

# A statistical theory of freedom<sup>α</sup>

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## Abstract

This paper is concerned with the definition and measurement of freedom. Specifically, we deal with the concept of social freedom. What do we mean by a "free society"? How free is a society? Can we say that society A exhibits a higher degree of freedom than society B? Can we rank societies on the basis of freedom? These are the kind of questions that motivate our work.

While the notion of freedom is given a central place in contemporary philosophical and political debates, it is surprising that not much effort has been devoted, until now, to the construction of a rigorous metric for comparing societies in terms of freedom.

The measures used by many organizations, among them the United Nations, in order to quantify the freedom of different countries can and have been legitimately criticized as over-crude, with unclear theoretical presuppositions.

On the other hand, the problem of measuring freedom has been addressed in a rigorous way recently by normative economists (see Pattanaik and Xu 1990, Sen 1991, and the related literature). However, this literature focuses only on the individual level; moreover, in our view, this literature is too abstract and, as a consequence, no empirical work has followed from it.

We propose a different approach. Following the line of enquiry suggested half a century ago by D. and A. Gabor (1954) (G-G henceforth), we adopt an approach which is phenomenological, because it is based on observable actions rather than individual preferences or ex-ante opportunities for choice; and statistical, in that it relates to large population rather than to single individuals. We identify some crucial aspects of freedom and propose a general framework which can be used for a rigorous quantitative discussion. We argue that this

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methodology is based on sound theoretical grounds, and we show by means of an empirical application that it is actually useful for operational purposes.

It might be useful to discuss our approach to defining freedom within the general conceptual framework proposed by MacCallum (1967). This will help to compare our approach with more recent attempts to give a quantitative dimension to discourses about freedom.

Following MacCallum (1967), a general description of liberty takes the form of a triadic relationship: an agent (individual or collective) is free from a given set of constraints to choose among a given set of options. A definition of freedom, and a measure consistent with it, is therefore obtained by specifying the set of options, the constraints and the agent they are referred to.

Thus the first aspect of freedom concerns the options people may choose: what are the actions open to an individual in a given situation? Given a set of options, though the act of choice only one of these options will materialize, depending on the desires and preferences of the agent. Therefore, looking only at the choices - i.e., adopting an objective approach - does not tell us anything about the ex-ante available options. However, this is only true at the individual level. If the individual is a member of a large population then an objective approach may be legitimate: it may be sensible to say that the real options individuals are free to choose in a society must manifest themselves in the actions or choices of at least a fraction of the population. As G-G say,

We take the view that what individuals "care to do" or "may care to do" must manifest itself in the actions of at least a fraction of the population. If there is no regulation which forbids bank clerks to wear boiler suits, yet among a thousand bank clerks on a thousand mornings there is not one who appears at work in a boiler suit, we conclude that they are de facto not free to wear that garment. (G-G, p.332)

With this methodology one uses information about ex-post choices in a society to infer the ex-ante opportunities of an individual in that society<sup>1</sup>. The approach is phenomenological rather than procedural and it is statistical rather than individual. Hence, G-G essentially propose to measure social freedom by measuring the freedom of a representative agent in the relevant society. This aspect of freedom is called the diversity aspect.

This phenomenological approach could be criticized on the basis of the following argument. Using information about ex-post choices instead of potential choices one basically overlooks the conditions in which decisions are made. But, it could be argued, the conditions under which decisions are made are relevant for the assessment of freedom; for, a society in which there is a plurality of accessible options, but where, for cultural reasons, only few of these options are chosen, is a free society. In other words, the pluralism should be assessed in the

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<sup>1</sup>More recently, this approach has been defended and used by Suppes (1995).

ex-ante distribution of opportunity for choice; not in the ex post distribution of choices.

Certainly there is something in this criticism. However, to appreciate the plurality of options in a society by looking at the ex-ante options, one should have detailed informations about the process of choice. For, to make some options which are only formally available genuine opportunities open to individuals it is necessary that the act of choosing is not restricted by external pressures or influences. But it is extremely difficult to have quantitative data about the ex-ante options and process leading to the final choices; on the contrary, excellent data can be obtained on the choices.

