California Institute of Technology
Division of Humanities and Social Sciences

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Fall 2004 — Investments

Example Exam Questions
Version with Solutions

1. Are the following statements True, False or Ambiguous? Provide a short justification for your answer. (You are evaluated on your justification.)
   (a) If a portfolio has a beta of 1.0 ($\beta_p = 1.0$), then it must be perfectly correlated with the market.
   Solution: False. If the portfolio has zero idiosyncratic risk, then it will be perfectly correlated with the market regardless of the value of beta. In general a portfolio could have a beta of 1.0, however, and substantial idiosyncratic risk that is uncorrelated with the market. In this case the correlation would not be perfect.
   (b) The current price of Digital stock is $44 per share. You are offered a forward price for Digital stock to be delivered in one year of $42. The forward price is lower than the spot price because the market anticipates a sharp decline in the price of Digital stock, and the contract offers a way to hedge this risk. There is no arbitrage opportunity.
   Solution: False. Consider taking a long position in the future and shorting the stock in the spot market. Invest the present value of $42 in the T-Bills. At the maturity of the future, unwind your short position by buying the stock for $42.
   (c) The stocks of Merck and Disney are traded at the same price of $37 a share. The historical returns of Merck are more volatile than those of Disney and exhibit higher systematic risk. In addition, given the coming health reform proposal, it is generally believed that Merck may have a negative alpha in the future. It is known that neither Merck nor Disney will pay dividends in a month. Consider now forward contracts on these two stocks with one month to maturity. The forward price of 100 shares of Merck should be lower than the forward price of 100 shares of Disney.
   Solution: False. The forward price of both stocks should be $37×(1+r).
   (d) Since investors are compensated for holding risk, two securities with the same standard deviation should have the same expected return.
   Solution: FALSE: If assets are priced according to the CAPM, then the asset's beta will determine the expected return. Two assets with the same standard deviation could have different betas since they could have different amounts of diversifiable risk. If they have different betas, they will have different expected returns.
   (e) The sales of retailing firms, such as department stores, are very seasonal. For example, a very large fraction of sales occur in November and December. Because of this we expect that the returns on the common stock of these firms will be high in November and December and low in January and February.
   Solution: False. This should already be capitalized in the stock price.
   (f) The current spot price of gold is $300 per ounce and the one-year futures price is $330 per ounce. (Don't argue with these "facts!") These numbers tell us that the market is expecting at most a 10% increase in the price of gold over the next year.
   Solution: Uncertain. The futures price does not help us figure out expectations. For example let S_0 be the current price of gold, *r* be the riskfree rate, and *k* the per ounce cost of storing gold Then by no arbitrage, the futures price must be:
   $$F_0 = S_0 \times (1 + r + k)$$
   regardless of expectations. For example suppose that *r = 8%, k = 2%* and $S_0 = 300$. Then:
   $$F_0 = \$300 \times (1 + 0.08 + 0.02) = \$330$$
   independent of our or the market's expectations about the price of gold in the future.
   (g) You just ran a regression relating the excess returns of a portfolio of small stocks with the excess returns on the S&P500. (Remember "excess returns" are returns in excess of the risk-free rate.) You found that the intercept in this regression is ***negative*** and statistically significant. This means that the S&P500 is not mean-variance efficient.
   Solution: True. If the S&P500 is on the mean-variance frontier it must be the tangency portfolio. For the tangency portfolio we have:
   $$E(r_i) - r_f = \frac{Cov(r_i, r_T)}{\sigma_T^2} \times \left[E(r_T) - r_f\right]$$
   hence the regression should produce an intercept of zero.
   (h) The price of ABC stock is currently $1 per share and you hold an in-the-money American put option on ABC stock. The options will expire in one-year, but you would like to get out of your option position now. Because ABC is not expected to pay a dividend over the next year you would sell the option instead of exercising it.
   Solution: Uncertain. It is possible that exercising the option now is the best think to do. If you wait the most you can make is $1. If there is a reasonable chance (according to the risk-neutral probabilities) then the put option will have a small value. In this case it is better to exercise the option now.
   (i) The return of XYZ's stock is perfectly correlated with the market return. According to the CAPM the expected return of XYZ's stock is equal to the market return.
   Solution: False in general. A stock can have low beta and yet be perfectly correlated with the market if the idiosyncratic risk is zero.

