x_V, x_E) \leq 0$ for all $x_V \in [\tilde{x}_V, x_E]$, i.e., there is still a pure strategy equilibrium if veto players are more extreme. Note that $\tilde{G}(y_0; x_V, x_E)$ is a strictly convex quadratic function of x_V. Thus if $\tilde{G}(y_0; \breve{x}_V, x_E) > 0$ for some $\breve{x}_V \in (\tilde{x}_V, x_E)$ (i.e., there isn't a pure strategy equilibrium at $\breve{x}_V$), then $\tilde{G}(y_0; x_V, x_E) > 0$, $\forall x_V > [\breve{x}_V, x_E]$, and in particular $\tilde{G}(y_0; x_E, x_E) > 0$. But this cannot be the case, because if $x_V = x_E$ then $y_L^{M*} > 0$ and $y_R^{R*} < 0$ and hence R strictly prefers not to pay the marginal cost of moving policy rightward from L's monopoly policy.

We also note that at $y_0 = \frac{\alpha-1}{2\alpha} x_V$, $\breve{y}(x_V) = \tilde{y}(x_V)$ (via straightforward but tedious algebra). Thus if x_V increases from a value $< \frac{\alpha-1}{2\alpha} x_V$ to a value $> \frac{\alpha-1}{2\alpha} x_V$, the arguments above imply that if there is a pure strategy equilibrium at the lower value of x_V there is a also pure strategy equilibrium at the higher value.

The final component of the result is that within a mixed strategy equilibrium region, the probability that the less-motivated developer is active is strictly decreasing in x_V. This follows from computational analysis of the equilibrium characterized in Proposition C.3, holding fixed all parameters except for x_V.

Part 2. Follows directly from Proposition 1's condition for a monopolist to invest in policy development.

Corollary 2

This result follows directly from Equation (3) and Footnote 4 of Hirsch and Shotts (2015).

Proposition 5

This result follows from a combination of analytical and computational results.

The analytical results are the following. First, absent veto players, decisionmaker utility EU_D^0 is characterized in Corollary 2. Second, if $y_0 = 0$, decisonmaker utility with veto players is characterized in Proposition C.1, and Proposition C.2 shows this is strictly less than EU_D^0. Third, for parameters where neither developer is active, decisionmaker utility is $s_0 = -y_0^2 < 0 < EU_D^0$. Fourth, for parameters where exactly one developer is active, decisionmaker utility is the monopoly score, s_i^{M*}, which from Corollary 1 is strictly increasing in $|y_0|$.

The final piece of the results is for parameters where both developers are active and $y_0 \neq 0$. In this case, we compute decisionmaker utility by numerical evaluation of the equilibrium in Proposition C.3.

Extremist veto players harm decisionmaker when $\alpha > \tilde{\alpha}$

In our discussion of decisionmaker welfare in the main paper, we note that if $\alpha > \tilde{\alpha} \approx 3.68$, and veto players are extreme then for any y_0 the decisionmaker's utility is higher without veto players than with veto players. We show this result for $x_V = x_E$, noting that by continuity for any $\alpha > \tilde{\alpha}$ it holds for a neighborhood of x_V below x_E.

We first argue that for $x_V = x_E$ and any y_0 any equilibrium must be in pure strategies. Suppose not, i.e., there is an equilibrium in mixed strategies. Note that the less-motivated developer k must weakly prefer the status quo over any policy in the support of $-k$'s strategy, because one of the veto players is at k's ideal point. Also, $sign\,(y_0) = sign\,(x_k)$ so because $\alpha > 2$ there is no policy that k weakly prefers to develop and enact over the status quo. Hence we have a contradiction and k must in fact be inactive.

Also note that for any y_0 at which neither developer is active the decisionmaker's utility is ≤ 0 and hence strictly less than his utility without veto players, $EU_D^0 = 4x_E^2 \int_0^1 2F \left(\int_0^F \frac{G}{\alpha(\alpha-G)} dG \right) dF$ from Corollary 2.

Thus we only need to consider equilibria with exactly one active developer. From Corollary 1, when policy development occurs, the monopoly score is increasing in $|y_0|$, so to characterize a bound on α we can set $y_0 = -x_V = -x_E < 0$, calculate decisionmaker utility with R acting as a monopolist s_R^{M*}, and compare it with EU_D^0.

From Proposition 1, if R develops policy as a monopolist

$$y_R^{M*} = \frac{1}{\alpha} x_E + \left(1 - \frac{1}{\alpha}\right)(-x_V) = \frac{2}{\alpha} x_E - x_E.$$

Also, because the left veto player is indifferent,

$$q_R^* = \left(y_R^{M*} - (-x_V)\right)^2 - (y_0 - (-x_V))^2 = \frac{4}{\alpha^2} x_E^2$$

so decisionmaker utility is

$$s_R^{M*} = -\left(y_R^{M*}\right)^2 + q_R^* = -\left(\frac{2}{\alpha} x_E - x_E\right)^2 + \frac{4}{\alpha^2} x_E^2 = x_E^2 \left(\frac{4}{\alpha} - 1\right).$$

Without veto players, from Corollary 2 decisionmaker utility is

$$\begin{aligned} EU_D^0 &= 4x_E^2 \int_0^1 2F \left(\int_0^F \frac{G}{\alpha(\alpha - G)} dG \right) dF \\ &= 4x_E^2 \left(\left(\alpha + \frac{1}{2} - \frac{2}{3\alpha} \right) - \left(\alpha^2 - 1\right) \ln \left(\frac{\alpha}{\alpha - 1} \right) \right). \end{aligned}$$

Both s_R^{M*} and EU_D^0 are strictly decreasing in α. Evaluating the expressions numerically, $EU_D^0 > s_R^{M*}$, $\forall \alpha > \tilde{\alpha} \approx 3.68$.

E Data Notes

Figure 6 uses Nominate data downloaded from voteview.com on February 26th, 2023. If more than two people served as Senators for a state during a given session of Congress (due to death or exit), we use Nominate scores for the two who served the longest within that session.

We calculate the left filibuster pivot for a session as the 41st most liberal Senator and the right filibuster pivot as the 60th most liberal.

For calculation of within-party medians (Median Republican and Median Democrat in the figure), Senators who were independent or members of minor parties but caucused with a major party (Democrat or Republican) are treated as members of that party.