CHAPTER 5: HISTORY OF INTEREST RATES AND RISK PREMIUMS

4. $E(r) = [0.35 \times 44\%] + [0.30 \times 14\%] + [0.35 \times (-16\%)] = 14\%$

   $\sigma^2 = [0.35 \times (44 - 14)^2] + [0.30 \times (14 - 14)^2] + [0.35 \times (-16 - 14)^2] = 630$

   $\sigma = 25.10\%$

   The mean is unchanged, but the standard deviation has increased, as the probabilities of the high and low returns have increased.

11. a [The expected dollar return on the investment in equities is $18,000 compared to the $5,000 expected return for T-bills.]

12. b

13. d

14. c

15. b

17. b

18. a. Probability Distribution of the HPR on the Stock Market and Put:

		STOCK		PUT	
State of the Economy	Probability	Ending Price + $4 Dividend	HPR	Ending Value	HPR
Boom	0.25	$144	44%	$ 0	–100%
Normal Growth	0.50	$114	14%	$ 0	–100%
Recession	0.25	$ 84	–16%	$ 30	150%

Remember that the cost of the index fund is $100 per share, and the cost of the put option is $12.

b. The cost of one share of the index fund plus a put option is $112. The probability distribution of the HPR on the portfolio is:

State of the Economy	Probability	Ending Price + Put + $4 Dividend	HPR	
Boom	0.25	$144	28.6%	= (144 – 112)/112
Normal Growth	0.50	$114	1.8%	= (114 – 112)/112
Recession	0.25	$114	1.8%	= (114 – 112)/112

c. Buying the put option guarantees the investor a minimum HPR of 1.8% regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

CHAPTER 7: CAPITAL ALLOCATION BETWEEN THE RISKY ASSET AND THE RISK-FREE ASSET

13. a. $E(r_C) = 8\% = 5\% + y(11\% - 5\%) \Rightarrow y = \frac{8-5}{11-5} = 0.5$

    b. $\sigma_C = y\sigma_P = 0.50 \times 15\% = 7.5\%$

    c. The first client is more risk averse, allowing a smaller standard deviation.

Reward to variability ratio $= \frac{10}{14} = 0.71$

CHAPTER 8: OPTIMAL RISKY PORTFOLIOS

1. The parameters of the opportunity set are:

   $E(r_S) = 20\%,\ E(r_B) = 12\%,\ \sigma_S = 30\%,\ \sigma_B = 15\%,\ \rho = 0.10$

   From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho\sigma_S\sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{Min}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{Min}) = (0.1739 \times 20) + (0.8261 \times 12) = 13.39\%$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

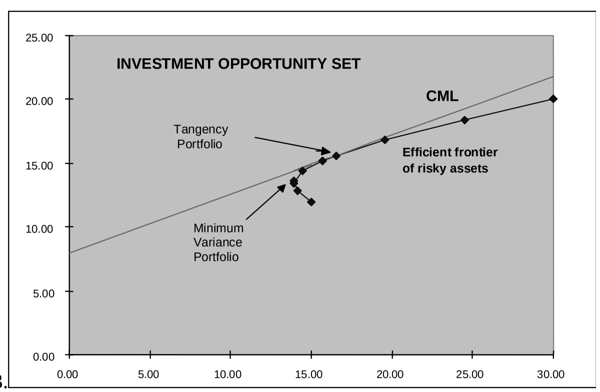
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

2.

Proportion in stock fund	Proportion in bond fund	Expected return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39%	82.61%	13.39%	13.92%	minimum variance
20.00%	80.00%	13.60%	13.94%	
40.00%	60.00%	15.20%	15.70%	
45.16%	54.84%	15.61%	16.54%	tangency portfolio
60.00%	40.00%	16.80%	19.53%	
80.00%	20.00%	18.40%	24.48%	
100.00%	0.00%	20.00%	30.00%	

Graph shown for next question.



3.

The graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

4. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f]Cov(r_S, r_B)}{[E(r_S) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f]Cov(r_S, r_B)}$$

$$= \frac{[(20-8)\times 225] - [(12-8)\times 45]}{[(20-8)\times 225] + [(12-8)\times 900] - [(20-8+12-8)\times 45]} = 0.4516$$

$w_B = 1 - 0.4516 = 0.5484$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P) = (0.4516 \times 20) + (0.5484 \times 12) = 15.61\%$

$\sigma_P = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2} =$ 16.54%

5. The reward-to-variability ratio of the optimal CAL is:

$$\frac{E(r_p) - r_f}{\sigma_p} = \frac{15.61 - 8}{16.54} = 0.4601$$

6. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_P}\sigma_C = 8 + 0.4601\sigma_C$$

Setting $E(r_C)$ equal to 14%, we find that the standard deviation of the optimal portfolio is 13.04%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$E(r_C) = (1 - y)r_f + yE(r_P) = r_f + y[E(r_P) - r_f] = 8 + y(15.61 - 8)$

Setting $E(r_C) = 14\%$ we find: $y = 0.7884$ and $(1 - y) = 0.2116$ (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = $0.7884 \times 0.4516 = 0.3560$
Proportion of bonds in complete portfolio = $0.7884 \times 0.5484 = 0.4324$

7. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

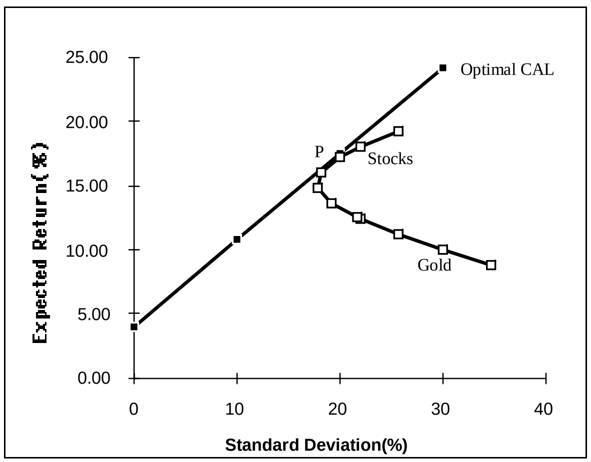
$14 = 20w_S + 12(1 - w_S) = 12 + 8w_S \Rightarrow w_S = 0.25$

So the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$

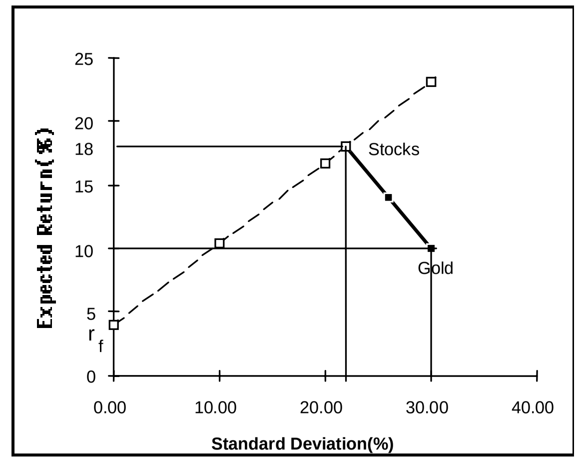
This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.

9. a.



Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.

b. If the correlation between gold and stocks equals +1, then no one would hold gold. The optimal CAL would be comprised of bills and stocks only. Since the set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph), these combinations would be dominated by the stock portfolio. Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.



10. Since Stock A and Stock B are perfectly negatively correlated, a risk-free portfolio can be created and the rate of return for this portfolio, in equilibrium, will be the risk-free rate. To find the proportions of this portfolio [with the proportion w_A invested in Stock A and $w_B = (1 - w_A)$ invested in Stock B], set the standard deviation equal to zero. With perfect negative correlation, the portfolio standard deviation is:

σ_P = Absolute value $[w_A\sigma_A - w_B\sigma_B]$

$0 = 5w_A - [10 \times (1 - w_A)] \Rightarrow w_A = 0.6667$

The expected rate of return for this risk-free portfolio is:
$E(r) = (0.6667 \times 10) + (0.3333 \times 15) = 11.667\%$

Therefore, the risk-free rate is 11.667%.

11. False. If the borrowing and lending rates are not identical, then, depending on the tastes of the individuals (that is, the shape of their indifference curves), borrowers and lenders could have different optimal risky portfolios.

12. False. The portfolio standard deviation equals the weighted average of the component-asset standard deviations *only* in the special case that all assets are perfectly positively correlated. Otherwise, as the formula for portfolio standard deviation shows, the portfolio standard deviation is *less* than the weighted average of the component-asset standard deviations. The portfolio *variance* is a weighted *sum* of the elements in the covariance matrix, with the products of the portfolio proportions as weights.