2. You are at lunch one day with a friend and the conversation turns to investment in stocks and bonds. Your friend has little experience with investment in stocks. You start by explaining the benefits of diversification. Your friend then says, “This is great! That means if I invest in 1,000 stocks I will be able to avoid risk altogether. Why have I been keeping my money in the bank all this time?" Under what situation would your friend be correct? Do you think that this situation is realistic? What further “instruction" would you give your friend?

Solution:

If all stock returns are uncorrelated, then by investing in a large number of stocks risk could be essentially eliminated. This is not realistic because all stock returns tend to move together to some extent. I would explain to my friend that firm specific risk can be eliminated, but not market wide movements in stock price.
My friend would still face a risk-return tradeoff between high-return with high-risk in the stock market and low-risk with low-return at the bank.

3. Consider the following properties of the returns of stock 1, of stock 2 and of the market (m):

$$E(\tilde{r}_m) = 0.10 \quad \sigma_1 = 0.20 \quad \rho_{1,m} = 0.4$$
$$\sigma_m = 0.10 \quad \sigma_2 = 0.30 \quad \rho_{2,m} = 0.7$$

Suppose further that the risk-free rate is 5%.
   (a) According to the Capital Asset Pricing Model, what should be the expected return of stock 1 and of stock 2?
   (b) Suppose that the correlation between the return of stock 1 and the return of stock 2 is 0.5. What is the expected return and the standard deviation of the return of a portfolio that has a 40% investment in stock 1 and a 60% investment in stock 2?
   (c) Assume that the Capital Asset Pricing Model is valid. How could you construct a new portfolio using the market portfolio and the riskfree asset that has the same expected return as the portfolio you considered in part 3b but has the lowest standard deviation possible? What is the standard deviation of the return of this portfolio?
   (d) Suppose that the correlation between the return of stock 1 and the return of stock 2 is -0.7. What is the expected return and the standard deviation of the return of a portfolio that has a 40% investment in stock 1 and a 60% investment in stock 2? Consider an investment that is made up of a combination of this risky portfolio and the risk-free asset. In this case, would it be advisable to switch to a portfolio made up of the market portfolio and the risk-free asset? In other words, is the market portfolio mean-variance efficient in this case?

Solution:

a) First compute the betas

$$\beta_1 = \frac{\sigma_{1,m}}{\sigma_m^2} = \frac{\rho_{1,m}\sigma_1\sigma_m}{\sigma_m^2} = \frac{0.4\times 0.2\times 0.15}{0.15^2} = 0.5333$$
$$\beta_2 = \frac{0.7\times 0.3\times 0.15}{0.15^2} = 1.4$$

$$E(\tilde{r}_1) = r_f + \beta_1 \times \lfloor E(\tilde{r}_m) - r_f \rfloor = 5\% + 0.5333\times[10\% - 5\%] = 7.67\%$$
$$E(\tilde{r}_2) = 5\% + 1.4\times[10\% - 5\%] = 12.00\%$$

b) Combining stocks 1 and 2 in the portfolio yields

$$E(\tilde{r}_p) = w_1E(\tilde{r}_1) + w_2E(\tilde{r}_2) = 0.4\times 7.67\% + 0.6\times 12\% = 10.27\%$$

$$SR_M = \frac{E(\tilde{r}_m) - r_f}{\sigma_m} = \frac{0.10 - 0.05}{0.15} = 0.3333$$

c) Let w_m be the weight on the market portfolio. We require:

$$E(\tilde{r}_p) = 10.27\% = w_mE(\tilde{r}_m) + (1 - w_m)r_f$$
$$\Leftrightarrow 10.27\% = w_m \times 10\% + (1 - w_m)\times 5\%$$
$$\Leftrightarrow 10.27\% = w_m \times 5\% + 5\%$$
$$\Leftrightarrow w_m = \frac{10.27\% - 5\%}{5\%} = 1.054$$

I would invest 105.4% in the market and –5.4% at the risk-free rate. The standard deviation to the return on this portfolio would be 1.054×0.15 = 0.1581.

d) $E(r_p)$ = 10.27%, as in part b. However now:

$$\sigma_p^2 = (w_1\sigma_1)^2 + (w_2\sigma_2) + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$
$$= (0.4\times 0.2)^2 + (0.6\times 0.3)^2 + 2\times 0.4\times 0.6\times(-0.7)\times 0.2\times 0.3 = 0.01864$$
$$\sigma_p = \sqrt{0.01864} = 0.137 = 13.7\%$$

Risk-return tradeoff on the market:

$$SR_M = \frac{E(\tilde{r}_m) - r_f}{\sigma_m} = \frac{0.10 - 0.05}{0.15} = 0.3333$$

Risk-return tradeoff on the portfolio:

$$SR_P = \frac{E(\tilde{r}_P) - r_f}{\sigma_P^2} = \frac{0.1027 - 0.05}{0.137} = 0.38$$

There is a better tradeoff with this portfolio, therefore the market is not mean-variance efficient.

