BEM 105, HMWK 2, Due Monday, Jan 30 2006

From Chapter 3.

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1. Give an example of a Cox-Ross-Rubinstein model with expected relative stock return equal to 0.2, $E[S(t)/S(t-1)] = 0.2$, and variance equal to 0.3, $\text{Var}[S(t)/S(t-1)] = 0.3$. That is, choose the values of parameters p, u, and d so that these conditions are satisfied.

From Chapter 6.

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2. Show that the risk-neutral probabilities in the Cox-Ross-Rubinstein model are given by equations (6.15).

3. In the context of Example 6.3, find the cost of replication $C(0)$ for the put option with the payoff function $g(s) = (100 - s)^+ = \max(100 - s, 0)$. Construct arbitrage strategies in the case that the option price is less than $C(0)$, and in the case that it is larger than $C(0)$. Compute the option price as a risk-neutral expected value.

4. Consider a non-dividend-paying stock worth \$9.00 today in a market with continuously compounded interest rate of 10% per year and an American put option on this stock, with maturity 5 months and strike price \$10.00, trading at the price of \$2.00. Find the upper and lower no-arbitrage bounds for the price of the corresponding European call, same maturity, same strike.

5. Assume that the future dividends on a given stock S are known, and denote their discounted value at the present time t by $\bar{D}(t)$. Argue the following:

$$c(t) + \bar{D}(t) + Ke^{-r(T-t)} = p(t) + S(t)$$

6. Prove using a no-arbitrage argument that a European put option on a non-dividend paying stock is a convex function of the strike price, that is, if we denote by $p(K)$ the put price when the strike price is K, then, for $0 < \alpha < 1$,

$$\alpha p(K_1) + (1-\alpha)p(K_2) > p[\alpha K_1 + (1-\alpha)K_2] \quad .$$