Hence, roughly speaking, the more the options chosen by at least some member of the society, the higher the degree of freedom individuals in that society enjoy. Thus freedom manifests itself in the plurality of the chosen options; and it is measured as statistical spread of choices.

The diversity aspect of freedom is only one of its features. A second aspect, G-G say, is the independence aspect. We can grasp the meaning and the role of this aspect of freedom by considering the constraints an individual faces in the context of a choice. From this viewpoint the question is: when is an agent free from "external circumstances" to do this and that, to choose this and that? Which factors or conditions influence her ability to choose a given option? This question evokes a problem of independence of the available options from external factors. Given a set of options, an individual is free, in the current sense, if the set of options open to her is independent from external circumstances. This is the independence aspect considered by G-G. Again, the approach to measurement they propose is statistical: the degree of independence is to be evaluated by looking at a large population: an individual is free from factor x to choose option y if, in the relevant society, we observe statistical independence of y and x:

To sum up, freedom in a society, according to Gabor and Gabor is defined as

"that part of the diversity displayed by the behaviour of the group, which cannot be accounted for by reference to the influence of a selected set of observable factors." (Gabor, 1955 p.392)

The concept of freedom as independence could be criticised on the basis of the following argument. Suppose we add more and more variables in the set of factors from which one individual must be independent to be free. In this case, each group of people would consist only of one individual and, in each group, there would be no diversity of choices. In the whole population there would be no diversity which cannot be accounted for by reference to the influence of the selected set of factors: i.e., the liberty would be nihil. The message of this criticism is that we should not confuse the observed statistical dependences with the restraints to freedom. Because if we do so, the argument goes, then any perfect metric of freedom would give as a result the complete absence of freedom.

There are two answers to this criticism.

The first is that this is a limit case, and it seems reasonable to say that, for all practical purposes, a perfect knowledge of all internal and external conditions of the individual will never be reached. Hence, this completely deterministic case seems to be not relevant.

The second answer consists in shifting the discussion from a positive to a normative dimension. The idea of freedom as independence, the way we have formulated it, leaves open the question of which external factors are relevant when measuring freedom. In fact, my set of options may be dependent from a plurality of factors: natural factors, social conditions, my own past choices and actions. Of which of these classes must one be independent to be free?

A guide in this search can be found in the discussion on the constraints which are relevant for the definition of freedom (see Berlin 1958, Bobbio 1956, Miller 1983, Oppenheim 1985). Recalling MacCallum's scheme, a conception of freedom must define three things: an agent, X, who is free from constraints, Y, to do an action (or to be something), Z. All disagreements about the definition of freedom therefore boil down to questions about the extension of factors X, Y and Z. Specifically relevant for the present discussion is the debate over whether the Y factor should indicate merely "external" or also "internal" constraints, and over whether Y includes only constraints imposed by other agents or also those imposed by nature. To use a classic example, does the poverty of a tramp who is unable to dine at the Ritz restrict her freedom?

One plausible solution is that of arguing that impersonal economic forces cannot reasonably be interpreted as constraints on freedom. At this point, the "Y factor" in MacCallum's formula gets narrowed down to only those obstacles that are imposed deliberately, or that are at least foreseeable, by other individuals. But an alternative answer would be that he is unfree only if the fact that he is poor - so the constraints that prevent him to dine at the Ritz - cannot be attributed to his choice, nor to fortune, but to an unjust distribution of resources in the society, so that we can attribute to the society the will that causally determines the situation of poverty.

Using a similar argument, we can argue that "the conditions from which one must be independent to be free" are to be identified by looking at "the dependences that are considered unjust in the given society". This consideration links the theory of freedom to the theory of justice.

In the rest of the paper: (i) we propose an analytical framework to represent the concept of freedom sketched above and we derive numerical measures of freedom and of its two constituent elements, plurality and independence; (ii) we show the usefulness of this approach by means of an empirical application.

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