4. Your current portfolio consists of three assets, the common stock of IBM and GM combined with an investment in the riskless asset. You know the following about the stocks ($\rho_{i,j}$ denotes the correlation between asset *i* and asset *j*, and *M* denotes the market portfolio):

$$\rho_{IBM,M} = 0.30 \quad \rho_{GM,M} = 0.40$$
$$\sigma_{IBM}^2 = 0.64 \quad \sigma_{GM}^2 = 0.25$$

You also have the following information about the market portfolio, and the riskless asset, *f*:

$$E(\tilde{r}_m) = 0.13 \quad \sigma_m^2 = 0.04 \quad r_f = 0.04$$

Assume that individuals can borrow and lend at r_f and that the Capital Asset Pricing Model (CAPM) describes expected returns on assets. You have $200; 000 invested in IBM, $200; 000 invested in GM, and $100; 000 invested in the riskless asset.

(a) What are the expected rates of return on IBM stock and GM stock?
(b) What are the R^2 values of the regressions of the returns of IBM and GM on the return on the market?
(c) Assume that the correlation between IBM and GM, $\rho_{IBM,GM}$ *is 0.10*. What is the variance of your portfolio? What is its beta, $\beta_{p,M}$?
(d) Find an efficient portfolio that has the same standard deviation as your portfolio, but has the highest expected return possible. What is the expected return on this portfolio?

Solution:

(a) First compute the stock's betas and then their expected returns:

$$\beta_{IBM} = \frac{\sigma_{IBM,m}}{\sigma_m^2} = \frac{\rho_{IBM,m}\sigma_{IBM}\sigma_m}{\sigma_m^2}$$
$$= \frac{0.3 \times 0.64^{1/2} \times 0.04^{1/2}}{0.04} = 1.2$$
$$\beta_{GM} = \frac{0.4 \times 0.25^{1/2} \times 0.04^{1/2}}{0.04} = 1.0$$

$$E(\tilde{r}_{IBM}) = r_f + \beta_{IBM} \times \left[E(\tilde{r}_m) - r_f\right]$$
$$= 4\% + 1.3 \times [13\% - 4\%] = 14.80\%$$
$$E(\tilde{r}_{GM}) = 4\% + 1.0 \times [13\% - 4\%] = 13.00\%$$

(b) The R^2 are computed as follows:

$$R_{IBM}^2 = \frac{\beta_{IBM}^2 \sigma_m^2}{\sigma_{IBM}^2} = \frac{1.2^2 \times 0.04}{0.64} = 0.09$$
$$R_{GM}^2 = \frac{\beta_{GM}^2 \sigma_m^2}{\sigma_{GM}^2} = \frac{1.0^2 \times 0.04}{0.25} = 0.16$$

(c) Here $w_f = 100/500 = 0.2$, $w_{IBM} = 200/500 = 0.4$ and $w_{GM} = 200/500 = 0.4$.

$$\sigma_p^2 = (w_{IBM}\sigma_{IBM})^2 + (w_{GM}\sigma_{GM})^2 + 2 w_{IBM} w_{GM} \rho_{IBM,GM} \sigma_{IBM} \sigma_{GM}$$
$$= (0.4)^2 \times 0.64 + (0.4)^2 \times 0.25 + 2 \times 0.4 \times 0.4 \times 0.1 \times (0.64 \times 0.25)^{1/2} = 0.155$$
$$\sigma_p = \sqrt{0.155} = 0.3937 = 39.37\%$$
$$\beta_p = w_{IBM} \times \beta_{IBM} + w_{GM} \times \beta_{GM} + w_f \times \beta_f$$
$$= 0.4 \times 1.2 + 0.4 \times 1.0 + 0.2 \times 0 = 0.88$$

(d) Efficient portfolios are made up of combinations of the riskfree asset and the market. Let w_M be the amount invested in the market portfolio. The standard deviation of an efficient portfolio is:

$$\sigma_{efficient} = w_M \sigma_M$$

The standard deviation of the portfolio is $\sigma_p = 0.40$. Therefore the efficient portfolio with the same standard deviation is determined by $w_m \times 0.04^{1/2} = 0.4 \Rightarrow w_m = 2$. The expected rate of return on the portfolio is $w_m \times 0.13 - 1 \times 0.04 = 22\%$

5. You have the following information about the prices of a one-year discount or zero-coupon bond and a two-year coupon bond (note that the two-year bond is not a discount bond).

