BEM 105, HMWK 3, Due Monday, Feb 13 2006

From Chapter 3.

1. Use Itô's rule to find the Stochastic Differential Equation for Brownian motion cubed, W^3. Do the same for S^3 where $S(t) = e^{(\mu - \sigma^2/2)t + \sigma W(t)}$ is the stock price in the Merton-Black-Scholes model.

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FROM CHAPTER 7

2. Suppose that the stock price today is $S(t) = 2.00$, the annual volatility is $\sigma = 0.1$, and the time to maturity is 3 months. Consider an option whose Black-Scholes price is given by the function

$$V(t, s) = s^3 e^{0.13(T-t)} \quad ,$$

where the time is in annual terms. What is the option price today? What is the interest rate equal to?

3. Consider a Merton-Black-Scholes model with $r = 0.07$, $\sigma = 0.3$, $T = 0.5$ years, $S(0) = 100$, and a put option with the strike price $K = 100$. Using the normal distribution table (or an appropriate software program), find the price of the put option, when there are no dividends.

4. Find the Black-Scholes formula for the option paying at maturity T the amount $\$A$ if $S(T) \leq K_1$ or if $S(T) \geq K_2$, and zero otherwise, in the Black-Scholes continuous-time model.

5. The exchange rate process Q, representing the value in dollars of one unit of a foreign currency, satisfies the SDE

$$dQ(t) = Q(t)[\mu dt + \sigma dW(t)].$$

The foreign stock is given by SDE

$$dS^f(t) = S^f(t)[\mu^f dt + \sigma_1 dW(t) + \sigma_2 dZ(t)],$$

where Z is a Brownian Motion independent of W.

(a) Find the SDE for the exchange rate of one dollar into the foreign currency.

(b) Find the SDE for the value of the foreign stock in domestic currency (dollars).

6. Find the price at time zero of the claim which pays $\frac{S(T_1)}{S(T_0)}$ at time T_1, $T_1 > T_0 > 0$. You can assume Black-Scholes model if you wish.

7. (**Due Wed February 22**) Write a computer code for a program that would compute the price for a European call option in a CRR model in which

$$1 + r = e^{r\Delta t}, \quad u = e^{\sigma\sqrt{\Delta t}}, \quad d = \frac{1}{u}, \quad \Delta t = T/n.$$

The input of the program should consist of $r, \sigma, K, S(0), T$, and n. The output should give the option price at time $t = 0$. Run the program for different values of n. You can check the correctness of your program by setting $K = 0$. What value should you get in this case? Finally, check your results by using the Black-Scholes formula and a table (or software) for the cumulative normal distribution function.

Remark: If you wish, you can use the Excel spread sheet for CH 11 on the book web page that contains the Black-Scholes formula computations and a CRR binomial tree with $n = 10$. See http://www.hss.caltech.edu/ cvitanic/EXCEL/ch11.xls