13. The probability distribution is:

Probability	Rate of Return
0.7	100%
0.3	–50%

Mean = $[0.7 \times 100] + [0.3 \times (-50)] = 55\%$

Variance = $[0.7 \times (100 - 55)^2] + [0.3 \times (-50 - 55)^2] = 4725$

Standard deviation = $4725^{1/2} = 68.74\%$

14. $\sigma_P = 30 = y\sigma = 40y \Rightarrow y = 0.75$

$E(r_P) = 12 + 0.75(30 - 12) = 25.5\%$

18. The correct choice is c. Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A, and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - Cov(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2Cov(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

`

Since all standard deviations are equal to 20%:

$$Cov(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of G(I, J) reduces to:

$$\sigma_{Min}(G) = [200(1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

19. No, the answer to Problem 18 would not change, at least as long as investors are not risk lovers. Risk neutral investors would not care which portfolio they held since all portfolios have an expected return of 8%.

CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

1. $E(r_P) = r_f + \beta_P[E(r_M) - r_f]$

$18 = 6 + \beta_P(14 - 6) \Rightarrow \beta_P = 12/8 = 1.5$

2. If the security's correlation coefficient with the market portfolio doubles (with all other variables such as variances unchanged), then beta, and therefore the risk premium, will also double. The current risk premium is: 14 – 6 = 8%
The new risk premium would be 16%, and the new discount rate for the security would be: 16 + 6 = 22%

If the stock pays a constant perpetual dividend, then we know from the original data that the dividend (D) must satisfy the equation for the present value of a perpetuity:

Price = Dividend/Discount rate

$50 = D/0.14 \Rightarrow D = 50 \times 0.14 = \$7.00$

At the new discount rate of 22%, the stock would be worth: $7/0.22 = $31.82
The increase in stock risk has lowered its value by 36.36%.

4. a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

b. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility includes diversifiable risk.

c. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then:

$$\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$$

5. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

$$\beta_A = \frac{-2-38}{5-25} = 2.00$$

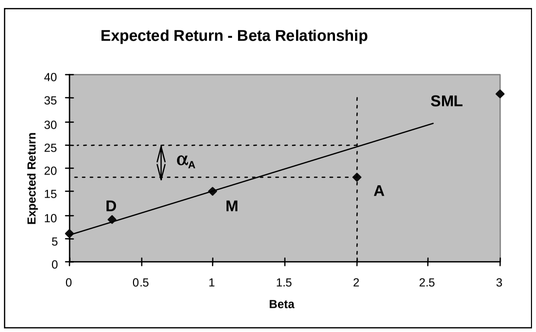
$$\beta_D = \frac{6-12}{5-25} = 0.30$$

b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$$E(r_A) = 0.5 \times (-2 + 38) = 18\%$$

$$E(r_D) = 0.5 \times (6 + 12) = 9\%$$

c. The SML is determined by the market expected return of [0.5(25 + 5)] = 15%, with a beta of 1, and the T-bill return of 6% with a beta of zero. See the following graph.



The equation for the security market line is:

$$E(r) = 6 + \beta(15 - 6)$$

d. Based on its risk, the aggressive stock has a required expected return of:

$$E(r_A) = 6 + 2.0(15 - 6) = 24\%$$

The analyst's forecast of expected return is only 18%. Thus the stock's alpha is:

α_A = actually expected return – required return (given risk)

= 18% – 24% = –6%

Similarly, the required return for the defensive stock is:

$$E(r_D) = 6 + 0.3(15 - 6) = 8.7\%$$

The analyst's forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

α_D = actually expected return – required return (given risk)

= 9 – 8.7 = +0.3%

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta (0.3), not the firm's beta. The correct discount rate is 8.7%, the fair rate of return for stock D.

6. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B. Thus, these two portfolios cannot exist in equilibrium.

7. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk, represented by beta, rather than for the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is less than B's.

8. Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market. This scenario is impossible according to the CAPM because the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{16-10}{12} = 0.5$$

$$S_M = \frac{18-10}{24} = 0.33$$

Portfolio A provides a better risk-reward tradeoff than the market portfolio.

9. Not possible. Portfolio A clearly dominates the market portfolio. Portfolio A has both a lower standard deviation and a higher expected return.