- The one-year discount bond pays $100 in one year and sells for a current price of $96.
- The two-year coupon bond has a principal of $1000 and an annual coupon of $60. The bond currently sells for a price of $1050.

(a) What are the implied yields to maturity on one- and two-year discount bonds?
(b) What is the implied forward rate between years 1 and 2?
(c) Consider a 2 year annuity with annual payments of $500. What is the most that you would be willing to pay for this annuity?

Solution:

(a) YTM_1 = 100/96 - 1 = 4.17%. YTM_2 satisfies:

$$1050 = \frac{60}{1.0417} + \frac{1060}{(1 + YTM_2)^2}$$

Or

$$YTM_2 = \left(\frac{1060}{1050-57.60}\right)^{1/2} - 1 = 3.35\%$$

(b) The forward rate is given by:

$$f_2 = \frac{(1+YTM_2)^2}{(1+YTM_1)} - 1 = \frac{(1+0.0335)^2}{(1+0.0417)} - 1 = 2.54\%$$

(c) The price of this annuity should be:

$$\frac{500}{1.0417} + \frac{500}{(1.0335)^2} = \$948.10$$

I would pay not more that $948.10.

6. The stock of Withit Incorporated has a β of 0.5 and that of Againstit Incorporated has a β of -0.5. The excess return on the market is 10% and the risk free return is currently 5%. A portfolio that has a 50% investment in Withit and a 50% investment in Againstit has an expected return of 6%. As a manager of a large portfolio, would this information affect your investment strategy? If so, in what manner and why?

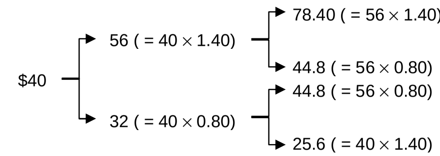
Solution:

The β of the portfolio is $0.5 \times 0.5 + 0.5 \times (-0.5) = 0$. According to the CAPM the expected return on this portfolio should be the risk free return which is 5%. Since this portfolio is expected to earn 6%, this portfolio represents a good opportunity. I would increase my investment in this portfolio relative the proportion that it represents in the market portfolio. This strategy implies that I will face the diversifiable risk in the two companies. As a result I would not necessarily invest all of my money in this portfolio, but I would think about how much of this company specific risk I would be willing to take.

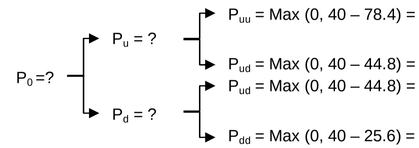
7. Consider a stock which is currently selling for $40. Each year you expect that the stock will either increase in value by 40% or decrease in value by 20%. Currently a one year pure discount bond with face value $1000 is selling for $980 and the term structure is flat. After analyzing the stock you believe that its prospects are not good and you believe that there is a 90% chance that it will decrease in value each year.
   (a) What is the price of a European put option written on this stock with two years to expiration and strike price of $40?
   (b) Suppose that this option is selling for $2. Would you take advantage of this price in some way? If so, describe in detail what you would do.

Solution:

(a) I know that I will need a risk-free interest rate to do this calculation. According to the discount bond price the risk free interest rate is 1000/980 -1 = 2%. The potential payoffs of the stock over time are given by:



The payoff on the put option is:



Where P_u is the price of the call when the stock goes up the first period and P_d is the price of the call when the stock goes down after the first period. Obviously $P_u = 0$.
To answer this question you could calculate risk-neutral probabilities and then calculate. Instead I will derive the strategy that replicates the call payoff since I know I will need it in part (b).
Let Δ_d and D_d be the respective positions in the stock and bonds needed to replicate the put option after the stock has down to 32. Then we need:

$$44.8\Delta_d + 1.02D_d = 0$$
$$25.6\Delta_d + 1.02D_d = 14.4$$

This implies that $\Delta_d = -0.75$ and $D_d = 32.94$. Hence $P_d = -0.75 \times 32 + 32.94 = 8.94$.
Let Δ and D be the respective positions in the stock and bonds needed to replicate the payoff to the put option over the first year. We need:

$$56\Delta + 1.02D = 0$$
$$32\Delta + 1.02D = 8.94$$

This implies that $\Delta = -0.37$ and $D = 20.31$. Hence $P = 5.51$.

