

**California Institute of Technology  
Division of Humanities and Social Sciences**

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BEM 104  
Investments

**Example Midterm Exam Questions  
Version without Solutions**

1. Are the following statements True, False or Ambiguous? Provide a short justification for your answer. (You are evaluated on your justification.)

- (a) (5 points) In order for investors to be willing to invest their money for the long run, the yield to maturity on a 15 year bond must always be greater than the yield to maturity on a 5 year bond.
- (b) (5 points) If two assets are negatively correlated, then there is a portfolio of the two assets with a variance of zero.
- (c) (5 points) Consider an investor who currently has substantial savings to devote to buying a house, but she would like to buy the house 5 years from now. A sequence of short term investments would mean that the investor would have to periodically reinvest her money. Therefore the investor should invest in any bond with a maturity of at least 5 years. For example, she could invest in a 10 year bond.
- (d) (5 points) Because of the potential for diversification if two stocks are not perfectly correlated a portfolio that invests 50% in each stock must be less risky than a portfolio invested just in one of the stocks.

2. (25 points:)

You are in charge of the bond trading and forward loan department of a large investment bank. Market prices for 1, 2 and 3 year pure discount bonds (zero coupon bonds) with face value of \$100 are displayed on your computer terminal as follows:

Years to Maturity	1	2	3
Price	98	96	90

(a) A new summer intern from Harvard has just told you that he thinks that 3 year treasury notes with annual coupons of \$30 and face value of \$1,000 are trading for \$1,000. Would you ask the intern to recheck the price of this coupon bond? If so, why?

(b) A customer approaches you looking for a quote on a loan of \$20 million dollars to be received by the customer one year from now. The customer

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will repay the loan two years from now. What forward interest rate would you quote for your customer?

(c) Suppose that your customer is willing to enter into the loan agreement of part 2b. How would you structure your holdings of pure discount bonds so that you can exactly match the future cash flows of this loan?

3. (25 points)

Suppose that the yield to maturity on all bonds is 5% (in other words, the yield curve is flat at 5%). You have an obligation to pay \$10M in 4 years. You have only two investment opportunities: a 1 year Treasury strip with face value of \$100 and a perpetuity that pays an annual coupon of \$10 each year.

(a) How much money do you have to invest today in the bond market to entirely fund your obligation?

(b) How would you structure your holdings of the 1 year Treasury strip and the perpetuity so that you are protected against the risk of interest rate fluctuations? How many 1 year Treasury strips and how many perpetuities would you buy?

4. (30 points)

Suppose that you are considering investing in two stocks. After analyzing the two stocks you think that there are two possible states for the economy over the next year: "Good" and "Bad." Each state is equally likely (probability 0.5). The returns of the two securities in each state are as follows:

State	Stock 1 Return	Stock 2 Return
Good	30%	5%
Bad	10%	10%

(a) What is the expected return and standard deviation of each stock return?

(b) What is the covariance and the correlation between the two stock returns?

(c) Draw a picture to illustrate the tradeoff between risk and return that is available by investing in these two stocks.

(d) Suppose that a risk-free investment of 5% is also available. Does this present a profit opportunity to you? Why or why not?

5. (30 points)

You work in the bond trading group of a large investment bank. You are told that 1, 2 and 3 year treasury strips (zero coupon bonds) are currently trading at the following implied yields to maturity

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Time to Maturity ( $t$ )	1	2	3
YTM	5%	4%	8%

- (a) The commercial loan department of your firm has asked for your help in constructing a forward loan for one of their customers. This customer would like to enter into a contract to borrow \$100 million from your firm a year from now to be repaid 2 years from now. The customer is willing to pay 7% interest on the loan. Should your firm enter into this contract? If so, how would you structure your holdings of Treasury strips so that your firm can exactly match the cash flows required by the loan.
- (b) You are told that a 3 year U.S. Treasury note is to be issued today with an annual coupon of \$50 and a face (principal) value of \$1,000. What is the highest price that your group should pay for this bond?
- (c) Suppose that you purchased the bond in part 5b at the price that you calculated. It is now one year later and you just received the first coupon payment on the bond. At this time the yields to maturity on 1, 2 and 3 year Treasury strips are:

Time to Maturity ( $t$ )	1	2	3
YTM	6%	7%	8%

If you were to sell the bond now, what rate of return would you realize on your investment in the bond?

6. (25 points)  
 You have been hired by a small pension fund to help them design a bond portfolio to fund a \$10 million obligation that will come due in 4 years. The managers of the fund would like to use a 2 year zero coupon bond along with an 8 year zero coupon bond to fund this obligation. Suppose that the yield curve is flat so that the yields to maturity on all zero coupon bonds are 5%.
- (a) Design a portfolio of the two bonds that will protect the pension fund from fluctuations in interest rates.
- (b) Suppose that right after you create this portfolio, the yield curve shifts to 6% at all maturities. Calculate what you expect the future value of the investment in the two bonds to be in year 4. Do you meet the obligation of the fund? Explain any difference.

7. (25 points)  
 You have \$1 million currently invested entirely in mutual fund A. You are considering switching into a combination of T-bills and mutual fund B for the next year. Mutual fund B is invested 50% in the stock of ABC and 50% in the stock of XYZ. A one-year T-bill with face value \$10,000 is currently selling for

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\$9,523. You have come up with the following assessments of the return to mutual fund A along with the returns to ABC and XYZ for the next year.

$$E(\tilde{r}_A) = 0.10 \quad E(\tilde{r}_{ABC}) = 0.10 \quad E(\tilde{r}_{XYZ}) = 0.18$$

$$\sigma_A = 0.20 \quad \sigma_{ABC} = 0.20 \quad \sigma_{XYZ} = 0.30$$

$$\rho_{ABC,XYZ} = 0.5$$

Using just the T-bill and mutual fund B show how you can construct an investment of your \$1 million such that the portfolio is better than your current investment in mutual fund A. You must show clearly why the new portfolio is better and indicate the dollar investment in T-bills and in mutual fund B.

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