10. Not possible. The SML for this scenario is: $E(r) = 10 + \beta(18 - 10)$
Portfolios with beta equal to 1.5 have an expected return equal to:

$$E(r) = 10 + [1.5 \times (18 - 10)] = 22\%$$

The expected return for Portfolio A is 16%; that is, Portfolio A plots below the SML ($\alpha_A = -6\%$), and hence, is an overpriced portfolio. This is inconsistent with the CAPM.

11. Not possible. The SML is the same as in Problem 10. Here, Portfolio A's required return is: $10 + (0.9 \times 8) = 17.2\%$
This is greater than 16%. Portfolio A is overpriced with a negative alpha:
$\alpha_A = -1.2\%$

12. Possible. The CML is the same as in Problem 8. Portfolio A plots below the CML, as any asset is expected to. This scenario is not inconsistent with the CAPM.

13. Since the stock's beta is equal to 1.2, its expected rate of return is:

$$6 + [1.2 \times (16 - 6)] = 18\%$$

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0}$$

$$0.18 = \frac{6 + P_1 - 50}{50} \Rightarrow P_1 = \$53$$

14. The series of $1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

$$6 + [0.5 \times (16 - 6)] = 11\%$$

PV = $1,000/0.11 = $9,090.91

If, however, beta is equal to 1, then the investment should yield 16%, and the price paid for the firm should be:

PV = $1,000/0.16 = $6,250

The difference, $2,840.91, is the amount you will overpay if you erroneously assume that beta is 0.5 rather than 1.

17. a. Since the market portfolio, by definition, has a beta of 1, its expected rate of return is 12%.

b. $\beta = 0$ means no systematic risk. Hence, the stock's expected rate of return in market equilibrium is the risk-free rate, 5%.

c. Using the SML, the *fair* expected rate of return for a stock with $\beta = -0.5$ is:

$$E(r) = 5 + [(-0.5)(12 - 5)] = 1.5\%$$

The *actually* expected rate of return, using the expected price and dividend for next year is:

$$E(r) = [(\$41 + \$1)/40] - 1 = 0.10 = 10\%$$

Because the actually expected return exceeds the fair return, the stock is underpriced.

22. d. From CAPM, the fair expected return = 8 + 1.25(15 – 8) = 16.75%

Actually expected return = 17%

$\alpha = 17 - 16.75 = 0.25\%$

25 d.

26. d. [You need to know the risk-free rate]

27. d. [You need to know the risk-free rate]

28. Under the CAPM, the only risk that investors are compensated for bearing is the risk that cannot be diversified away (systematic risk). Because systematic risk (measured by beta) is equal to 1.0 for both portfolios, an investor would expect the same rate of return from both portfolios A and B. Moreover, since both portfolios are well diversified, it doesn't matter if the specific risk of the individual securities is high or low. The firm-specific risk has been diversified away for both portfolios.

CHAPTER 14: BOND PRICES AND YIELDS

12. a. The bond pays $50 every 6 months. The current price is:

[$50 × Annuity factor (4%, 6)] + [$1,000 × PV factor (4%, 6)] = $1,052.42

Assuming the market interest rate remains 4% per half year, price six months from now is:

[$50 × Annuity factor (4%, 5)] + [$1,000 × PV factor (4%, 5)] = $1,044.52

b. $$\text{Rate of return} = \frac{\$50 + (\$1{,}044.52 - \$1{,}052.42)}{\$1{,}052.42} = \frac{\$50 - \$7.90}{\$1{,}052.42}$$

= 0.04 = 4.0% per six months

15. If the yield to maturity is greater than the current yield, then the bond offers the prospect of price appreciation as it approaches its maturity date. Therefore, the bond must be selling below par value.

21. The stated yield to maturity, based on promised payments, equals 16.075%. [n = 10; PV = –900; FV = 1000; PMT = 140]

Based on *expected* coupon payments of $70 annually, the expected yield to maturity is 8.526%.

CHAPTER 15: THE TERM STRUCTURE OF INTEREST RATES

6.