(b) Since the option is sell for less than it is worth, I would buy the option now and purchase 0.37 shares of the stock and borrow 20.31 in the bond market today. I would then readjust my holdings of stocks and bonds as in part (a) to allow me to deliver the option payoff at time 2. This hedges my risk and I can keep the difference between 5.51 and 2.

8. You are a manager in a brokerage house which makes markets in a variety of securities including stocks, bonds, and options. A customer has called one of your salesmen to find out if the firm would be willing to write (i.e., sell to) him a two-year European call option on 100 shares of ABC stock with an exercise

price of $48 per share. The customer is willing to pay $2000 for the call option, and your salesman wants to know if the firm will do it or not. ABC stock is currently selling for $50 per share. Although call options on ABC are not traded, European put options on ABC are actively traded on the Chicago Board Options Exchange. A two-year put option on 100 shares of ABC with an exercise price of $48 per share is currently selling for $500. In the government bond market, a Treasury bill with a principal value of $100 and a maturity date of one year from now is currently trading at $85. A Treasury Note which promises a coupon payment of $10 one year from now and a principal payment of $100, along with a $10 coupon payment two years from now is currently trading at $83.5.
Answer the following questions:
(a) Should the firm write the call option? Why?
(b) What is the lowest price for the call option at which you would recommend that your firm write the option?
(c) Suppose that in addition to the above information, you discovered that your research department viewed ABC stock favorably and was recommending its purchase. Would your answer to part b change? Why or why not?

Solution:

(a) To answer this question we first need the yield to maturity on a two-year zero. We can calculate YTM_1 from the one-year zero:

$$YTM_1 = 100/85 - 1 = 17.65\%$$

The coupon bond price must satisfy:

$$835.5 = \frac{10}{1.1765} + \frac{110}{(1+YTM_2)^2}$$

Hence YTM_2 = 21:11%.
According to Put-Call Parity the price of the call (per share) should be:

$$C = S_0 + P - \frac{K}{(1+YTM_2)^2}$$
$$= 50 + 5 - \frac{48}{1.2111^2} = 22.27$$

For 100 shares, the call price should be $2,227. Since the customer is willing to pay only $2,000, you shouldn't write the call.
(b) The lowest price is $2,227.
(c) This doesn't change my answer since we are pricing the call from arbitrage.

9. You are approached by two portfolio managers. The track records of these two managers in the past five years are as follows: The first had an average annual excess return of 10% and a standard deviation of 25%; and the second had an average excess return of 12% and a standard deviation of 32%. With

this information, could you decide who is a better manager? Would you entrust some of your money with either of these two managers and why?

Solution:

I would like information about their betas in order to assess whether any of the risk is due to market variation and to assess whether I could combine these portfolios in my larger portfolio to improve the portfolio performance.

10. You conclusively demonstrate that a multifactor model describes stock returns better than the CAPM. You are building portfolios for a client. He says, "That's all very nice, but I am only interested in mean and variance. Therefore, the two-fund theorem applies. I don't believe in active management, so I should just hold the market portfolio." How would you respond?

Solution:

If the CAPM does not work, the market is not mean-variance efficient. The analysis implies that we could combine the multifactor portfolios together to create a portfolio that will beat the market.

11. Suppose that the risk-free rate is 5%, and that the expected return on the market is 12%. You are advising a friend who has identified a stock which he expects will have a rate of return of 15%. Your analysis of the stock indicates that the stock has a beta of 0.5. What advice would you offer to your friend and why?

Solution:

The required return on the stock is:

$$r^e_{required} = 5\% + 0.5 \times (12\% - 5\%) = 8.5\%$$

Since the return expected by my friend is greater than the required return, this stock represents a good investment. My friend should consider increasing his investment in the stock. He should be careful about increasing this investment too much, however, since he will be exposed to idiosyncratic risk as he becomes less diversified.

12. You have an obligation to pay out $10 million in 5 years and you would like to invest today in the bond market to protect your investment. You can invest in 2 year pure discount bonds and 10 year pure discount bonds. What combination of bonds would you choose to invest in.

Solution:

The duration of the obligation is 5. The duration of the 2-year zero and 2 and the duration of the 10-year zero is 10. Let *x* be the proportion of my investment in the 2-year zero, then to protect myself against interest rate fluctuations I need:

$$2x + 10(1 - x) = 5$$

Hence *x = 62:5%*. I would invest 62.5% of my money in the 2-year zero and 37.5

13. You are an investor in ***Canada***. The current exchange rate between U.S. and Canadian dollars is 0.7 U.S. dollars for each Canadian dollar. The one-year forward exchange rate is 0.72 U.S. dollars for each Canadian dollar. The two-year forward exchange rate is 0.7 U.S. dollars for each Canadian dollar. The yields to maturity on zero coupon bonds of all maturities in Canada are 6% (in Canadian dollars).
    (a) What is the yield to maturity (in U.S. dollars) on a one-year pure discount bond in the United States? What is the yield to maturity (in U.S. dollars) on a two-year pure discount bond in the United States?
    (b) A U.S. bank that you work with is willing to sign a forward contract with you in which you agree to invest $US 100 million in the bank in one year. A year later (i.e. two years from now) you will receive $US 105 million. Alternatively you could sign a forward contract with this same bank in which you borrow $US 100 million in one year and repay $US 105 million in two years. Are these terms consistent with your calculations in part (a)? If not, is there a way to take advantage of the bank? Is so, how?

Solution:

(a) Let E_0 be the current exchange rate, $F_{0,1}$ the one-year forward rate and $F_{0,2}$ the two-year forward rate. Then

$$F_{0,j} = E_0 \frac{(1+YTM_j^{US})^j}{(1+YTM_j^{CDN})^j}$$

where YTM_j^{US} is the yield to maturity on a pure discount bond with *j* years to maturity in the US and YTM_j^{CDN} is the yield to maturity on a pure discount bond with *j* years to maturity in Canada. Hence:

$$YTM_j^{US} = \left[\frac{F_{0,j}}{E_0}(1+YTM_j^{CDN})^j\right]^{\frac{1}{j}} - 1$$

In our case this gives

$$YTM_1^{US} = \left[\frac{0.72}{0.7}(1+06)^1\right]^{\frac{1}{1}} - 1 = 9.03\%$$

$$YTM_2^{US} = \left[\frac{0.7}{0.7}(1+06)^2\right]^{\frac{1}{2}} - 1 = 6\%$$

(b) The implied forward rate in the US between year 1 and year 2 is:

$$f_2^{US} = \frac{1.06^2}{1.0903} - 1 = 3.05\%$$

The bank is quoting a forward rate of 5% hence the terms are not consistent with my calculations in part (a). I would do the following:

- Agree to lend the funds to the bank. That is, invest $US 100 M in year 1 and receive $US 105 M in year 2.
- Buy $US 100 M in the one-year forward currency market. This will cost me 100/0.72 = $CDN 138.89 M in year 1.
- Finance the $CDN 138.89 M in the Canadian bond market through a forward loan at 6%. If this is not available then I would use the one-year and two-year zero coupon bonds in Canada to create this contract. I will owe $CDN 138.89 M × 1.06 = $CDN 147.22 M.
- Sell $US 105 M in the two-year forward currency market. I will receive 105/0.7 = $CDN 150 M in year two.

This generates $CDN 150 M - $CDN 147.22 M = $CDN 2.78 M in year 2 with no expenditure anywhere.

14. Consider the following information about the return on two stocks:

$$E(\tilde{r}_1) = 0.08 \quad \sigma_1 = 0.14 \quad r_f = 0.04$$
$$E(\tilde{r}_2) = 0.10 \quad \sigma_2 = 0.18 \quad \rho_{1,2} = 0$$

   (a) Suppose that you currently invest 20% in the risk-free security, 10% in stock #1 and 70% in stock #2. What is the expected return on your portfolio? What is the standard deviation of the return? What is the Sharpe ratio of your portfolio?
   (b) Is it possible to invest in a combination of the risk-free security, stock #1 and stock #2 to give you a portfolio with a higher expected return but with the same standard deviation as your portfolio in part (a)? If so, construct this new portfolio. The new portfolio you construct must also have the best possible Sharpe ratio of portfolios invested in combinations of stocks #1 and #2, and the risk-free security.