Maturity	Price	YTM	Forward Rate
1	$943.40	6.00%	
2	$898.47	5.50%	$(1.055^2/1.06) - 1 = 5.0\%$
3	$847.62	5.67%	$(1.0567^3/1.055^2) - 1 = 6.0\%$
4	$792.16	6.00%	$(1.06^4/1.0567^3) - 1 = 7.0\%$

16. a. The current bond price is:

($85 × 0.94340) + ($85 × 0.87352) + ($1,085 × 0.81637) = $1,040.20

This price implies a yield to maturity of 6.97%, as shown by the following:

[$85 × Annuity factor (6.97%, 3)] + [$1,000 × PV factor (6.97%, 3)] = $1,040.17

b. If one year from now y = 8%, then the bond price will be:

[$85 × Annuity factor (8%, 2)] + [$1,000 × PV factor (8%,2)] = $1,008.92

The holding period rate of return is:

[$85 + ($1,008.92 – $1,040.20)]/$1,040.20 = 0.0516 = 5.16%

20. a. <u>Five-year Spot Rate</u>:

$$\$1{,}000 = \frac{\$70}{(1+y_1)^1} + \frac{\$70}{(1+y_2)^2} + \frac{\$70}{(1+y_3)^3} + \frac{\$70}{(1+y_4)^4} + \frac{\$1{,}070}{(1+y_5)^5}$$

$$\$1{,}000 = \frac{\$70}{(1.05)} + \frac{\$70}{(1.0521)^2} + \frac{\$70}{(1.0605)^3} + \frac{\$70}{(1.0716)^4} + \frac{\$1{,}070}{(1+y_5)^5}$$

$$\$1{,}000 = \$66.67 + \$63.24 + \$58.69 + \$53.08 + \frac{\$1{,}070}{(1+y_5)^5}$$

$$\$758.32 = \frac{\$1{,}070}{(1+y_5)^5}$$

$$(1+y_5)^5 = \frac{\$1{,}070}{\$758.32} \Rightarrow y_5 = \sqrt[5]{1.411} - 1 = 7.13\%$$

<u>Five-year Forward Rate</u>:

$$\frac{(1.0713)^5}{(1.0716)^4} - 1 = 1.0701 - 1 = 7.01\%$$

b. The yield to maturity is the single discount rate that equates the present value of a series of cash flows to a current price. It is the internal rate of return.

The spot rate for a given period is the yield to maturity on a zero-coupon bond that matures at the end of the period. A spot rate is the discount rate for each period. Spot rates are used to discount each cash flow of a coupon bond in order to calculate a current price. Spot rates are the rates appropriate for discounting future cash flows of different maturities.

A forward rate is the implicit rate that links any two spot rates. Forward rates are directly related to spot rates, and therefore to yield to maturity. Some would argue (as in the expectations hypothesis) that forward rates are the market expectations of future interest rates. A forward rate represents a break-even rate that links two spot rates. It is important to note that forward rates link spot rates, not yields to maturity.

Yield to maturity is not unique for any particular maturity. In other words, two bonds with the same maturity but different coupon rates may have different yields to maturity. In contrast, spot rates and forward rates for each date are unique.

c. The 4-year spot rate is 7.16%. Therefore, 7.16% is the theoretical yield to maturity for the zero-coupon U.S. Treasury note. The price of the zero-coupon note discounted at 7.16% is the present value of $1,000 to be received in 4 years. Using annual compounding:

$$PV = \frac{\$1{,}000}{(1.0716)^4} = \$758.35$$

CHAPTER 16: MANAGING BOND PORTFOLIOS

1. The percentage change in the bond's price is:

$$-\frac{\text{Duration}}{1+y} \times \Delta y = -\frac{7.194}{1.10} \times 0.005 = -0.0327 = -3.27\% \text{ or a 3.27\% decline.}$$

2. a. YTM = 6%

(1) Time until Payment (years)	(2) Cash Flow	(3) PV of CF (Discount rate = 6%)	(4) Weight	(5) Column (1) × Column (4)
1	$60.00	$56.60	0.0566	0.0566
2	$60.00	$53.40	0.0534	0.1068
3	$1,060.00	$890.00	0.8900	2.6700
	Column Sums	$1,000.00	1.0000	2.8334

Duration = 2.833 years

b. YTM = 10%

(1) Time until Payment (years)	(2) Cash Flow	(3) PV of CF (Discount rate = 10%)	(4) Weight	(5) Column (1) × Column (4)
1	$60.00	$54.55	0.0606	0.0606
2	$60.00	$49.40	0.0551	0.1102
3	$1,060.00	$796.39	0.8844	2.6532
	Column Sums	$900.53	1.0000	2.8240

Duration = 2.824 years, which is less than the duration at the YTM of 6%.

5. a.

(1) Time until Payment (years)	(2) Cash Flow	(3) PV of CF (Discount rate = 10%)	(4) Weight	(5) Column (1) × Column (4)
1	$10 million	$9.09 million	0.7857	0.7857
5	$4 million	$2.48 million	0.2143	1.0715
	Column Sums	$11.57 million	1.0000	1.8572

D = 1.8572 years = required maturity of zero coupon bond.

b. The market value of the zero must be $11.57 million, the same as the market value of the obligations. Therefore, the face value must be:

$$\$11.57 \text{ million} \times (1.10)^{1.8572} = \$13.81 \text{ million}$$

11. a. PV of the obligation = [$10,000 × Annuity factor (8%, 2)] = $17,832.65
Duration = 1.4808 years, which can be verified using Rule 6 or a table such as Table 15.3.

b. A zero-coupon bond maturing in 1.4808 years would immunize the obligation. Since the present value of the zero-coupon bond must be $17,832.65, the face value (i.e., the future redemption value) must be:

$$\$17{,}832.65 \times 1.08^{1.4808} = \$19{,}985.26$$

c. If the interest rate increases to 9%, the zero-coupon bond would decrease in value to:

$$\frac{\$19{,}985.26}{1.09^{1.4808}} = \$17{,}950.92$$

The present value of the tuition obligation would decrease to $17,591.11. The net position decreases in value by $0.19.

If the interest rate decreases to 7%, the zero-coupon bond would increase in value to:

$$\frac{\$19{,}985.26}{1.07^{1.4808}} = \$18{,}079.99$$

The present value of the tuition obligation would increase to $18,080.18. The net position decreases in value by $0.19.

The reason the net position changes at all is that, as the interest rate changes, so does the duration of the stream of tuition payments.

16. a. PV of obligation = $2 million/0.16 = $12.5 million
Duration of obligation = 1.16/0.16 = 7.25 years
Call w the weight on the 5-year maturity bond (which has duration of 4 years). Then:

$$(w \times 4) + [(1 - w) \times 11] = 7.25 \Rightarrow w = 0.5357$$

Therefore: $0.5357 \times \$12.5 = \$6.7$ million in the 5-year bond, and;

$0.4643 \times \$12.5 = \$5.8$ million in the 20-year bond.

b. The price of the 20-year bond is:

[$60 × Annuity factor (16%, 20)] + [$1,000 × PV factor (16%, 20)] = $407.12

Therefore, the bond sells for 0.4071 times its par value, and:

Market value = Par value × 0.4071

$5.8 million = Par value × 0.4071 ⇒ Par value = $14.25 million

Another way to see this is to note that each bond with par value $1,000 sells for $407.12. If total market value is $5.8 million, then you need to buy approximately 14,250 bonds, resulting in total par value of $14.25 million.

17. a. The duration of the perpetuity is: 1.05/0.05 = 21 years

Call w the weight of the zero-coupon bond. Then:

$$(w \times 5) + [(1 - w) \times 21] = 10 \Rightarrow w = 11/16 = 0.6875$$

Therefore, the portfolio weights would be as follows: 11/16 invested in the zero and 5/16 in the perpetuity.

b. Next year, the zero-coupon bond will have a duration of 4 years and the perpetuity will still have a 21-year duration. To obtain the target duration of nine years, which is now the duration of the obligation, we again solve for w:

$$(w \times 4) + [(1 - w) \times 21] = 9 \Rightarrow w = 12/17 = 0.7059$$

So, the proportion of the portfolio invested in the zero increases to 12/17 and the proportion invested in the perpetuity falls to 5/17.

23. a. % price change = (–Effective duration) × Change in YTM (%)

CIC: (–7.35) × (–0.50%) = 3.675%

PTR: (–5.40) × (–0.50%) = 2.700%

b. Since we are asked to calculate horizon return over a period of only one coupon period, there is no reinvestment income.

$$\text{Horizon return} = \frac{\text{Coupon payment} + \text{Year-end price} - \text{Initial Price}}{\text{Initial price}}$$

CIC: $$\frac{\$31.25 + \$1{,}055.50 - \$1{,}017.50}{\$1{,}017.50} = 0.06806 = 6.806\%$$

PTR: $$\frac{\$36.75 + \$1{,}041.50 - \$1{,}017.50}{\$1{,}017.50} = 0.05971 = 5.971\%$$

c. Notice that CIC is non-callable but PTR is callable. Therefore, CIC has positive convexity, while PTR has negative convexity. Thus, the convexity correction to the duration approximation will be positive for CIC and negative for PTR.