Solution:

(a) The expected return on this portfolio is:

$$E(\tilde{r}_p) = 0.2\times 4\% + 0.1\times 8\% + 0.7\times 10\% = 8.6\%$$

The variance and standard deviation of the return on this portfolio are:

$$\sigma_p^2 = (0.1\times 0.14)^2 + (0.7\times 0.18)^2 = 0.0161$$

$$\sigma_p = \sqrt{0.0161} = 0.13 = 13\%$$

The sharpe ratio for the portfolio is:

$$SR_P = \frac{E(\tilde{r}_p) - r_f}{\sigma_m} = \frac{0.086 - 0.04}{0.13} = 0.35$$

(b) To answer this question we need the tangency portfolio. First we find the weights w1 and w2 on stock 1 and 2 respectively that solve:

$$w_1\sigma_{1,1} + w_2\sigma_{1,2} = E(r_1) - r_f$$
$$w_1\sigma_{1,2} + w_2\sigma_{2,2} = E(r_2) - r_f$$

Solving yields:

$$w_1 = \frac{(0.08-0.04)}{0.14^2} = 2.04$$
$$w_2 = \frac{(0.10-0.04)}{0.18^2} = 1.85$$

The actual weights are determined by normalizing these so that they add to one. Hence

$$w_1 = \frac{2.04}{2.04+1.85} = 0.52$$
$$w_2 = \frac{1.85}{2.04+1.85} = 0.48$$

The expected return of the tangency portfolio is:

$$E(\tilde{r}_T) = 0.52\times 8\% + 0.48\times 10\% = 8.96\%$$

The variance of the tangency portfolio return is:

$$\sigma_T^2 = (0.52\times 0.14)^2 + (0.48\times 0.18)^2 = 0.0128$$
$$\sigma_p = \sqrt{0.0128} = 0.113 = 11.3\%$$

Our goal is to construct a portfolio combining the tangency portfolio and the risk free asset in such a fashion as to match the standard deviation of the portfolio constructed in (a). Let w_T be the weight on the Tangency portfolio. Then w_T = 0.13/0.113 = 1.15. The expected return on this portfolio is:

$$E(\tilde{r}_{Match}) = -0.15\times 4\% + 1.15\times 8.96\% = 9.74\%$$

15. ABC incorporated is currently trading for $50 per share. After examining the stock of ABC you have determined that in one year the price of ABC will either increase to $65 per share with probability 85% or decrease to $45 per share with probability 15%. ABC does not pay dividends. A one-year T-bill with a face value of $100 is also trading for $98.
   (a) What should be the price of a European call option with one year to maturity and a strike price of $60 per share?
   (b) Suppose that the call option of part (a) is actually trading for $2. Is this an arbitrage opportunity? If so, how would you exploit it?
   (c) Suppose now that the call option of part (a) is trading for the price you calculated in part (a). A European put option with a strike price of $60 and one year to maturity is trading for $8, however. Could you use the call option (and other securities) to exploit this price in any way? If so, how?

Solution:

(a) The one-year risk-free rate is 100/98 – 1 = 2%. The potential payoff of the stock and of the call option are given by:

S_0 = 56 → S_u = 65 ; S_d =

C_0 =? → C_u = Max(0,65-60) = 5 ; C_d = Max(0,45-60) =

We need a Δ and a D such that:

$$65\Delta + 1.02D = 5$$
$$45\Delta + 1.02D = 0$$

These imply that Δ = 0.25 and D = –11.03. The price of the call should be 0.25 × 50 – 11.03 = 1.47.

(b) Since the option is overvalued I would sell the option and hedge this obligation by buying 0.25 shares of the stock and borrowing $11.03 bonds. For each time I did this I would make 0.54. I would do this a lot!!

(c) According to put-call parity, the put option should be worth:

$$P = C - S + K/(1+r)$$
$$= 1.47 - 50 + 60/1.02 = 10.29$$

Since the put is underpriced in the market I would buy the put option and hedge this position by selling a call, buying a share of the stock and borrowing 60/1.02 = 58.82 using a one-period pure discount bond.

16. Consider the following information about the returns to stocks #1 and #2 along with the market return,

$$\sigma_1 = 0.4 \quad \rho_{1,m} = 0.2 \quad \sigma_2 = 0.5 \quad \rho_{2,m} = 0.4$$
$$E(\tilde{r}_m) = 0.10 \quad \sigma_m = 0.15 \quad r_f = 0.05$$

   (a) According to the CAPM, what is the expected return on a portfolio that is invested 40% in stock #1 and 40% in stock #2 and the rest in the risk-free security?
   (b) Assuming that the idiosyncratic risk in stock #1 is uncorrelated with the idiosyncratic risk in stock #2, what percentage of the total variance of the return to the portfolio in part (a) is explained by the market return?
   (c) Suppose that you are currently 50% invested in the market portfolio and 50% in the risk-free security. After analyzing stock #1 you expect that the return on this stock will be 10%. Would this information make you change your investment strategy? If so, how?

Solution:

(a) The beta of the stock #1 is:

$$\beta_1 = \frac{\sigma_{1,m}}{\sigma_m^2} = \frac{\rho_{1,m}\sigma_1\sigma_m}{\sigma_m^2}$$
$$= \frac{0.2\times 0.4\times 0.15}{0.15^2} = 0.5333$$
$$\beta_2 = \frac{0.4\times 0.5\times 0.15}{0.15^2} = 1.33$$

The beta of the portfolio is 0.4 × 0.53 + 0.4 × 1.33 = 0.74. Hence the expected return of the portfolio is:

$$E(\tilde{r}_p) = r_f + \beta_p \times \left[E(\tilde{r}_m) - r_f\right]$$
$$= 5\% + 0.74\times\left[10\% - 5\%\right] = 8.72\%$$

(b) The covariance between the two stock returns is given by:

$$\sigma_{1,2} = \beta_1\beta_2\sigma_m^2 = 0.53\times 1.33\times 0.15^2 = 0.016$$

The variance of the portfolio is:

$$\sigma_p^2 = (0.4\times 0.4)^2 + (0.4\times 0.5)^2 + 2\times 0.4\times 0.4\times 0.016 = 0.0707$$
$$\sigma_p = \sqrt{0.0707} = 0.2659 = 26.59\%$$

Hence the percentage of total variance explained by the market is:

$$R^2 = \frac{\beta_p^2\sigma_m^2}{\sigma_p^2} = \frac{0.74^2\times 0.15^2}{0.0707} = 0.176 \text{ or } 17.6\%$$

(c) According to the CAPM it should return:

$$E(\tilde{r}_1) = r_f + \beta_1\times\left[E(\tilde{r}_m) - r_f\right] = 5\% + 0.533\times\left[10\% - 5\%\right] = 7.67\%$$

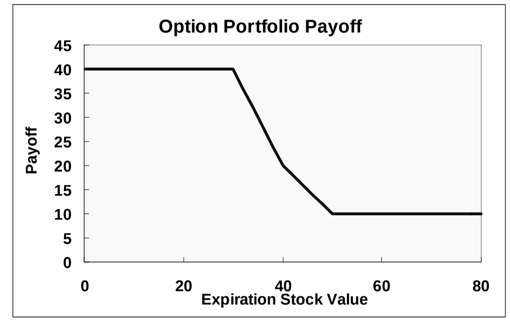
Hence this is a good stock. I would buy more of it but I would be careful to control my exposure to idiosyncratic risk. In other words I would not put all of my money in this stock.

17. You have a portfolio that consists of the following:
- One share of ABC stock.
- A long position in a European put option on shares of ABC with strike price of $40 and one year to maturity.
- A short position in two European call options on shares of ABC with strike price of $30 and one year to maturity.
- A long position in a European call option on shares of ABC with strike price of $50 and one year to maturity.

(a) Draw a diagram of the payoff to this portfolio at the end of the year.
(b) What should be the minimum price of this portfolio?

Solution:

(a) The payoff looks like:



(b) Since the payoff is always above 10, the minimum price for this portfolio should be the price of a bond with face value of $10 and maturity equal to the maturity date of the options.

18. Your uncle has asked for investment advice. Currently his entire savings are invested in one-year treasury bills. He has been listening to his friends, however, and they are advising him about some good stock picks. Using all of the lessons from this course what issues would you think about in designing a better portfolio for your uncle. Provide a discussion of why these issues are important. Clearly indicate what advice you would give your uncle.

Solution:

Discussion points:
- Go into stocks possibly because of the equity premium.
- Diversification with the market.
- Interest rate exposure current: reinvestment risk. Match Duration.
- Possibly use small stocks or value strategy.
- Ignore “stock picks" because of market efficiency.
- How much risk can he tolerate? Present some hypothetical return performance measures to your uncle.